Transformer, Signaler och System, D3
SSY080 141031

1.

$$G(s) = \frac{2s}{s+a}$$

Frekvenssvar $G(j\omega) = 2 \cdot \frac{j\omega}{j\omega + a}$

a) Amplitudförändring $|G(j\omega)| = 2 \frac{\omega}{\sqrt{\omega^2 + a^2}}$

Insignal $x(t) = \sin(100t) \Rightarrow \omega = 100$ r/s
Från figur ser vi att $|G(j\omega)|_{\omega=100} = 1$
(In- och utsignal har samma ampl.)

$$|G(j\omega)|_{\omega=100} = \frac{2 \cdot 100}{\sqrt{100^2 + a^2}} = 1$$

$200 = \sqrt{100^2 + a^2} \Rightarrow 200^2 = 100^2 + a^2$
$a^2 = 200^2 - 100^2 = (4-1) \cdot 100^2 = 3 \cdot 100^2$
$a = (\pm) \sqrt{3} \cdot 100$ $(a > 0$ ty pol i VHP för stabilitet$)$

b) $\phi = \arg\{G(j\omega)\} = \arg\{j\omega\} - \arg\{j\omega + a\} =$
$= 90^\circ - \arctan\left\{\frac{\omega}{a}\right\} = \left\{\begin{matrix} \omega = 100 \\ a = \sqrt{3} \cdot 100 \end{matrix}\right\} =$
$= 90^\circ - \arctan(1/\sqrt{3}) = 90^\circ - 30^\circ = 60^\circ$

Svar: a) $a = \sqrt{3} \cdot 100$

$\phi = 60^\circ = \left\{\frac{\pi}{3} \text{ rad}\right\}$

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2. $y[n] - 0{,}1\,y[n-1] - 0{,}2\,y[n-2] = x[n]$

z-transf.

a) $Y(z)\left[1 - 0{,}1z^{-1} - 0{,}2z^{-2}\right] = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0{,}1z^{-1} - 0{,}2z^{-2}} = \frac{z^2}{z^2 - 0{,}1z - 0{,}2}$$

Poler $z_{1,2} = \frac{0{,}1}{2} \pm \sqrt{\left(\frac{0{,}1}{2}\right)^2 + 0{,}2} = 0{,}05 \pm 0{,}45 = \begin{cases} 0{,}5 \\ -0{,}4 \end{cases}$

b)

$$H(z) = \frac{z^2}{(z-0{,}5)(z+0{,}4)} \quad ; \quad \frac{H(z)}{z} = \frac{z}{(z-0{,}5)(z+0{,}4)} = \frac{A}{z-0{,}5} + \frac{B}{z+0{,}4}$$

Partialbråksuppdelning

$z = A(z+0{,}4) + B(z-0{,}5)$; $z = 0{,}5 \Rightarrow 0{,}5 = A(0{,}5+0{,}4)$

$A = \frac{5}{9}$

$z = -0{,}4 \Rightarrow -0{,}4 = B(-0{,}4-0{,}9)$

$B = \frac{4}{9}$

$$H(z) = \frac{5}{9}\cdot\frac{z}{z-0{,}5} + \frac{4}{9}\cdot\frac{z}{z+0{,}4}$$

$h[n] = \frac{1}{9}\left[5\cdot 0{,}5^n + 4(-0{,}4)^n\right]u[n]$ Impulssvar

Insättning ger $h[n] = [\underset{n=0}{1}, 0{,}1, 0{,}21, 0{,}041, 0{,}0461, \dots]$

c) Poler innanför enhetscirkeln + kausalt system

$\Rightarrow$ Stabilt system

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3. $h(t) = \delta(t) + \left(\cos(t) + 2\sin(t)\right) u(t)$

$x(t) = e^{-2t} u(t)$ $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

Laplace transf.

$$\mathcal{L}\{h(t)\} = H(s) = 1 + \frac{s}{s^2+1} + \frac{2}{s^2+1} =$$
$$= 1 + \frac{s+2}{s^2+1}$$

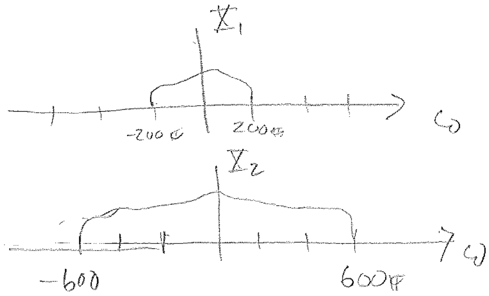
$$\mathcal{L}\{x(t)\} = X(s) = \frac{1}{s+2}$$

$$Y(s) = X(s) \cdot H(s) = \frac{1}{s+2}\left(1 + \frac{s+2}{s^2+1}\right) =$$
$$= \frac{1}{s+2} + \frac{1}{s^2+1}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[e^{-2t} + \sin(t)\right] u(t)$$

4.(i) $y_1(t) = x_1(t) * x_2(t)$

$X_1(j\omega) = 0 \ , \ |\omega| > 200\pi$ r/s
$X_2(j\omega) = 0 \ , \ |\omega| > 600\pi$ r/s



Fouriertransf.

$Y_1(j\omega) = X_1(j\omega) \cdot X_2(j\omega)$

Multiplikation

Då blir $Y_1(j\omega) = 0 \ , \ |\omega| > 200\pi$

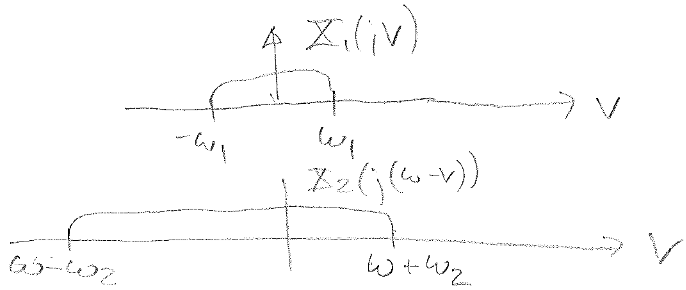
(Begränsas av $X_1(j\omega)$)

Samplingsteoremet ger

$\omega_s > 2 \cdot \omega_{max} = 2 \cdot 200\pi$ r/s

Samplingsintervall $T_s = \frac{2\pi}{\omega_s} \Rightarrow T_{1s} < \frac{2\pi}{2 \cdot 200\pi} = \frac{1}{200} = 5 \cdot 10^{-3}$

(ii) $y_2(t) = x_1(t) \cdot x_2(t) \Rightarrow Y_2(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega) =$



$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(jv) X_2(j(\omega - v)) \, dv$$

Låt ω variera från $-\infty$ till ∞.

Överlapp startar för $\omega = -\omega_1$ och slutar då $\omega = +\omega_1$

$\Rightarrow Y_2(j\omega) = 0$ för $\omega < -(\omega_1 + \omega_2)$
$Y_2(j\omega) = 0$ för $\omega > (\omega_1 + \omega_2)$

$\Rightarrow \omega_{max} = \omega_1 + \omega_2$

$\omega_1 = 200\pi$
$\omega_2 = 600\pi$

$$\omega_s > 2\,\omega_{max} = 2(\omega_1 + \omega_2)$$

$$T_{s2} < \frac{2\pi}{2\,\omega_{max}} = \frac{2\pi}{2(200+600)\pi} = \frac{1}{800} = 1{,}25 \cdot 10^{-3}$$

Svar: (i) $T < 5{,}0$ ms (ii) $T < 1{,}25$ ms

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5. $x(t) = \sin(\omega t) + b(t)$

$\uparrow$ brus

$\omega = 2\pi f$, $f = 250$ Hz

$x(t) \xrightarrow{\text{samplas}} x(nT) = x[n]$ $n = 0, 1, 2, \ldots, N-1$

$N = 64$

$\text{DFT}\{x[n]\} = X[k]$, $k = 0, 1, 2, \ldots, N-1$

Samband $\dfrac{k}{N} = \dfrac{f}{f_s}$

Reell sinusformad signal ger bidrag vid k och N−k

f_s : Samplingsfrekv [Hz]

$T_s = 1/f_s$: Sampelintervall i [s]

$k = f \cdot N \cdot T_s$

Fall	T_s	"k"	Svarar mot
1	0,90 ms	$250 \cdot 64 \cdot 0{,}90 \cdot 10^{-3} = 14{,}4$	B
2	1,4 ms	$250 \cdot 64 \cdot 1{,}4 \cdot 10^{-3} = 22{,}4$	C
3	3,4 ms	$250 \cdot 64 \cdot 3{,}4 \cdot 10^{-3} = 54{,}4$ $N - k = 9{,}6$	A

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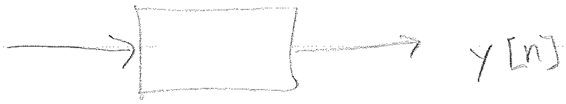
1a) Samband: $\delta[n] = u[n] - u[n-1]$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k] = \{\text{Alt.}\} =$$

$$= \delta[n] + \delta[n-1] + \delta[n-2] + \ldots$$



Insignal	Utsignal
$\delta[n]$	$h[n]$
$u[n]$	$\sum_{k=-\infty}^{n} h[k] = \{\text{Alt.}\} = h[n] + h[n-1] + h[n-2] + \ldots$

Impulssvar	Stegsvar ($y[n]$)	
C	I	ty $h[0]=1$ och $y[0]=1$
D	III	$h[0]=0$ och $y[0]=0$, $h[1]=y[1]=1$ $h[1], h[2], h[3] > 0 \quad h[4]=0$ $y[1] < y[2] < y[3] = y[4]$
A	IV	$h[0]=0$ och $y[0]=0$ $h[1] = y[1] \approx 0{,}5$ $h[6]=0$ och $y[5]=y[6]$
B	II	$h[0]=0$ och $y[0]=0$ Stämmer med $y[n] = \sum_{k=-\infty}^{n} h[k]$

Svar:

A	B	C	D
IV	II	I	III

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1b)

$$x(t) = 2\cos(12t) + 4\sin(48t) + 6\cos\left(8t + \frac{\pi}{3}\right)$$

$$\omega_1 = 12 \Rightarrow T_1 = \frac{2\pi}{12}$$

$$\omega_2 = 48 \Rightarrow T_2 = \frac{2\pi}{48}$$

$$\omega_3 = 8 \Rightarrow T_3 = \frac{2\pi}{8}$$

$$T_0 = k_1 \cdot T_1 = k_2 \cdot T_2 = k_3 \cdot T_3 \quad , k_i \in \mathbb{Z}$$

$$2\pi\left(\frac{k_1}{\omega_1} = \frac{k_2}{\omega_2} = \frac{k_3}{\omega_3}\right) = 2\pi\left(\frac{k_1}{12} = \frac{k_2}{48} = \frac{k_3}{8}\right)$$

Multiplicera med 96 $(= 48 \cdot 2 = 12 \cdot 8 = 8 \cdot 12)$

$$\frac{96k_1}{12} = \frac{96k_2}{48} = \frac{96k_3}{8} \Rightarrow 8k_1 = 2k_2 = 12k_3$$

Dela med 2 $\quad 4k_1 = k_2 = 6k_3$

$$k_3 = 2 \Rightarrow k_2 = 12 \Rightarrow k_1 = \frac{6 \cdot 2}{4} = 3$$

$$\left.\begin{aligned} T_0 &= 3 \cdot T_1 = 3 \cdot \frac{2\pi}{12} = \frac{\pi}{2} \\ T_0 &= 12 \cdot T_2 = 12 \cdot \frac{2\pi}{48} = \frac{\pi}{2} \\ T_0 &= 2 \cdot T_3 = 2 \cdot \frac{2\pi}{8} = \frac{\pi}{2} \end{aligned}\right\} T_0 = \frac{\pi}{2} \text{ s}$$

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2. Poler: $s_1 = -2$, $s_2 = -1+j3$, $s_3 = s_2^* = -1-j3$

$$H(s) = \frac{H_0}{(s-s_1)(s-s_2)(s-s_3)} = \frac{H_0}{(s+2)(s+1-j3)(s+1+j3)} =$$

$$= \frac{H_0}{(s+2)\left[(s+1)^2+3^2\right]} = \frac{H_0}{(s+2)(s^2+2s+10)} =$$

$$= \frac{H_0}{s^3+4s^2+14s+20}$$

Frekvenssvar: $H(j\omega) = H(s)\big|_{s=j\omega}$

$$H(j\omega) = \frac{H_0}{(2+j\omega)(10-\omega^2+j2\omega)}$$

$$|H(j\omega)| = \frac{H_0}{\sqrt{4+\omega^2}\cdot\sqrt{(10-\omega^2)^2+4\omega^2}} \to 2 \text{ då } \omega \to 0$$

$$\Rightarrow |H(j\omega)|_{\omega=0} = \frac{H_0}{2\cdot 10} = 2 \Rightarrow H_0 = 40$$

Insignal $x(t) = 5\cos(4t)$

$$|H(j\omega)|_{\omega=4} = \frac{40}{\sqrt{4+16}\,\sqrt{(10-16)^2+4\cdot 16}} =$$

$$= \frac{40}{\sqrt{20}\cdot\sqrt{(16+64)}} = \frac{40}{\sqrt{20\cdot 80}} = \frac{40}{40} = 1$$

$$\arg\{H(j\omega)|_{\omega=4}\} = -\arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{2\omega}{10-\omega^2}\right) = \{\omega=4\} =$$

$$= -\arctan(2) - \arctan(2) = -2\arctan(2) =$$

$$= -127^\circ$$

Utsignalen $y(t) = |H(j\omega)|\, 5\cos(\omega t + \arg\{H(j\omega)\}) =$

$$= 5\cos(4t - 127^\circ)$$

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3. $h[n] = \frac{1}{2}\left[(0{,}6)^n + (-0{,}2)^n\right] u[n]$ z-transf.

$$H(z) = \frac{1}{2} \cdot \frac{z}{z-0{,}6} + \frac{z}{z+0{,}2}$$

Insignal ett steg $x[n] = u[n] \overset{Z}{\longleftrightarrow} X(z) = \frac{z}{z-1}$

Utsignalens transform $Y(z) = H(z) \cdot X(z)$

$$\frac{Y(z)}{z} = \frac{1}{2}\left[\underbrace{\frac{z}{(z-0{,}6)(z-1)}}_{Y_1(z)} + \underbrace{\frac{z}{(z+0{,}2)(z-1)}}_{Y_2(z)}\right] \quad : \text{P.B.U.}$$

Y_1: $\frac{z}{(z-0{,}6)(z-1)} = \frac{A_1}{z-0{,}6} + \frac{B_1}{z-1}$

$z = A_1(z-1) + B_1(z-0{,}6)$, $z = 0{,}6 \Rightarrow 0{,}6 = A_1(0{,}6-1) \Rightarrow A_1 = -\frac{3}{2}$

$z = 1 \Rightarrow 1 = B_1 \cdot 0{,}4 \Rightarrow B_1 = \frac{5}{2}$

Y_2: $\frac{z}{(z+0{,}2)(z-1)} = \frac{A_2}{z+0{,}2} + \frac{B_2}{z-1}$

$z = A_2(z-1) + B_2(z+0{,}2)$, $z = -0{,}2 \Rightarrow -0{,}2 = A_2(-1{,}2) \Rightarrow A_2 = \frac{1}{6}$

$z = 1 \Rightarrow 1 = B_2 \cdot 1{,}2 \Rightarrow B_2 = \frac{5}{6}$

$$Y(z) = \frac{A_1}{2} \cdot \frac{z}{z-0{,}6} + \frac{B_1}{2} \cdot \frac{z}{z-1} + \frac{A_2}{2}\,\frac{z}{z+0{,}2} + \frac{B_2}{2} \cdot \frac{z}{z-1} =$$

$$= \frac{B_1+B_2}{2} \cdot \frac{z}{z-1} + \frac{A_1}{2}\,\frac{z}{z-0{,}6} + \frac{A_2}{2} \cdot \frac{z}{z+0{,}2} =$$

$$= \frac{5}{3} \cdot \frac{z}{z-1} - \frac{3}{4} \cdot \frac{z}{z-0{,}6} + \frac{1}{12}\,\frac{z}{z+0{,}2} \xrightarrow{Z^{-1}}$$

$$y[n] = \left[\frac{5}{3} - \frac{3}{4}(0{,}6)^n + \frac{1}{12}(-0{,}2)^n\right] u[n]$$

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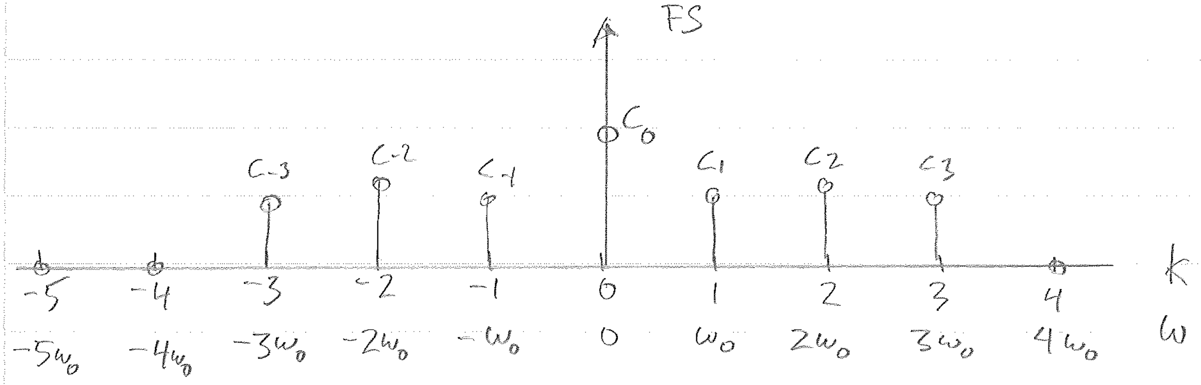
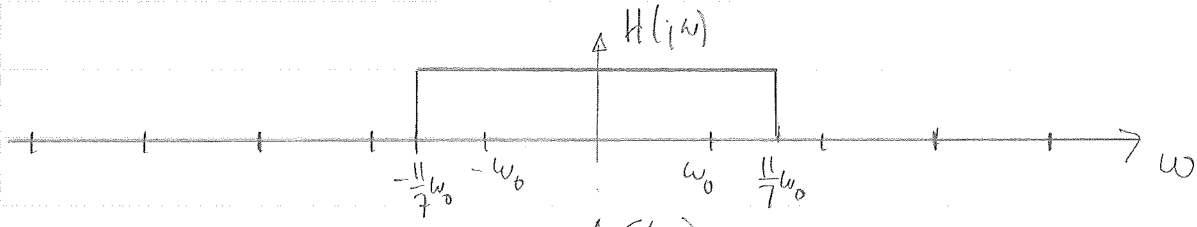
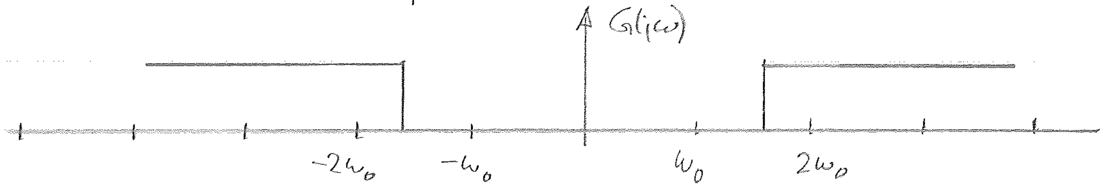
4.







$$x(t) \longrightarrow \boxed{G(j\omega)} \longrightarrow y(t)$$

$G(j\omega)$ släpper endast igenom vinkelfrekv. $|\omega| > \frac{11}{7}\omega_0$

FS-koeff till $y(t)$ blir då $\begin{cases} c_2, c_{-2}, c_3 \text{ och } c_{-3} \\ \text{övriga } c_k = 0 \end{cases}$

Medeleffekt $\quad P = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$P_y = 2|c_2|^2 + 2|c_3|^2 = 2 \cdot 0{,}5^2 + 2 \cdot 0{,}2^2 = 0{,}58$$

$$P_x = P_y + 2|c_1|^2 + |c_0|^2 = P_y + 2 \cdot 1^2 + 2^2 =$$
$$= P_y + 6 = 6{,}58$$

$$\frac{P_y}{P_x} = \frac{P_y}{P_y + 6} = \frac{1}{1 + \frac{6}{0{,}58}} = 0{,}088$$

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5. $x[n] = (1, 2, 1, 0, 1, 2, 1, 0)$

$N = 8$, $f_s = 200$ Hz, $\Delta f = \frac{200}{8} = 25$ Hz

$f = 75$ Hz $= k \cdot \Delta f = 3 \cdot 25 \Rightarrow k = 3$

Alltså, beräkna $X[3]$

$$X[k] = \sum_{n=0}^{7} x[n] e^{-j\frac{2\pi}{8} \cdot k \cdot n}$$

$$X[3] = \sum_{n=0}^{7} x[n] e^{-j\frac{3\pi}{4} \cdot n} =$$

$$= \sum_{n=0}^{7} x[n] \left(\cos \frac{3\pi}{4} \cdot n - j \sin\left(\frac{3\pi}{4} \cdot n\right)\right)$$

Beräkna varje term i summan:

$n = 0$: $1 \cdot (\cos(0) - j \sin(0)) = 1 + 0$

$n = 1$: $2 \cdot \left(\cos \frac{3\pi}{4} - j \sin \frac{3\pi}{4}\right) = 2\left(-\frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}}\right)$

$n = 2$: $1 \cdot (\cos(3\pi/2) - j \sin(3\pi/2)) = 1 \cdot (0 + j)$

$n = 3$: $0 \cdot \left(\cos\left(\frac{6\pi}{4}\right) - j \sin\left(\frac{6\pi}{4}\right)\right) = 0$

$n = 4$: $1 \cdot (\cos(3\pi) - j \sin(3\pi)) = -1 + 0$

$n = 5$: $2 \cdot \left(\cos\left(\frac{3\pi}{4} \cdot 5\right) - j \sin\left(\frac{3\pi}{4} \cdot 5\right)\right) = 2\left(\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}\right)$

$n = 6$: $1 \cdot \left(\cos\left(\frac{9\pi}{2}\right) - j \sin\left(\frac{9\pi}{2}\right)\right) = 1 \cdot (0 - j)$

$n = 7$: $0 \cdot (\ldots\ldots) = 0$

$\sum = 0$

Svar: $X[3] = 0$

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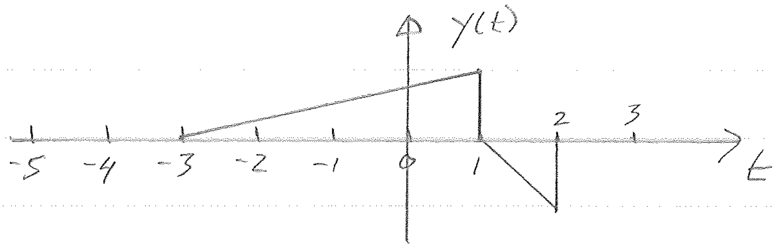

b) Minsta gemensamma period $N = 20$

2. $$H(z) = \frac{1.6 \cdot z^{-1}}{1 - 0.2z^{-1}}$$

$$y[n] - 0.2y[n-1] = 1.6\, x[n-1]$$

3. $$H(s) = \frac{25}{s^2 + 10s + 125} = \frac{25}{(s+5)^2 + 10^2}$$

$$h(t) = 2.5\, e^{-5t} \sin(10t) \cdot u(t)$$

4. 400 Hz svarar mot $k = 2$

$$X[2] = \ldots\ldots = -2(1+j)$$

5. $n=1$ $$B_1^Y = |B_1| \cdot |G(j\omega_0)| = \frac{2}{\pi} \cdot \frac{400}{10^2+20^2} = \frac{8}{5\pi}$$

$n=2$ $$B_2^Y = |B_2| \cdot |G(j2\omega_0)| = \frac{1}{\pi} \cdot \frac{400}{20^2+20^2} = \frac{1}{2\pi}$$

$n=3$ $$B_3^Y = |B_3| \cdot |G(j3\omega_0)| = \frac{2}{3\pi} \cdot \frac{400}{20^2+30^2} = \frac{2}{3\pi} \cdot \frac{4}{13}$$

$$\left\{ \omega_0 = 10, \quad G(j\omega) = \frac{400}{(j\omega+20)^2} \right\}$$