SSY080 D3
16-12-21

1. $x(t) = \sin(100t)$, $\omega = 100$ r/s

Enligt Bodediagram $|H_1(j\omega)|_{\omega=100} = 0\,\text{dB} \hat{=} 1$ ggr

$\arg\{H_1(j\omega)\}_{\omega=100} = -45^\circ$

$y(t) = \sin(100t - 45^\circ)$

2. $$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n} = 2 + \left(-\frac{j}{4}\right)e^{j\omega_0 t} + \frac{j}{4}e^{-j\omega_0 t} =$$

$$= 2 + \frac{1}{4}e^{-j\frac{\pi}{2}}e^{j\omega_0 t} + \frac{1}{4}e^{j\frac{\pi}{2}}e^{-j\omega_0 t} =$$

$$= 2 + \frac{1}{4}\cdot 2 \cdot \frac{e^{j(\omega_0 t - \frac{\pi}{2})} + e^{-j(\omega_0 t - \frac{\pi}{2})}}{2} =$$

$$= 2 + \frac{1}{2}\cos\left(\omega_0 t - \frac{\pi}{2}\right)$$

3. $$y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\,u(t-\tau)\,d\tau =$$

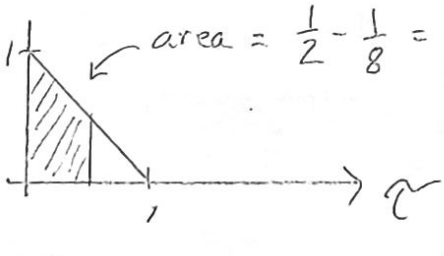
$$= \{t = 0{,}5\} = \int_0^{\infty} h(\tau)\,u\left(\tfrac{1}{2}-\tau\right)d\tau =$$

$$= \int_0^{0{,}5} h(\tau)\,d\tau = \int_0^{0{,}5} (1-\tau)\,d\tau =$$

$$= \left[\tau - \frac{\tau^2}{2}\right]_0^{0{,}5} = 0{,}5 - \frac{0{,}5^2}{2} = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

Fås även direkt ur figur

area $= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$



SSY080 D3

2016-12-21

4. $x(t) = e^{-5t} u(t)$ Samplas $t = nT$

$T = 20$ ms

$$x[n] = e^{-5 \cdot Tn} u[n] = e^{(-0,1)n} = \left(\frac{1}{e^{0,1}}\right)^n$$

Z-transf. $$X(z) = \frac{z}{z - e^{-0,1}} = \frac{1}{1 - e^{-0,1} \cdot z^{-1}}$$

5. $$y[n] - \frac{1}{4} y[n-1] = x[n]$$ Z-transf.

$$Y(z)\left(1 - \frac{1}{4} z^{-1}\right) = X(z) \quad , \quad H(z) = \frac{Y(z)}{X(z)}$$

$$x[n] = \delta[n-1] \Rightarrow X(z) = z^{-1}$$

$$Y(z) = H(z) X(z) = \frac{1}{1 - \frac{1}{4} z^{-1}} \cdot z^{-1}$$

$$\Rightarrow y[n] = \left(\frac{1}{4}\right)^{n-1} \cdot u[n-1]$$

6. $y(t) = \cos(x(t))$

Linjärt? Nej

Tidsinv.? Ja

Stabilt? Ja

In	Ut
x_1	$\cos(x_1)$
x_2	$\cos(x_2)$
$x_3 = x_1 + x_2$	$\cos(x_1 + x_2) \neq \cos(x_1) + \cos(x_2)$. Ej Linjärt

SSY080 D3
2016-12-21

7. $F(t)$ insignal ; $x(t)$ utsignal
Laplacetransf.

$m = 10$ kg
$k = 0{,}4$ N/m

$$(ms^2 + ds + k)X(s) = F(s)$$

Överföringsfkn: $H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+ds+k} = \frac{1/m}{s^2+\frac{d}{m}s+\frac{k}{m}}$

Inga oscillatoriska inslag ⇒ inga cos eller sin termer i stegsvaret ⇒ $H(s)$ har reella poler !

$$s^2 + \frac{d}{10}s + \frac{0{,}4}{10} = 0 \; ; \; s_{1,2} = -\frac{d}{20} \pm \sqrt{\underbrace{\frac{d^2}{400} - \frac{0{,}4}{10}}_{\geq 0}}$$

$$\frac{d^2}{400} \geq \frac{0{,}4}{10} \; ; \; d^2 \geq \frac{0{,}4 \cdot 400}{10} = 16$$

$$\therefore \; d \geq 4 \; Ns/m$$

8. ii) $X(j\omega)$ är kontinuerlig i ω
iv) $X(j\omega)$ är icke periodisk

9. $s^2 + 100 = 0 \Rightarrow s = \sqrt{-100} = \pm j10 = \sigma_1 \pm j\omega_1 \; (\sigma_1 = 0)$
$s^2 + 400 = 0 \Rightarrow s = \sqrt{-400} = \pm j20 = \sigma_2 \pm j\omega_2 \; (\sigma_2 = 0)$
$\omega = 10$ och 20 rad/s släcks ut

10. $x(t) = \sin(2\pi f_1 t)$ med $f_1 = 24$ kHz
$f_s = 40$ kHz $\qquad f > \frac{1}{2}f_s \Rightarrow$ Aliasing !
Rekonstruktion ger $f = f_s - f_1 = 16$ kHz
$\omega = 2\pi f = 2\pi \cdot 16 \cdot 10^3 = 32\pi \cdot 10^3$ rad/s

SSY080 D3

2016-12-21

B11 $x(t) \to \boxed{H_1} \to \boxed{H_2} \to y(t)$

$$Y(s) = H_1(s) \cdot H_2(s) \cdot X(s)$$

$$x(t) = 0{,}6 \cdot e^{-2t} u(t) \longleftrightarrow X(s) = \frac{0{,}6}{s+2}$$

$$h_2(t) = 0{,}5 e^{-0{,}5t} \cdot u(t) \longleftrightarrow H_2(s) = \frac{0{,}5}{s+0{,}5}$$

$$y(t) = \left(2{,}0 \cdot e^{-0{,}2t} - 3e^{-0{,}3t} + 1 \cdot e^{-0{,}5t}\right) u(t)$$

Laplace $Y(s) = \frac{2}{s+0{,}2} - \frac{3}{s+0{,}3} + \frac{1}{s+0{,}5} =$

$$= \frac{2(s+0{,}3)(s+0{,}5) - 3(s+0{,}2)(s+0{,}5) + (s+0{,}2)(s+0{,}3)}{(s+0{,}2)(s+0{,}3)(s+0{,}5)} =$$

$$= \ldots = \frac{0{,}06}{(s+0{,}2)(s+0{,}3)(s+0{,}5)}$$

$$H_1(s) = \frac{Y(s)}{H_2(s) X(s)} = \frac{0{,}06\,(s+2)\,\cancel{(s+0{,}5)}}{(s+0{,}2)(s+0{,}3)\cancel{(s+0{,}5)} \cdot 0{,}6 \cdot 0{,}5} =$$

$$= \frac{0{,}2\,(s+2)}{(s+0{,}2)(s+0{,}3)} = \{\text{P.B.U.}\} = \frac{A}{(s+0{,}2)} + \frac{B}{(s+0{,}3)} =$$

$$= \ldots = \frac{3{,}6}{s+0{,}2} - \frac{3{,}4}{s+0{,}3}$$

$$h_1(t) = \mathcal{L}^{-1}\{H_1(s)\} = \left(3{,}6 \cdot e^{-0{,}2t} - 3{,}4 e^{-0{,}3t}\right) u(t)$$

SSY080 D3
2016-12-21

B12 $y[n] = 6x[n] - 0{,}5\,y[n-1]$

$$y[n] + 0{,}5\,y[n-1] = 6x[n] \xleftrightarrow{\mathcal{Z}} Y(z)\left(1+0{,}5z^{-1}\right) = 6X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{6}{1+0{,}5z^{-1}} = \frac{6z}{z+0{,}5}$$

$$x[n] = u[n] \xleftrightarrow{\mathcal{Z}} X(z) = \frac{z}{z-1}$$

$$Y(z) = H(z)X(z) = z \cdot \underbrace{\frac{6z}{(z+0{,}5)(z-1)}}_{\text{P.B.U.}}$$

$$\frac{6z}{(z+0{,}5)(z-1)} = \frac{A}{z+0{,}5} + \frac{B}{z-1}$$

$$6z = A(z-1) + B(z+0{,}5)$$

z^0: $0 = -A + 0{,}5B$

z^1: $6 = A + B$

$6 = 1{,}5B \Rightarrow B = 4$

$A = 2$

$$Y(z) = 2\frac{z}{z+0{,}5} + 4\frac{z}{z-1}$$

Inv. z-transform

$$y[n] = \left(4 + 2(-0{,}5)^n\right)\cdot u[n] =$$

$$= 2\left(2 + (-0{,}5)^n\right)u[n]$$

SSY080 D3
16-12-21

B 13.

$$DFT\{x[n]\} = X[k]$$

$x[n]$, $n = 0, 1, 2, \ldots, N-1$
$X[k]$, $k = 0, 1, 2, \ldots, N-1$

Frekvensupplösning hos DFT : $\Delta f = \frac{f_s}{N} = \frac{1}{NT}$

$$k \cdot \Delta f = f \Rightarrow \frac{f}{f_s} = \frac{k}{N}$$

$$f = 50 \text{ Hz} \Rightarrow k = \frac{f}{f_s} \cdot N = f \cdot T \cdot N$$

: Vilket k-värde svarar mot 50 Hz ?

(1) $T = 1{,}5 \text{ ms}$, $N = 96 \Rightarrow k = 50 \cdot 1{,}5 \cdot 10^{-3} \cdot 96 \approx 7{,}2$

(2) $T = 1{,}6 \text{ ms}$, $N = 128 \Rightarrow k = 50 \cdot 1{,}6 \cdot 10^{-3} \cdot 128 = 10{,}2$

(3) $T = 4{,}0 \text{ ms}$, $N = 64 \Rightarrow k = 50 \cdot 4{,}0 \cdot 10^{-3} \cdot 64 \approx 12{,}8$

Svar: $x_1[n]$ — C
$x_2[n]$ — B
$x_3[n]$ — A