Transformer, Signaler & System
D3, SSY080 2018-08-29

A1. A - 4 C - 1
B - 2 D - 3

A2. $x[n] = \delta[n] + 2\delta[n-1] - 4\delta[n-2] + 8\delta[n-3]$

A3. $a_1 = 5\sin\left(\frac{\pi}{6}\right) = 2{,}5$

$b_1 = 5\cos(\pi/6) = 5\cdot\sqrt{3}/2 \approx 4{,}33$

$a_3 = 2\cos\left(\frac{\pi}{3}\right) = 1$

$b_3 = 2\sin(\pi/3) = 2\cdot\sqrt{3}/2 = \sqrt{3}$

övriga a_n och $b_n = 0$

A4. $|H(j\omega)| = \left|\dfrac{\sqrt{10}}{1+\frac{8}{j\omega}}\right| = 3 \Rightarrow \omega = 24$ rad/s

A5. $x[n] = \delta[n] + \delta[n-2] + \delta[n-3] \Rightarrow y[n] = h[n] + h[n-2] + h[n-3]$

$\Rightarrow y[4] = \frac{27}{12} = 2{,}25$

A6. $\omega_1 = 2\pi\cdot 50$ rad/s ; $T(s) = (s-j\omega_1)(s+j\omega_1) = s^2 + \omega_1^2$

$\Rightarrow a = 0$, $b = \omega_1^2 = \pi^2\cdot 10^4 \approx 98{,}7\cdot 10^3$

A7. $1/s^2$ (tabell)

A8. (2) DTFT

A9. $h_2(t)$

A10. $\Omega = \omega\cdot T = \dfrac{\omega}{\frac{\omega_s}{2\pi}} = \{\omega_s = 16\omega\} = \dfrac{\pi}{8}$

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B11.

Stegsvar $y_s(t) = (1-e^{-2t})\,u(t)$

$$Y_s(s) = \mathcal{L}\{y_s(t)\} = \frac{1}{s} - \frac{1}{s+2} = \frac{s+2-s}{s(s+2)} = \frac{1}{s} \cdot \underbrace{\frac{2}{s+2}}_{H(s)}$$

$$Y_s(s) = \frac{1}{s} \cdot H(s) \Rightarrow H(s) = \frac{2}{s+2}$$

$x(t) = e^{-t}\sin(3t)\,u(t)$ Laplacetransf.

$$X(s) = \frac{3}{(s+1)^2+3^2} = \frac{3}{s^2+2s+10}$$

$$Y(s) = H(s)\cdot X(s) = \frac{2}{s+2} \cdot \frac{3}{s^2+2s+10} = \{P.B.U\} =$$

$$= \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+10}$$

$$6 = A(s^2+2s+10) + (Bs+C)(s+2)$$

s^2: $0 = A + B \Rightarrow A = -B$

s^1: $0 = 2A + 2B + C \Rightarrow C = 0$

s^0: $6 = 10A + 2C \Rightarrow A = \frac{6}{10} = 0{,}6 = -B$

$$Y(s) = \frac{0{,}6}{s+2} - 0{,}6\frac{s}{s^2+2s+10} = \frac{0{,}6}{s+2} - 0{,}6 \cdot \frac{s+1-1}{(s+1)^2+3^2} =$$

$$= \frac{0{,}6}{s+2} - 0{,}6\frac{s+1}{(s+1)^2+3^2} + \frac{0{,}6}{3} \cdot \frac{3}{(s+1)^2+3^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[0{,}6e^{-2t} - e^{-t}\left(0{,}6\cos(3t) - 0{,}2\sin(3t)\right)\right]u(t)$$

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$$H_1(z) = \frac{6z}{z^2 - 0{,}4z - 0{,}05}$$

$$h_2[n] = \left[5(0{,}5)^{n-1} + (-0{,}1)^{n-1}\right] u[n-1]$$

$$H_2(z) = \mathcal{Z}\{h_2[n]\} = 5 \cdot \frac{z}{z-0{,}5} \cdot z^{-1} + \frac{z}{z+0{,}1} \cdot z^{-1} =$$

$$= \frac{5}{z-0{,}5} + \frac{1}{z+0{,}1} = \frac{5(z+0{,}1) + 1(z-0{,}5)}{(z-0{,}5)(z+0{,}1)} =$$

$$= \frac{6z}{z^2-0{,}4z-0{,}05} \quad \text{Notera} = H_1(z)$$

Alternativt:

$$H_1(z) = \frac{6z}{z^2-0{,}4z-0{,}05} = \frac{6z}{(z-0{,}5)(z+0{,}1)} =$$

$$= \frac{A}{z-0{,}5} + \frac{B}{z+0{,}1} \; ; \quad 6z = A(z+0{,}1) + B(z-0{,}5)$$

z^1: $6 = A + B$

z^0: $0 = 0{,}1A - 0{,}5B \Rightarrow A = 5B \Rightarrow B = 1$ och $A = 5$

$$H_1(z) = \frac{5}{z-0{,}5} + \frac{1}{z+0{,}1} = 5\frac{z}{z-0{,}5} \cdot z^{-1} + \frac{z}{z+0{,}1} \cdot z^{-1}$$

$$h_1[n] = \mathcal{Z}^{-1}\{H_1(z)\} = \left[5(0{,}5)^{n-1} + (-0{,}1)^{n-1}\right] u[n-1]$$

$$\left.\begin{aligned} H_1(z) &= H_2(z) \\ h_1[n] &= h_2[n] \end{aligned}\right\} \quad h[n] = h_1[n] - h_2[n] = 0 \;, \; \forall n$$

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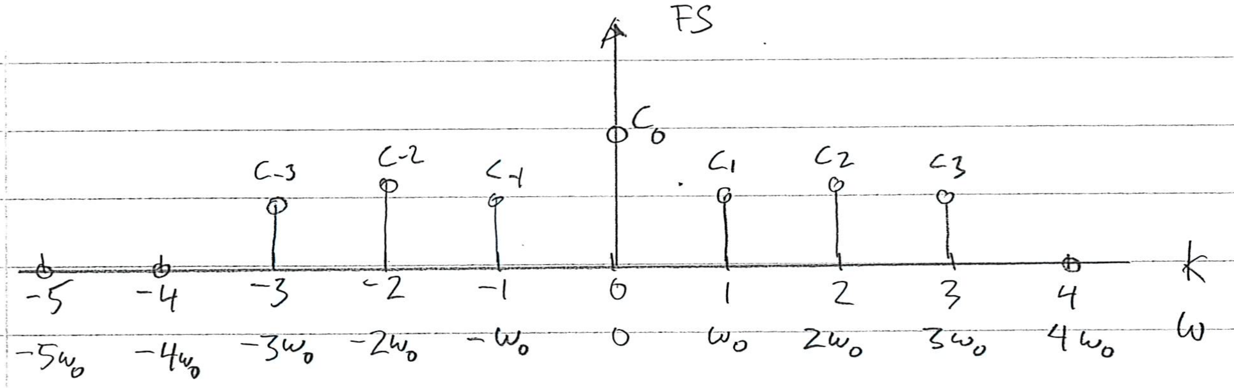

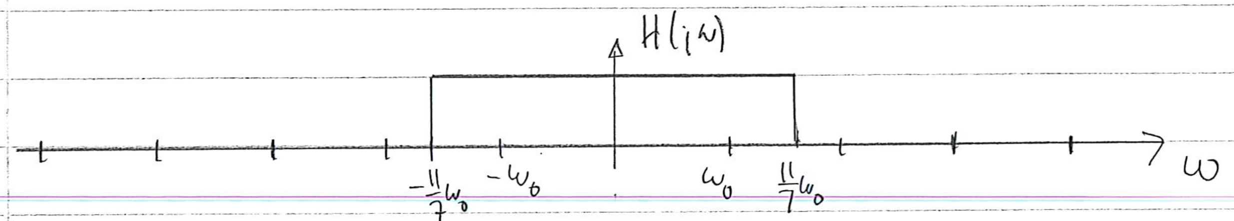

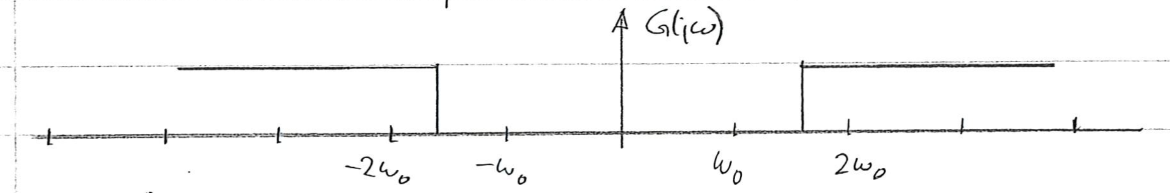

$$x(t) \longrightarrow \boxed{G(j\omega)} \longrightarrow y(t)$$

$G(j\omega)$ släpper endast igenom vinkelfrekv. $|\omega| > \frac{11}{7}\omega_0$

FS-koeff till $y(t)$ blir då $\begin{cases} c_2, c_{-2}, c_3 \text{ och } c_{-3} \\ \text{övriga } c_k = 0 \end{cases}$

Medeleffekt $\quad P = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$P_y = 2|c_2|^2 + 2|c_3|^2 = 2\cdot 0{,}5^2 + 2\cdot 0{,}2^2 = 0{,}58$$

$$P_x = P_y + 2|c_1|^2 + |c_0|^2 = P_y + 2\cdot 1^2 + 2^2 =$$
$$= P_y + 6 = 6{,}58$$

$$\frac{P_y}{P_x} = \frac{P_y}{P_y+6} = \frac{1}{1+\frac{6}{0{,}58}} = 0{,}088$$