Transformer, Signaler & System
03 SS4080 16-10-28

1. $h(t) = Ae^{-bt} \cdot u(t)$

$A = 600$
$b = \sqrt{3} \cdot 100$
$\omega = 300$ rad/s

$$H(s) = \frac{A}{s+b} = \frac{A}{b} \cdot \frac{1}{1+\frac{s}{b}}$$

Frekvenssvar, $s = j\omega$

$$H(j\omega) = \frac{A}{b} \cdot \frac{1}{1+j\frac{\omega}{b}}$$

$$|H(j\omega)|_{\omega=300} = \frac{600}{\sqrt{3}\cdot 100} \cdot \frac{1}{\sqrt{1+\left(\frac{300}{\sqrt{3}\cdot 100}\right)^2}} = 2\sqrt{3}\cdot\frac{1}{2} = \sqrt{3}$$

$$\arg\{H(j\omega)\}|_{\omega=300} = -\arctan\left(\frac{300}{\sqrt{3}\cdot 100}\right) = -60^\circ \text{ alt. } -\frac{\pi}{3} \text{ rad}$$

Insignal $x(t) = \cos(300t)$

Utsignal $y(t) = |H(j300)| \cos(300t + \arg\{H(j300)\}) =$

$$= \sqrt{3}\cos\left(300t - \frac{\pi}{3}\right)$$

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A2. $x(t) = 5 + 2\cos(500t + \frac{\pi}{6}) =$

$= 5 + 2 \cdot \frac{1}{2}\left(e^{j500t + \frac{\pi}{6}} + e^{-j(500t + \frac{\pi}{6})}\right) = \{\omega_0 = 500\} =$

$= 5 + e^{j\frac{\pi}{6}} \cdot e^{j\omega_0 t} + e^{-j\frac{\pi}{6}} e^{-j\omega_0 t} = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

Svar: $c_0 = 5$, $c_1 = e^{j\frac{\pi}{6}}$, $c_{-1} = e^{-j\frac{\pi}{6}} = c_1^*$
övriga $c_k = 0$

A3. $x[n] = \delta[n] + \delta[n-2] + \delta[n-4]$

$\Rightarrow y[n] = h[n] + h[n-2] + h[n-4]$

n	0	1	2	3	4	5	6
h[n]	0	2	1	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$
h[n-2]	0	0	0	2	1	$\frac{2}{3}$	$\frac{2}{4}$
h[n-4]	0	0	0	0	0	2	1
Σ					1,5		

$y[4] = 1{,}5$

A4. $y[n] + 0{,}5\, y[n-1] = 10\, x[n]$

$Y(z)\left(1 + 0{,}5 z^{-1}\right) = 10\, X(z)$

$H(z) = \frac{10}{1 + 0{,}5 z^{-1}} = 10 \frac{z}{z + 0{,}5}$

$h[n] = \mathcal{Z}^{-1}\{H(z)\} = 10 \cdot (-0{,}5)^n \cdot u[n]$

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A5. $y(t) = e^{-\pi t} \cdot x(t)$

Amplitud tidsberoende → Ej tidsinvariant

Insignal	Utsignal
$x_1(t)$	$y_1(t) = e^{-\pi t} \cdot x_1(t)$
$x_2(t)$	$y_2(t) = e^{-\pi t} \cdot x_2(t)$
$x_3(t) = a x_1(t) + b x_2(t)$	$y_3(t) = e^{-\pi t} \cdot x_3(t) =$

$$= e^{-\pi t}(a x_1(t) + b x_2(t)) =$$
$$= a e^{-\pi t} x_1(t) + b e^{-\pi t} \cdot x_2(t) =$$
$$= a y_1(t) + b y_2(t)$$

Linjärt? Ja!

A6. $x(t)$ reell

Tio hela perioder av en samplad sinus-formad signal ger högt värde på $|X[k]|$ för $k = 10$ och $k = N - 10 = 80 - 10 = 70$

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A7.

$$Y(s) = \frac{s+200}{s^2 + s\cdot 200 + 2\cdot 10^4}$$ komplexa poler, kvadratkomplettera

$$Y(s) = \frac{s+100+100}{(s+100)^2 + 100^2} =$$

$$= \frac{s+100}{(s+100)^2+100^2} + \frac{100}{(s+100)^2+100^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = e^{-100t}\left(\cos 100t + \sin 100t\right)\cdot u(t)$$

A8. ii) $X(e^{j\Omega})$ är kontinuerlig i Ω
iii) $X(e^{j\Omega})$ är periodisk (i Ω)

A9. $(s-s_1)(s-s_2) = \{s=j\omega\} = (j\omega - s_1)(j\omega - s_2)$

Nollställe för $\omega = 700$ $\quad j700 - s_1 = 0 \Rightarrow s_1 = j700$

Låt $s_2 = s_1^* = -j700$

$(s-j700)(s+j700) = s^2 + 700^2$

$\Rightarrow a = 0, \quad b = 700^2 = 49\cdot 10^4$

A10. $\omega_1 = 2\pi 36 \quad \omega_s = 2\pi\cdot 60 \quad 2\omega_1 > \omega_s \Rightarrow$ Aliasing

Perfekt rekonstruktion med idealt LP-filter

Frekvenser inom $-\frac{\omega_s}{2} < \omega < \frac{\omega_s}{2}$ passerar

Vikning ger $\omega = \omega_s - \omega_1 = 2\pi(60-36) = 2\pi\cdot 24 =$

$= 48\pi$ rad/s

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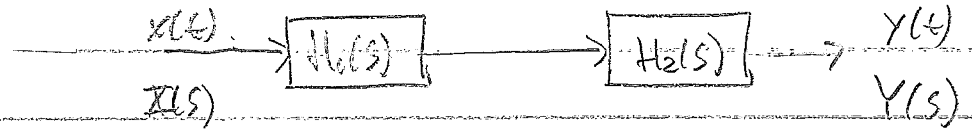
B1.



$$Y(s) = H_1(s) H_2(s) X(s)$$

$$x(t) = 6e^{-3t} u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{6}{s+3}$$

$$\cancel{H_1(s) = \frac{4}{s+2}}$$

Stegsvar till H_2 : $y_2(t) = (1 - e^{-5t}) u(t)$ $\mathcal{L}\{u(t)\}$

$$y_2(t) \xleftrightarrow{\mathcal{L}} Y_2(s) = \frac{1}{s} - \frac{1}{s+5} = \frac{s+5-s}{s(s+5)} = \frac{5}{s(s+5)} = \frac{1}{s} \cdot H_2(s)$$

$$\therefore H_2(s) = \frac{5}{s+5}$$

$$Y(s) = \frac{4}{(s+2)} \cdot \frac{5}{(s+5)} \cdot \frac{6}{s+3} = \{\text{P.B.U.}\} = \frac{A}{s+2} + \frac{B}{s+5} + \frac{C}{s+3}$$

$$A = \frac{4 \cdot 5 \cdot 6}{(-2+5)(-2+3)} = \frac{4 \cdot 5 \cdot 6}{3} = 40$$

$$B = \frac{4}{(-3)} \cdot \frac{5 \cdot 6}{(-2)} = 20 \quad ; \quad C = \frac{4 \cdot 5 \cdot 6}{(-1)(+2)} = -60$$

$$Y(s) = \frac{40}{s+2} + \frac{20}{s+5} - \frac{60}{s+3}$$ Inverstransformera

$$y(t) = 20(2e^{-2t} + e^{-5t} - 3e^{-3t}) u(t)$$

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B2.

$$h[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-2} u[n-1]$$

skriv om

$$h[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^{-1}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

Förskjutning "(n-1)".

a) z-transformera !

$$H(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + 2\cdot \frac{1}{1-\frac{1}{2}z^{-1}}\cdot z^{-1}$$

$$H(z) = \frac{z}{z-\frac{1}{3}} + \frac{2}{z-\frac{1}{2}} = \frac{z\left(z-\frac{1}{2}\right)+2\left(z-\frac{1}{3}\right)}{\left(z-\frac{1}{3}\right)\left(z-\frac{1}{2}\right)} =$$

$$= \frac{z^2+z\left(2-\frac{1}{2}\right)-\frac{2}{3}}{z^2-z\left(\frac{1}{2}+\frac{1}{3}\right)+\frac{1}{6}} = \frac{z^2+z\cdot\frac{3}{2}-\frac{2}{3}}{z^2-z\cdot\frac{5}{6}+\frac{1}{6}}$$

eller

$$H(z) = \frac{1+\frac{3}{2}z^{-1}-\frac{2}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

b) $H(z) = \frac{Y(z)}{X(z)} \Rightarrow Y(z)\left(1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}\right) = X(z)\left(1+\frac{3}{2}z^{-1}-\frac{2}{3}z^{-2}\right)$

Inv. z-transform

$$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2] = x[n]+\frac{3}{2}x[n-1]-\frac{2}{3}x[n-2]$$

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B3

a/ Signalernas (vinkel) frekvenser $\omega < \omega_M = 10\pi$ r/s

Enligt samplingsteoremet
Samplingsfrekvens $\omega_s \geq 2\omega_M = 20\pi$ r/s

Sampelintervall $T = \frac{2\pi}{\omega_s}$ s

b/ "Avstånd" mellan ingående frekvenser

$$\omega_r = 2\pi(0{,}45 - 0{,}40) = 0{,}1\pi \text{ r/s}$$

Frekvensupplösning hos DFT ; $\Delta\omega = \frac{\omega_s}{N}$
N = antal sampel

Krav: $|k_1 - k_2| \geq 10 \Rightarrow \omega_r \geq 10 \cdot \Delta\omega = \frac{10 \cdot \omega_s}{N}$

$$N \geq \frac{10 \cdot \omega_s}{\omega_r} = \left\{ \text{Välj } \omega_s = 2\omega_M = 20\pi \text{ r/s} \right\} =$$

$$= \frac{10 \cdot 20\pi}{0{,}1\pi} = 2000$$

Tid signalen samplas $t_{tot} = N \cdot T$

$$t_{tot} = N \cdot T = \frac{10\,\omega_s}{\omega_r} \cdot \frac{2\pi}{\omega_s} = \frac{2\pi}{\frac{\omega_r}{10}} = \frac{2\pi}{\Delta\omega} =$$

$$= \frac{2\pi}{0{,}1\pi} \cdot 10 = 200 \text{ s}$$