SSY080 , D3

2018-11-01

A1 $x[n] = (0,5)^n$

$x[n+2] = (0,5)^{n+2} = 0,5^2 \cdot 0,5^n$

$x[n+2]\,u[n] = 0,5^2 \cdot 0,5^n\, u[n]$

$$X(z) = \mathcal{Z}\{0,5^2 \cdot 0,5^n \cdot u[n]\} =$$

$$= 0,5^2\, \mathcal{Z}\{0,5^n \cdot u[n]\} = 0,25 \cdot \frac{z}{z-0,5}$$

A2. Stegsvar: $s[n]$, Impulssvar: $h[n]$

$$s[n] = \sum_{k=0}^{n} h[k]$$ ger

A - 2.

B - 1

C - 4

D - 3

A3. $x(t) = e^{-5t}\, u(t)$, $T = 20\,ms$

$$x[n] = x(nT) = e^{-5T \cdot n} \cdot u[n] = \{5 \cdot T = 0,1\}$$

$$= \left(e^{-0,1}\right)^n \cdot u[n]$$

z-transformera

$$X(z) = \frac{z}{z - e^{-0,1}} = \frac{z}{z - \frac{1}{e^{0,1}}}$$

A4. En period om 2π uppdelad i $N = 11$ st intervall (delar)

$$\Delta\Omega = \frac{2\pi}{11}$$

A5. Periodtid $T = \frac{\pi}{500}$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} \cdot 500 = 1000 \text{ rad/s}$$

Magnitude = 0dB vid 1000 rad/s (0dB $\triangleq$ 1ggr)
Phase $\approx -70°$ (negativ $\Rightarrow$ "fördröjning")

Svar: A

A6.

$$G(s) = \frac{K}{s+5} \quad ; \quad G(j\omega) = \frac{K}{j\omega + 5} = \frac{K/5}{1 + j\frac{\omega}{5}}$$

$$|G(j\omega)| = \frac{K}{5} \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{5}\right)^2}} \approx \frac{K}{5} \cdot \frac{5}{\omega} \quad \text{för } \omega \gg 5$$

$$\frac{|G(j1000)|}{|G(j100)|} \approx \frac{100}{1000} = \frac{1}{10}$$

Alt: $|G(j\omega)|$ fallar med 20dB/dekad för $\omega \gg \omega_0 = 5$

och $20 \, {}^{10}\log\left\{\frac{|G(j1000)|}{|G(j100)|}\right\} = -20$

$${}^{10}\log\left\{\frac{|G(j1000)|}{|G(j100)|}\right\} = -1 \Rightarrow \frac{|G(j1000)|}{|G(j100)|} = 10^{-1} = \frac{1}{10}$$

Svar: $\frac{8}{10} = 0{,}8$

A7. $x(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi t}{L}\right)$ eller allmänt

$$x(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\omega_0 t)$$

$$\omega_0 = \frac{\pi}{L} \Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{\pi}{L}} = 2L$$

A8. $x(t) = \cos(\omega t)$, $T = \frac{\pi}{60} \Rightarrow \omega_s = \frac{2\pi}{T} = 120$ rad/s

$\omega_1 = 10 < \frac{\omega_s}{2}$ Ingen aliasing $\Omega_1 = \omega_1 T = \frac{\pi}{6}$

$\omega_2 = 50 < \frac{\omega_s}{2}$ Ingen aliasing $\Omega_2 = \omega_2 T = \frac{5\pi}{6}$

$\omega_3 = 70 > \frac{\omega_s}{2}$ Aliasing $\Omega_3 = \omega_3 T = \frac{7\pi}{6}$

Nokra $\omega_s - \omega_3 = 120 - 70 = 50 = \omega_2$

$\omega_4 = 170 > \frac{\omega_s}{2}$ Aliasing $\omega_4 = \omega_s + \omega_2$ $\Omega_4 = (\omega_s + \omega_2)T =$

$= 2\pi + \Omega_2$

$x[n] = \cos(\Omega_2 n)$ Undersöks

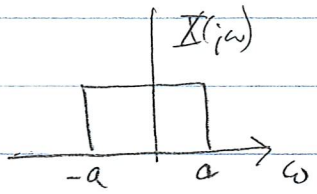
$x_2[n]$ och $x_4[n]$ lika ty $\Omega_4 = 2\pi + \Omega_2$

$x_2[n] = (1, -0{,}866, 0{,}5, 0, -0{,}5, 0{,}866, \ldots)$

$x_3[n] = (1, -0{,}866, 0{,}5, 0, -0{,}5, 0{,}866, \ldots$

S var: $x_2[n]$, $x_3[n]$ och $x_4[n]$ är lika

A9. $$X(j\omega) = \begin{cases} 1 & , \ |\omega| < a \\ 0 & , \ \text{annars} \end{cases}$$



Tabell ger $x(t) = \mathcal{FT}^{-1}\{X(j\omega)\} =$

$$= \frac{\sin(at)}{\pi t} = \frac{a}{\pi} \cdot \frac{\sin(at)}{at} =$$

$$= \frac{a}{\pi} \operatorname{sinc}(at)$$

A10. $\omega_s = 1000$ rad/s

$N = 2^{11} = 2048$

$k = 7$ (ur figur, ger grundvinkelfrekvens ω_0)

$$\omega_0 = \frac{k}{N} \cdot \omega_s$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \cdot N}{k \cdot \omega_s} =$$

$$= \frac{2\pi \cdot 2048}{7 \cdot 1000} \approx 1{,}84 \text{ s}$$

B11. $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 5x(t)$

Laplace transf. $Y(s)(s^2+4s+3) = X(s)(s+5)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+5}{s^2+4s+3} = \dots = \frac{s+5}{(s+1)(s+3)}$$

a/ $x(t) = e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s) = \frac{1}{s+2}$

$$Y(s) = H(s)\cdot X(s) = \frac{s+5}{(s+1)(s+3)(s+2)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$s+5 = A(s+3)(s+2) + B(s+1)(s+2) + C(s+1)(s+3)$$

$s=-1$: $4 = A(2)(1) \Rightarrow A=2$

$s=-3$: $2 = B(-2)(-1) \Rightarrow B=1$

$s=-2$: $3 = C(-1)(1) \Rightarrow C=-3$

$$Y(s) = \frac{2}{s+1} + \frac{1}{s+3} - \frac{3}{s+2}$$ Inv. Laplace transf.

$$y(t) = \left(2e^{-t} + e^{-3t} - 3e^{-2t}\right)u(t)$$

b/ $$H(s) = \frac{s+5}{(s+1)(s+3)} = \frac{D}{s+1} + \frac{E}{s+3}$$

$s+5 = D(s+3) + E(s+1)$ $\quad s=-1$: $4 = D\cdot 2 \Rightarrow D=2$

$s=-3$: $2 = E(-2) \Rightarrow E=-1$

$$H(s) = \frac{2}{s+1} - \frac{1}{s+3} = H_1(s) - H_2(s)$$

$$H_2(s) = \frac{1}{s+3}$$

B12. $y[n] + y[n-1] + 0{,}16\,y[n-2] = x[n-1] + 0{,}32\,x[n-2]$

Z-transformera

$$Y(z)\left(1 + z^{-1} + 0{,}16\,z^{-2}\right) = X(z)\left(z^{-1} + 0{,}32\,z^{-2}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} + 0{,}32\,z^{-2}}{1 + z^{-1} + 0{,}16\,z^{-2}} = \frac{z + 0{,}32}{z^2 + z + 0{,}16}$$

a/ $x[n] = \left(-\frac{1}{2}\right)^n u[n] \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad X(z) = \frac{z}{z + 0{,}5}$

Poler till $H(z)$: $z_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 0{,}16} = -0{,}5 \pm \sqrt{0{,}09} = \begin{cases} -0{,}2 \\ -0{,}8 \end{cases}$

$$Y(z) = H(z) \cdot X(z) = \frac{(z + 0{,}32)\,z}{(z + 0{,}2)(z + 0{,}8)(z + 0{,}5)}$$

$$\frac{Y(z)}{z} = \frac{z - 0{,}32}{(z + 0{,}2)(z + 0{,}8)(z + 0{,}5)} = \frac{A}{z + 0{,}2} + \frac{B}{z + 0{,}8} + \frac{C}{z + 0{,}5}$$

$$z + 0{,}32 = A(z + 0{,}8)(z + 0{,}5) + B(z + 0{,}2)(z + 0{,}5) + C(z + 0{,}2)(z + 0{,}8)$$

$z = -0{,}2$: $0{,}12 = A(0{,}6)(0{,}3)$ $\quad A = \frac{0{,}12}{0{,}6 \cdot 0{,}3} = \frac{12}{6 \cdot 3} = \frac{2}{3}$

$z = -0{,}8$: $-0{,}48 = B(-0{,}6)(-0{,}3)$ $\quad B = \frac{-0{,}48}{0{,}6 \cdot 0{,}3} = -\frac{48}{18} = -\frac{8}{3}$

$z = -0{,}5$: $-0{,}18 = C(-0{,}3)(0{,}3)$ $\quad C = \frac{0{,}18}{0{,}3 \cdot 0{,}3} = \frac{18}{9} = 2$

$$Y(z) = \frac{2}{3} \cdot \frac{z}{z + 0{,}2} - \frac{8}{3}\,\frac{z}{z + 0{,}8} + 2\,\frac{z}{z + 0{,}5}$$

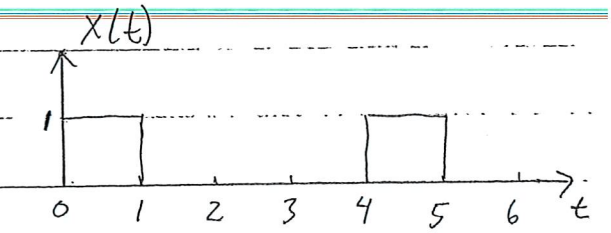
$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \left(\frac{2}{3} \cdot (-0{,}2)^n - \frac{8}{3}(-0{,}8)^n + 2(-0{,}5)^n\right) u[n]$$

b/ $y[0] = \frac{2}{3} - \frac{8}{3} + 2 = 0$

Stämmer! Se diff.ekv vid $n = 0$ med $y[n-1] = y[n-2] = 0$

och $x[n-1] = x[n-2] = 0$

B13. $x(t) = \sum_{n=-\infty}^{\infty} u(t-4n) - u(t-4n-1)$



a/ Periodtid $T = 4$ s

Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$ r/s

c/ $c_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{4}\int_0^1 e^{-jk\omega_0 t}\, dt =$

$= \frac{1}{4}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_0^1 = \frac{1}{4(-jk\omega_0)}\left[e^{-jk\omega_0} - 1\right] =$

$= \frac{j\,2}{k\cdot 4\cdot \pi}\left[e^{-jk\frac{\pi}{2}} - 1\right] = \frac{j}{2k\pi}\left[e^{-j\frac{k\pi}{2}} - 1\right]$ $k \neq 0$

b/ Studera $k=0$ separat

$c_0 = \frac{1}{4}\int_0^1 1\cdot dt = \frac{1}{4}\left[t\right]_0^1 = \frac{1}{4}$