SSY080, D3

2019-01-08

A1

$x_1[n]$ och $x_2[n]$ lika men har olika medelvärde. Medelvärdet speglas i $X[k]|_{k=0}$

$x_3[n]$: Två perioder sinus $\rightarrow X_A[k]$

$x_4[n]$: Sinusform men ej fullt ut två perioder

Svar:

x[n]	X[k]
1	B
2	C
3	A
4	D

A2.

$$H(s) = \frac{\omega_0^2}{s^2 + sk\omega_0 + \omega_0^2}$$

Poler: $$s_{1,2} = -k\frac{\omega_0}{2} \pm \sqrt{k^2\frac{\omega_0^2}{4} - \omega_0^2} =$$

$$= \omega_0\left(-\frac{k}{2} \pm \sqrt{\frac{k^2}{4} - 1}\right)$$

$k = -1$ och -2 ger poler i HHP $\Rightarrow$ instabilt och $h(t) \rightarrow \infty$

$k = 2$ dubbelpol i $s = -\omega_0$

$$H(s) = \frac{\omega_0^2}{(s+\omega_0)^2} \xleftrightarrow{\mathcal{L}} h(t) = \omega_0^2 t e^{-\omega_0 t} u(t)$$

Stämmer ej med fig.

$k = 1$ Komplexa poler $s_{1,2} = -\frac{\omega_0}{2} \pm j\frac{\omega_0}{2}$

$$H(s) = \frac{\omega_0^2}{(s + \frac{1}{2}\omega_0)^2 + \frac{3\omega_0^2}{4}} \Rightarrow h(t) \text{ dämpad sinus!}$$

$\boxed{\text{Svar: } k = 1}$

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A3.

$$x[n] = u[n] - (0{,}4)^n u[n]$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-0{,}4} = z \cdot \left[\frac{z-0{,}4-(z-1)}{(z-1)(z-0{,}4)}\right] =$$

$$= z\left[\frac{z-0{,}4-z+1}{z^2-0{,}4z-z+0{,}4}\right] =$$

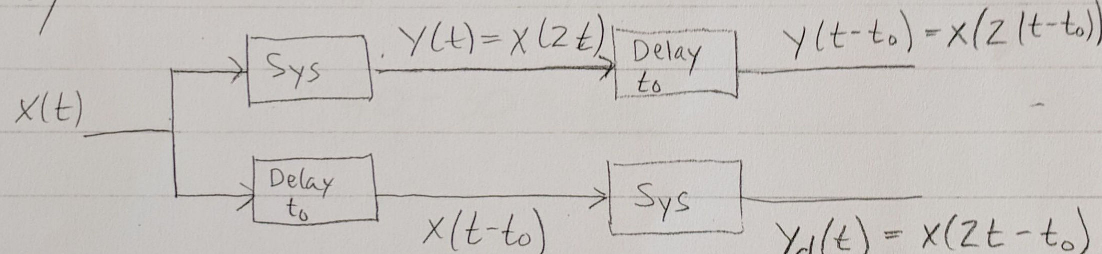
$$= \frac{0{,}6z}{z^2-1{,}4z+0{,}4}$$

$b_2 = 0$, $b_1 = 0{,}6$, $b_0 = 0$

$a_2 = 1$, $a_1 = -1{,}4$, $a_0 = 0{,}4$

A4

a)



$y(t-t_0) \neq y_d(t)$ Tidsinv? <u>Nej</u>

b)

In	Ut
$x_1(t)$	$y_1(t) = x_1(t)\cdot\cos(t)$
$a x_1(t)$	$a x_1(t)\cdot\cos(t) = a y_1(t)$
$b x_2(t)$	$b x_2(t)\cdot\cos(t) = b y_2(t)$
$a x_1(t) + b x_2(t) = x_3(t)$	$y_3(t) = x_3(t)\cdot\cos(t) =$
	$= [a x_1(t) + b x_2(t)]\cos(t) =$
	$= a_1 x_1(t)\cos(t) + b x_2(t)\cos(t) =$
	$= a_1 y_1(t) + b_2 y_2(t)$

Linjärt? <u>Ja!</u>

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A5. $h(t) = \delta(t) - 10\sqrt{3}e^{-(10\sqrt{3})t}u(t) = \delta(t) - ae^{-at}u(t)$

$H(s) = \mathcal{L}\{h(t)\} = 1 - \frac{a}{s+a} = \frac{s+a-a}{s+a} = \frac{s}{s+a}$

Frekvenssvar $H(j\omega) = H(s)\big|_{s=j\omega} = \frac{j\omega}{a+j\omega}$

$\omega = 10$ $\quad A = |H(j10)| = \frac{10}{\sqrt{a^2+\omega^2}} = \{a = 10\sqrt{3}\} =$

$= \frac{10}{\sqrt{300+100}} = \underline{\underline{0.5}}$

$\phi = \arg\{H(j10)\} = \arg\{j10\} - \arg\{a+j\omega\} =$

$= 90° - \arctan\{\frac{\omega}{a}\} = 90 - \arctan\{\frac{1}{\sqrt{3}}\} =$

$= 90 - 30 = \underline{\underline{60°}}$

A6: $|G(j\omega)|_{dB}$ faller med $n \cdot 20$ dB/dekad vid höga frekvenser. I figur (Magnitud) sjunker $|G(j\omega)|$ med 60dB mellan $\omega = 10^2$ och $\omega = 10^3$

$\Rightarrow n = 3$

A7 $\quad G(j\omega) = \frac{K}{(j\omega+10)^3} = \frac{K}{10^3}$ då $\omega \to 0$

$20 \log \frac{K}{10^3} = 20$ dB (från diagram)

$20 \log \frac{K}{10^3} = 20$; $\log \frac{K}{10^3} = 1$; $\frac{K}{10^3} = 10^1$

Svar: $\underline{\underline{K = 10^4}}$

554080, 03
2019-01-02

A8 Man ser att det krävs 25 sampel för två hela perioder = 4π

Alltså $x[n] = \sin(\Omega n) = \sin\left(\frac{4\pi}{25} \cdot n\right)$

$$\Omega = \frac{4\pi}{25}$$

A9. Grundvinkelfrekvensen f_0 speglas i $k=7$

$f_s = 1220$ Hz, $N = 2^{10}$

$$\frac{f_0}{f_s} = \frac{k}{N} \; ; \; T_0 = \frac{1}{f_0} = \frac{N}{k \cdot f_s} = \frac{2^{10}}{7 \cdot 1220} \approx 0{,}125$$

A10 $x(t) = A\sin(\omega_0 t)$, $T = \frac{2\pi}{\omega_0}$

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

Vår signal har $a_0, a_k = 0$ $b_1 = A$, övriga $b_k = 0$

Parseval ger $P = \frac{b_1^2}{2} = \frac{A^2}{2}$

ty $P = a_0^2 + \frac{1}{2}\sum_{k=1}^{\infty}(a_k^2 + b_k^2)$

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B11. Stegsvar $s(t) = (1 - 0{,}8e^{-t} - 0{,}2e^{-6t})u(t)$

Laplacetransf.

$$S(s) = \frac{1}{s} - \frac{0{,}8}{s+1} - \frac{0{,}2}{s+6} =$$

$$= \frac{(s+1)(s+6) - 0{,}8s(s+6) - 0{,}2s(s+1)}{s(s+1)(s+6)} =$$

$$= \frac{s^2 + 7s + 6 - s^2(0{,}8+0{,}2) - s(0{,}8 \cdot 6 + 0{,}2)}{s(s+1)(s+6)} =$$

$$= \frac{s^2 + 7s + 6 - s^2 - 5s}{s(s+1)(s+6)} = \frac{2s+6}{s(s+1)(s+6)} = \frac{2(s+3)}{s(s+1)(s+6)}$$

$X(s) \cdot H(s) = Y(s)$, vi har $Y(s) = S(s)$ och $X(s) = \frac{1}{s}$

$$\therefore H(s) = \frac{2(s+3)}{(s+1)(s+6)} = \ldots = \frac{\frac{4}{5}}{s+1} + \frac{\frac{6}{5}}{s+6}$$

Inv. Laplacetransform ger impulssvar:

$$h(t) = \left(\frac{4}{5}e^{-t} + \frac{6}{5}e^{-6t}\right) \cdot u(t)$$

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812 $\quad x[n] \rightarrow \boxed{H_1} \longrightarrow \boxed{H_2} \longrightarrow y[n]$

$$H_1(z) = \frac{z}{z+\frac{1}{2}} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$h_2[n] = \left(\frac{1}{2}\right)^n u[n] \Rightarrow H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$H(z) = H_1(z)H_2(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} = \{\text{P.B.U.}\}$$

$$= \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}} \qquad A = \frac{1}{1-\frac{1}{2}(-2)} = \frac{1}{2}$$

$$B = \frac{1}{1+\frac{2}{2}} = \frac{1}{2}$$

$$H(z) = \frac{\frac{1}{2}}{1+\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}} \qquad \left\{\begin{array}{l}\text{Se även} \\ \text{Beta Z.16}\end{array}\right\}$$

Inv. z-transform Kausalt system

$$h[n] = \frac{1}{2}\left[\left(-\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n\right] u[n] =$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)^n \left[1+(-1)^n\right] u[n] = \left(\frac{1}{2}\right)^{n+1}\left[1+(-1)^n\right] u[n]$$

eller $\left(\frac{1}{2}\right)^{n+1}\left[1-(-1)^{n+1}\right] u[n]$

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a) $\omega_0 = \frac{2\pi}{T} = \{T = 1{,}0 \cdot 10^{-3}\} = \boxed{2\pi \cdot 10^3 \quad r/s}$

c) $x_a(t) = \sum_{k=-\infty}^{\infty} c_{ak} e^{jk\omega_0 t}$

$x_b(t) = \sum_{k=-\infty}^{\infty} c_{bk} e^{jk\omega_0 t} = -x_a(t) = -\sum_{k=-\infty}^{\infty} c_{ak} e^{jk\omega_0 t} =$

$= \sum_{k=-\infty}^{\infty} -c_{ak} e^{jk\omega_0 t}$ $\boxed{\text{Svar: } c_{bk} = -c_{ak}}$

b) $c_{ak} = \frac{1}{T}\int_0^T x_a(t) e^{-jk\omega_0 t} dt = \{k=0\} = \frac{1}{T}\int_0^T x_a(t)\, dt =$

$= \frac{1}{T}\int_0^{T/2} t \cdot \frac{2}{T} dt = \frac{2}{T^2}\left[\frac{t^2}{2}\right]_0^{T/2} = \frac{2}{T^2} \cdot \frac{T^2}{2 \cdot 4} = \boxed{\frac{1}{4}}$; för $k=0$

d) $c_{ak} = \frac{1}{T}\int_0^{T/2} \frac{2}{T} t e^{-jk\omega_0 t} dt = \frac{2}{T^2}\int_0^{T/2} t e^{-jk\omega_0 t} dt$

$c_{ck} = \frac{1}{2T}\int_0^T \frac{\tau}{T} e^{-jk\omega_0' \tau} d\tau = \left\{ \begin{array}{l} \tau = 2t \\ t = \tau/2 \end{array} \right.$ $\left\{ \begin{array}{c|c|c} t & 0 & T/2 \\ \hline \tau & 0 & T \end{array} \right\}$

$d\tau = 2dt$

$= \frac{1}{2T}\int_0^{T/2} \frac{2t}{T} e^{-jk\omega_0' 2t} 2 \cdot dt =$

Svar:

$= \frac{2}{T^2}\int_0^{T/2} t e^{-jk2\omega_0' t} dt = \{2\omega_0' = \omega_0\} \Rightarrow \boxed{c_{ck} = c_{ak}}$