A1

	Linjärt ?	Nej
	Tidsinvariant ?	Ja
	Kausalt ?	Ja

A2 $X(s) = \frac{1}{s^2+4}$

A3. $\mathcal{Z}\{x[n] * x[n-n_0]\} = z^{-n_0} X^2(z)$

A4. $a=2$, $b=1$

A5. $y[3] = \frac{5}{32}$

A6. $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2\frac{dx(t)}{dt} + 5x(t)$

A7. $c_0 = 3\pi$, $c_1 = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{3}}$, $c_{-1} = \frac{1}{\sqrt{2}} e^{-i\frac{\pi}{3}} = c_1^*$

övriga $c_k = 0$

A8. ii och iii gäller ($X(i\omega)$ icke periodisk och kontinuerlig i ω)

A9. $c_k = \frac{1}{T}$ $\forall k$

A10. 100π r/s och 200π r/s

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B11

$$H(s) = \frac{3}{s^2+3s+3}$$

Poler $s_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - 3}$

Komplexa! Kvadrat-
komplettera

$$H(s) = \frac{3}{\left(s+\frac{3}{2}\right)^2 + 3 - \frac{9}{4}} =$$

$$= \frac{3}{(s+1{,}5)^2 + 0{,}75} = \frac{3}{\sqrt{0{,}75}} \cdot \frac{\sqrt{0{,}75}}{(s+1{,}5)^2 + (\sqrt{0{,}75})^2}$$

Impulssvar $h(t) = \mathcal{L}^{-1}\{H(s)\} = \ldots = 2\sqrt{3}\, e^{-1{,}5t} \sin(0{,}866t)\, u(t)$

Stegsvar: $Y(s) = X(s) \cdot H(s)$ Insignal $x(t) = u(t)$
$\Rightarrow X(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s} \cdot \frac{3}{s^2+3s+3} = \frac{A}{s} + \frac{Bs+C}{s^2+3s+3}$$

$$3 = A(s^2+3s+3) + s(Bs+C)$$

s^0: $3 = A \cdot 3 \Rightarrow A = 1$

s^1: $0 = 3A + C \Rightarrow C = -3$

s^2: $0 = A + B \Rightarrow B = -1$

$$Y(s) = \frac{1}{s} - \frac{s+3}{(s+1{,}5)^2 + 0{,}75} = \frac{1}{s} - \frac{s+1{,}5+1{,}5}{(s+1{,}5)^2 + 0{,}75}$$

$$Y(s) = \frac{1}{s} - \frac{s+1{,}5}{(s+1{,}5)^2 + (\sqrt{0{,}75})^2} - \frac{1{,}5}{\sqrt{0{,}75}} \cdot \frac{\sqrt{0{,}75}}{(s+1{,}5)^2 + (\sqrt{0{,}75})^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[1 - e^{-1{,}5t}\left\{\cos(0{,}866t) - \sqrt{3}\sin(0{,}866t)\right\}\right] u(t)$$

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B12 $y[n] = -y[n-1] + y[n-2] + x[n]$

$$y[n] + y[n-1] - y[n-2] = x[n]$$

z - transf.

$$Y(z) + z^{-1}Y(z) - z^{-2}Y(z) = X(z)$$

$$Y(z)\left(1 + z^{-1} - z^{-2}\right) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + z^{-1} - z^{-2}} = \frac{z^2}{z^2 + z - 1}$$

Poler: $z_{1,2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1} = -\frac{1}{2} \pm \sqrt{\frac{5}{4}} = \begin{cases} 0{,}618 \\ -1{,}618 \end{cases}$

PBU: $\frac{H(z)}{z} = \frac{z}{(z - 0{,}618)(z + 1{,}618)} = \frac{A}{z - 0{,}618} + \frac{B}{z + 1{,}618}$

$$A = \frac{0{,}618}{0{,}618 + 1{,}618} = 0{,}276 \qquad B = \frac{-1{,}618}{-1{,}618 - 0{,}618} = 0{,}724$$

$$H(z) = 0{,}276 \frac{z}{z - 0{,}618} + 0{,}724 \frac{z}{z + 1{,}618}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \left\{0{,}276\,(0{,}618)^n + 0{,}724\,(-1{,}618)^n\right\} u[n]$$

Instabilt ty kausalt system och pol utanför enhetscirkeln.

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B13 $f_1 = 329{,}6$ Hz
$f_2 = 246{,}9$ Hz

Samplingsfrekvens $f_s = \frac{1}{T} = \frac{1}{200\mu s} = 5000$ Hz

Frekvensupplösning
$\Delta f = \frac{f_s}{N}$ svarar mot $(k+1)-k$ i index hos DFT

$f_1 \curvearrowright k_1$ i DFT
$f_2 \curvearrowright k_2$ i DFT

$k_1 - k_2 > 8$

$8 \cdot \Delta f < f_1 - f_2$

$8 \cdot \frac{f_s}{N} < f_1 - f_2$

$N > 8 \frac{f_s}{f_1 - f_2} = \frac{8 \cdot 5000}{329.6 - 246.9} \approx 484$

Svar: $N > 484$