Transformer, Signaler & System
D3, SSY080 160105

1\. a) Studera frekvenssvar ($s=j\omega$).

$$H_1(j\omega) = \frac{\sqrt{2}\,1000}{j\omega+1000} = \frac{\sqrt{2}}{1+j\frac{\omega}{1000}}$$

$$|H_1(j\omega)| = \frac{\sqrt{2}}{\sqrt{1+\left(\frac{\omega}{1000}\right)^2}} = \{\omega=1000\} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\arg\{H_1(j\omega)\} = -\arctan\left\{\frac{\omega}{1000}\right\} = \{\omega=1000\} = -45^\circ$$

$$H_2(j\omega) = \frac{\sqrt{2}\,j\omega}{j\omega+1000} = \frac{\sqrt{2}}{1+\frac{1000}{j\omega}}$$

$$|H_2(j\omega)| = \frac{\sqrt{2}}{\sqrt{1+\left(\frac{1000}{\omega}\right)^2}} = \{\omega=1000\} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\arg\{H_2(j\omega)\} = \arg\{j\sqrt{2}\omega\} - \arg\left\{\frac{\omega}{1000}\right\} = \{\omega=1000\} = 90^\circ-45^\circ = 45^\circ$$

$$H_3(j\omega) = \frac{1000-j\omega}{1000+j\omega}$$

$$|H_3(j\omega)| = \sqrt{\frac{1000^2+\omega^2}{1000^2+\omega^2}} = 1$$

$$\arg\{H_3(j\omega)\} = \arctan\left\{\frac{-\omega}{1000}\right\} - \arctan\left\{\frac{\omega}{1000}\right\} = \{\omega=1000\} =$$

$$= -45^\circ - 45^\circ = -90^\circ$$

Alla $|H(j\omega)| = 1$ vid $\omega = 1000$

Från figur	Fasförskjutn.
A	$\approx +90^\circ$
B	$\approx +45^\circ$
C	$\approx -45^\circ$
D	$\approx -90^\circ$

Svar:

H_1	—	C
H_2	—	B
H_3	—	D

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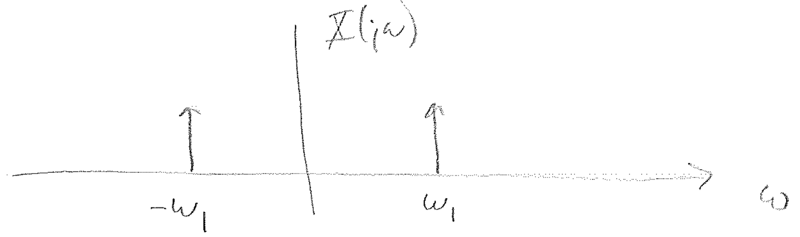
1b $x(t) = \sin(\omega_1 t) \xleftrightarrow{\mathcal{F}} X(j\omega) = \frac{\pi}{j}\left[\delta(\omega-\omega_1) - \delta(\omega+\omega_1)\right]$



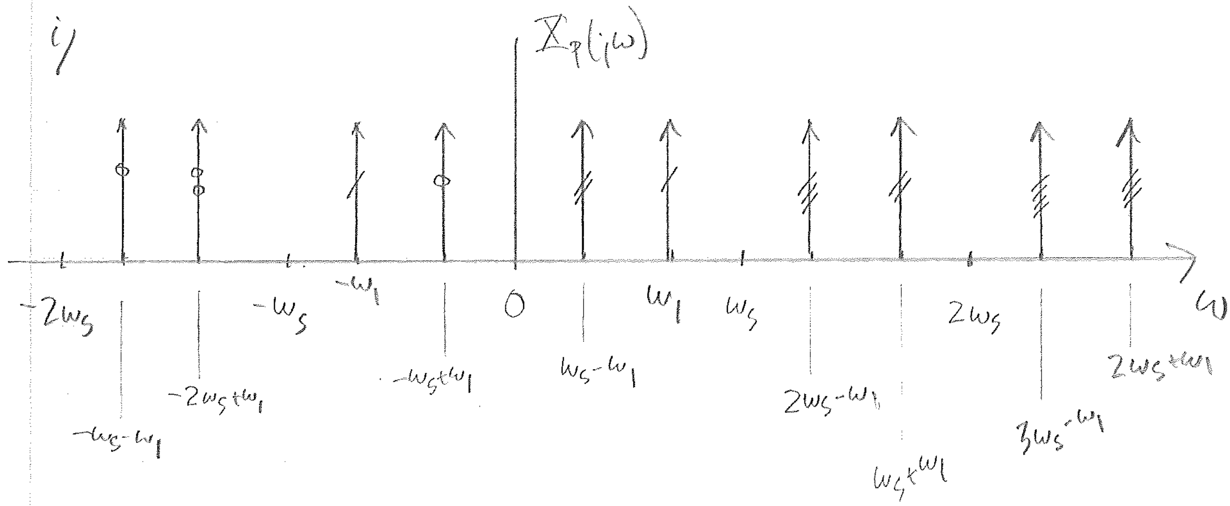
$T = \frac{32\pi}{23\omega_1}$; $\omega_s = \frac{2\pi}{T} = \frac{2\pi \cdot 23\omega_1}{32\pi} = \frac{23}{16}\omega_1$

$\omega_s \approx 1{,}44\,\omega_1 \Rightarrow \omega_1 \approx 0{,}70\,\omega_s$

ii) Vid sampling uppkommer aliasing $\left(\omega_1 > \frac{\omega_s}{2}\right)$

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

i)



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2. $$x(t) = \sum_{k=1}^{10} b_k \sin(k\omega_0 t + \phi_k) \quad , \quad \omega_0 = \frac{2\pi}{T}$$

a/ Frekvens $k\omega_0$ ger k perioder sinusformad signal i intervallet $[0,T]$

b_k bestämmer amplitud på signal med frekv. $k\omega_0$

Figur 3: I b_8, b_9, b_{10} dominerar ⇒ "höga" frekvenser
$\omega_8, \omega_9, \omega_{10}$ dominerar
II b_4, b_5, b_6 dominerar ⇒ "mellan" frekvenser
$\omega_4, \omega_5, \omega_6$ dominerar
III b_2, b_3 dominerar ⇒ "låga" frekvenser
ω_2, ω_3 dominerar

Alltså:
A – II
B – III
C – I

b/ Parsevals formel ger

$$\bar{P} = \frac{1}{T} \int_a^{a+T} x^2(t) = \frac{1}{2} \sum_{k=1}^{10} b_k^2 \qquad \text{(Beta)}$$

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3. $x(t) = \left[e^{-4t}\cos(8t)\right]u(t) \xrightarrow{\mathcal{L}} X(s) = \dfrac{s+4}{(s+4)^2+8^2}$

System: $y(t) + \frac{1}{8}\frac{dy(t)}{dt} = x(t)$ Laplacetransf.

$$Y(s) + \frac{1}{8}\,sY(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1+\frac{s}{8}} = \frac{8}{s+8}$$

$$Y(s) = H(s)\cdot X(s) = \frac{8}{s+8}\cdot\frac{s+4}{s^2+8s+80}$$

Partial-bråks-uppdela

$$\frac{8(s+4)}{(s+8)(s^2+8s+80)} = \frac{A}{s+8} + \frac{Bs+C}{s^2+8s+80}$$

$$8(s+4) = A(s^2+8s+80) + (Bs+C)(s+8)$$

s^0: $32 = 80A + 8C$ | $32 = 80A + 8\cdot 8 \Rightarrow A = -\frac{32}{80} = -0{,}4$

s^1: $8 = 8A + 8B + C$ | $C = 8$

s^2: $0 = A + B$ | $A = -B$

$$Y(s) = -0{,}4\cdot\frac{1}{s+8} + \frac{0{,}4s+8}{s^2+8s+80} =$$

$$= \frac{-0{,}4}{s+8} + 0{,}4\,\frac{s+20}{(s+4)^2+8^2} =$$

$$= -\frac{0{,}4}{s+8} + 0{,}4\left(\frac{s+4}{(s+4)^2+8^2} + \frac{16\cdot 8}{8\,((s+4)^2+8^2)}\right)$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 0{,}4\left[e^{-4t}(\cos 8t + 2\sin 8t) - e^{-8t}\right]u(t)$$

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4. $\frac{k}{N} = \frac{\omega}{\omega_s}$; $\omega_s = \frac{2\pi}{T}$

$$k = \frac{\omega N}{\omega_s} = \frac{\omega N \cdot T}{2\pi} = \frac{\omega T}{2\pi} \cdot N$$

$N=16$ ω och T varierar enligt tabell
Beräkna k. Reell sinusformad signal ger
DFT bidrag vid k och $N-k$.

A: $k = \frac{280 \cdot 2{,}8 \cdot 10^{-3}}{2\pi} \cdot 16 \approx 2$ $N-k=14$

B: $k = \frac{560 \cdot 3{,}5 \cdot 10^{-3}}{2\pi} \cdot 16 \approx 5$ $N-k=11$

C: $k = \frac{390 \cdot 7{,}1 \cdot 10^{-3} \cdot 16}{2\pi} \approx 7$ $N-k=9$

D: $k = \frac{430 \cdot 9{,}2 \cdot 10^{-3} \cdot 16}{2\pi} \approx 10$ $N-k=6$

E: $k = \frac{525 \cdot 11{,}2 \cdot 10^{-3} \cdot 16}{2\pi} \approx 15$ $N-k=1$

Svar:
A – X_3
B – X_5
C – X_2
D – X_4 (Aliasing)
E – X_1 (Aliasing)

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5. $h[n] = \left[(0{,}5)^n + (-0{,}4)^n\right]u[n] \overset{Z}{\longleftrightarrow} H(z) = \frac{z}{z-0{,}5} + \frac{z}{z+0{,}4}$

a) $$H(z) = \frac{z(z+0{,}4)+z(z-0{,}5)}{(z-0{,}5)(z+0{,}4)} = \frac{z(2z-0{,}1)}{z^2+z(-0{,}5+0{,}4)-0{,}2} =$$
$$= \frac{2z^2-0{,}1z}{z^2-0{,}1z-0{,}2} = \frac{2-0{,}1z^{-1}}{1-0{,}1\cdot z^{-1}-0{,}2\cdot z^{-2}}$$

b) $x[n] = 3\cdot(0{,}2)^n\cdot u[n] \overset{Z}{\longleftrightarrow} X(z) = \frac{3z}{z-0{,}2}$

$$Y(z) = H(z)\cdot X(z) = \frac{3z\cdot(2z^2-0{,}1z)}{(z-0{,}2)(z-0{,}5)(z+0{,}4)}$$

Partialbråksuppdela

$$\frac{2z^2-0{,}1z}{(z-0{,}2)(z-0{,}5)(z+0{,}4)} = \frac{A}{z-0{,}2} + \frac{B}{z-0{,}5} + \frac{C}{z+0{,}4}$$

$$2z^2-0{,}1z = A(z-0{,}5)(z+0{,}4) + B(z-0{,}2)(z+0{,}4) + C(z-0{,}2)(z-0{,}5)$$

$z=0{,}2:\quad 2\cdot0{,}2^2-0{,}1\cdot0{,}2 = A(0{,}2-0{,}5)(0{,}2+0{,}4) \Rightarrow A = \frac{0{,}06}{-0{,}18} = -\frac{1}{3}$

$z=0{,}5:\quad 2\cdot0{,}5^2-0{,}1\cdot0{,}5 = B(0{,}5-0{,}2)(0{,}5+0{,}4) \Rightarrow B = 0{,}45/0{,}27 = \frac{5}{3}$

$z=-0{,}4:\quad 2(-0{,}4)^2+0{,}1\cdot0{,}4 = C(-0{,}4-0{,}2)(-0{,}4-0{,}5) \Rightarrow C = 0{,}36/0{,}54 = \frac{2}{3}$

$$Y(z) = \frac{3zA}{z-0{,}2} + \frac{3zB}{z-0{,}5} + \frac{3zC}{z+0{,}4} = -\frac{z}{z-0{,}2} + 5\frac{z}{z-0{,}5} + 2\frac{z}{z+0{,}4}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = -(0{,}2)^n + 5(0{,}5)^n + 2(-0{,}4)^n \quad , n \geq 0$$