D3

Transformer, Signaler & System

SSY080 131025

1. Samplingsvinkelfrekv. $\omega_s = \frac{2\pi}{T}$

a) Signal $x_1(t)$: Periodtid $T_1 = 3T = \frac{2\pi}{\omega_1} = 3 \cdot \frac{2\pi}{\omega_s}$

$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{3T} = \frac{\omega_s}{3}$$

Signal $x_2(t)$ Periodtid T_2: $8T_2 = 12T$, $T_2 = \frac{12}{8}T = \frac{3}{2}T$

$$\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi \cdot 2}{3T} = \frac{2}{3}\omega_s$$

$T = \frac{1}{3} \cdot 10^{-3}$ s $\Rightarrow \omega_s = \frac{2\pi}{T} = 3 \cdot 10^3 \cdot 2\pi$ r/s

$\omega_1 = \frac{\omega_s}{3} = 2\pi \cdot 10^3$ r/s ("Ingen Aliasing") $\omega_1 < \frac{\omega_s}{2}$

$\omega_2 = \frac{2}{3}\omega_s = 4\pi 10^3$ r/s ("Aliasing") $\omega_2 > \frac{\omega_s}{2}$

b) $\omega_M = 10 \cdot 10^3$ r/s

$\omega_s = 4\omega_M = 40 \cdot 10^3$ r/s

Frekvensupplösning $\Delta\omega = \frac{\omega_s}{N} \leq 10$

$\therefore N \geq \frac{\omega_s}{10} = \frac{40 \cdot 10^3}{10} = 4000$

Närmast större jämna tvåpot. $2^{12} = 4096$

Svar: Välj $N = 4096$ och $m = 12$

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2.

$$H(s) = K \cdot \frac{(s-c_1)(s-c_2)}{(s-p_1)(s-p_2)} =$$

$$= K \frac{(s-1-3j)(s-1+3j)}{(s+1-3j)(s+1+3j)} = K \frac{(s-1)^2-(3j)^2}{(s+1)^2-(3j)^2} =$$

$$= K \frac{s^2-2s+10}{s^2+2s+10}$$

$s = j\omega$

$$H(j\omega) = K \frac{-\omega^2 - j2\omega + 10}{-\omega^2 + j2\omega + 10} = K \cdot \frac{10-\omega^2 - j2\omega}{10-\omega^2 + j2\omega}$$

$$H(j\omega)\Big|_{\omega \to 0} = K \cdot \frac{10}{10} = 5 \Rightarrow K = 5$$

a) $$H(s) = 5 \cdot \frac{s^2-2s+10}{s^2+2s+10}$$

b) $$H(j\omega) = 5 \frac{10-\omega^2 - j2\omega}{10-\omega^2 + j2\omega}$$

Amplitudkar: $$|H(j\omega)| = 5 \cdot \frac{\sqrt{(10-\omega^2)^2 + (2\omega)^2}}{\sqrt{(10-\omega^2)^2+(2\omega)^2}} = 5$$

$\therefore$ Allpass: Samma amplitudförst. för alla ω

Faskar: $$\arg\{H(j\omega)\} = \arctan\left(\frac{-2\omega}{10-\omega^2}\right) - \arctan\left(\frac{2\omega}{10-\omega^2}\right) =$$

$$= -\arctan\left(\frac{2\omega}{10-\omega^2}\right) - \arctan\left(\frac{2\omega}{10-\omega^2}\right) =$$

$$= -2\arctan\left(\frac{2\omega}{10-\omega^2}\right)$$

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3. $y[n] - 0{,}5\,y[n-1] = 5x[n] - 4x[n-1]$

z-transformera

$$Y(z)(1 - 0{,}5z^{-1}) = X(z)(5 - 4z^{-1})$$

Insignal: $x[n] = -\left(\frac{1}{2}\right)^n = -\dfrac{1}{1-\frac{1}{2}z^{-1}}$

$$Y(z) = X(z)\frac{5-4z^{-1}}{1-0{,}5z^{-1}} = -\frac{5-4z^{-1}}{(1-0{,}5z^{-1})(1-0{,}5z^{-1})} =$$

$$= -z \cdot \underbrace{\frac{5z-4}{(z-0{,}5)^2}}_{\text{P.B.U}}$$

$$\frac{5z-4}{(z-0{,}5)^2} = \frac{A}{z-0{,}5} + \frac{B}{(z-0{,}5)^2}$$

$5z-4 = A(z-0{,}5) + B$, $\quad z^1: \; 5 = A$

$z^0: \; -4 = -0{,}5A + B$

$B = -4 + 0{,}5A = -1{,}5 = -\frac{3}{2}$

$$Y(z) = \frac{3}{2}\frac{z}{(z-0{,}5)^2} - 5\frac{z}{z-0{,}5} =$$

$$= \frac{3}{2 \cdot 0{,}5} \cdot \frac{0{,}5z}{(z-0{,}5)^2} - 5\frac{z}{z-0{,}5}$$

Inv. z-transf. (Beta)

$$y[n] = \left[3n(0{,}5)^n - 5(0{,}5)^n\right]u[n] =$$

$$= (3n-5)0{,}5^n\, u[n]$$

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4. $y(t) = 10e^{-t}\cos(4t)\,u(t)$

$x(t) = e^{-t}\,u(t)$

Laplacetransformera!

$$y(t) \xleftrightarrow{\mathcal{L}} Y(s) = 10\,\frac{s+1}{(s+1)^2+16}$$

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s+1}$$

$$\underset{X(s)}{x(t)} \longrightarrow \boxed{H(s)} \longrightarrow \underset{Y(s)}{y(t)} \qquad Y(s) = H(s)\,X(s)$$

a/

$$H(s) = \frac{Y(s)}{X(s)} = 10\cdot\frac{(s+1)^2}{(s+1)^2+16} = 10\cdot\frac{s^2+2s+1}{s^2+2s+17}$$

b/ Impulssvar $h(t) = \mathcal{L}^{-1}\{H(s)\}$

$$H(s) = 10\cdot\frac{(s+1)^2+16-16}{(s+1)^2+16} = 10\left(1-\frac{16}{(s+1)^2+4^2}\right)$$

$$H(s) = 10\left(1-4\cdot\frac{4}{(s+1)^2+4^2}\right)$$

Inv. Laplace

$$h(t) = 10\,\delta(t) - 40e^{-t}\sin(4t)\cdot u(t)$$

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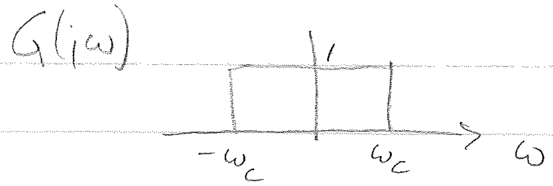
5.

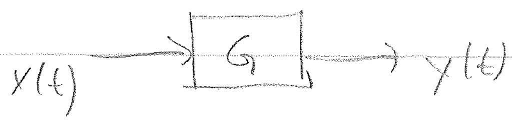
Signal: $x(t) = 2e^{-0{,}2t}\, u(t)$

Total energi $E = \int_0^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$ (Parseval)

där $X(j\omega) = \mathcal{F}\{x(t)\} = \frac{2}{j\omega + 0{,}2}$

$$E = \int_0^{\infty} (2e^{-0{,}2t})^2 dt = 4\left[\frac{e^{-0{,}4t}}{-0{,}4}\right]_0^{\infty} = 4\left(0 - \frac{1}{-0{,}4}\right) = 10$$





Utsignalens energi
$E_y = \frac{1}{2}E$

Från Parsevals relation

$$\frac{E}{2} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} |X(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{4}{0{,}2^2 + \omega^2} d\omega = \{\text{"jämn"}\} =$$

$$= \frac{4}{2\pi}\cdot 2\int_0^{\omega_c} \frac{1}{0{,}2^2 + \omega^2} d\omega = \frac{4}{\pi}\cdot\frac{1}{0{,}2}\left[\operatorname{atan}\frac{\omega}{0{,}2}\right]_0^{\omega_c} =$$

$$= \frac{20}{\pi}\operatorname{atan}\frac{\omega_c}{0{,}2} = 5$$

$$\operatorname{atan}\frac{\omega_c}{0{,}2} = \frac{5\pi}{20} = \frac{\pi}{4}$$

$$\tan\frac{\pi}{4} = 1 \Rightarrow \frac{\omega_c}{0{,}2} = 1$$

Svar:

$\omega_c = 0{,}2$ rad/s

1a/ $x(t) = 5\cos\left(\omega_0 t + \frac{\pi}{4}\right) + 2\sin(3\omega_0 t)$

Euler:

$$x(t) = 5 \cdot \frac{1}{2}\left(e^{j(\omega_0 t + \frac{\pi}{4})} + e^{-j(\omega_0 t + \frac{\pi}{4})}\right) + \frac{2}{2j}\left(e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right)$$

$$= \frac{5}{2} e^{j\frac{\pi}{4}} e^{j\omega_0 t} + \frac{5}{2} e^{-j\frac{\pi}{4}} e^{-j\omega_0 t} + \frac{1}{j} e^{j3\omega_0 t} - \frac{1}{j} e^{-j3\omega_0 t}$$

Jämför $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

Vi ser att $c_1 = \frac{5}{2} e^{j\frac{\pi}{4}}$ [k=1]

$c_{-1} = \frac{5}{2} e^{-j\frac{\pi}{4}} = c_1^*$ [k=-1]

$c_3 = \frac{1}{j} = -j$ [k=3]

$c_{-3} = -\frac{1}{j} = j = c_3^*$ [k=-3]

övriga $c_k = 0$

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1b)
$$H(s) = \frac{\omega_0}{s+\omega_0} = \frac{1}{1+\frac{s}{\omega_0}}$$

Frekvenssvar $H(j\omega) = H(s)\Big|_{s=j\omega}$

$$H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$$

Amplitud påverkan: $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$

Fasbidrag $\phi = \arg\{H(j\omega)\} = -\arctan\left(\frac{\omega}{\omega_0}\right)$

$\omega = \omega_0$: $H_0 = |H(j\omega_0)| = \frac{1}{\sqrt{2}}$

$\phi_0 = -\arctan(1) = -\frac{\pi}{4}$ (-45°)

$\omega = 3\omega_0$ $H_3 = |H(j3\omega_0)| = \frac{1}{\sqrt{10}}$ $(-71{,}6^\circ)$

$\phi_3 = -\arctan(3) = -1{,}25 \text{ rad} \approx -0{,}4\pi$

$$y(t) = 5 \cdot H_0 \cos\left(\omega_0 t + \frac{\pi}{4} + \phi_0\right) + 2 \cdot H_3 \sin(3\omega_0 t + \phi_3) =$$

$$= \frac{5}{\sqrt{2}} \cos\Big(\omega_0 t + \underbrace{\frac{\pi}{4} - \frac{\pi}{4}}_{=0}\Big) + \frac{2}{\sqrt{10}} \sin(3\omega_0 t - 0{,}4\pi)$$

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2. $$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 10y(t) = 8x(t) + \frac{dx(t)}{dt}$$

Laplacetransformera!

$$s^2Y(s) + 7sY(s) + 10Y(s) = 8X(s) + sX(s)$$

$$Y(s)(s^2+7s+10) = X(s)(s+8)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+8}{s^2+7s+10} = \ldots = \frac{s+8}{(s+2)(s+5)}$$

Insignal $u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} = X(s)$

PBU!

$$Y(s) = X(s)\cdot H(s) = \frac{s+8}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$s+8 = A(s+2)(s+5) + Bs(s+5) + Cs(s+2)$$

$s=0 \Rightarrow 8 = A\cdot 2\cdot 5 = 10A \quad \therefore A = 0.8 = \frac{4}{5}$

$s=-2 \Rightarrow 6 = B(-2)(3) \quad \therefore B = -1$

$s=-5 \Rightarrow 3 = C(-5)(-3) \quad \therefore C = \frac{1}{5} = 0.2$

$$Y(s) = \frac{4}{5}\cdot\frac{1}{s} - \frac{1}{(s+2)} + \frac{1}{5}\frac{1}{(s+5)}$$

Inv. Laplacetransf.

$$y(t) = \left(\frac{4}{5} - e^{-2t} + \frac{1}{5}e^{-5t}\right)u(t)$$

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3. $y[n] - 0{,}25\, y[n-1] = 2x[n] - x[n-1]$

z-transformera! Antag $y[-1] = 0$

$$Y(z)\left(1 - 0{,}25 z^{-1}\right) = X(z)\left(2 - z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-1}}{1 - 0{,}25 z^{-1}}$$

Insignal $x[n] = \left(-\frac{1}{4}\right)^n u[n] \stackrel{Z}{\longleftrightarrow} X(z) = \dfrac{1}{1 + 0{,}25 z^{-1}}$

$$Y(z) = H(z) \cdot X(z)$$

$$Y(z) = \frac{2 - z^{-1}}{(1 - 0{,}25 z^{-1})(1 + 0{,}25 z^{-1})} = z \cdot \frac{2z - 1}{(z - 0{,}25)(z + 0{,}25)}$$

P.B.U. $\dfrac{2z - 1}{(z - 0{,}25)(z + 0{,}25)} = \dfrac{A}{z - 0{,}25} + \dfrac{B}{z + 0{,}25}$

$$2z - 1 = A(z + 0{,}25) + B(z - 0{,}25)$$

$z = 0{,}25 \Rightarrow -0{,}5 = A(0{,}5) \Rightarrow A = -1$

$z = -0{,}25 \Rightarrow -1{,}5 = B(-0{,}5) \Rightarrow B = 3$

$$Y(z) = 3\,\frac{z}{z + 0{,}25} - \frac{z}{z - 0{,}25}$$

Inv. z-transf.

$$y[n] = \left[3\left(-\frac{1}{4}\right)^n - \left(\frac{1}{4}\right)^n\right] u[n]$$

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4.

Studera $|X[k]|$, $N=32$

$N=32$ Vi har fyra värden på $|X[k]|$ som är betydligt större än övriga: se $k = 8, 13, 19$ och 24

$k=32$ svarar mot samplingsfrekvensen $f_s = 200$ Hz

Skillnad i frekvens mellan två intill liggande värden på $X[k]$ är

$$\Delta f = \frac{f_s}{N} \quad \text{(frekvensupplösning)}$$

Ett k-värde svarar då mot frekvensen $f_k = \frac{k}{N} \cdot f_s$

$k = 8$ och 24 $(N-8=24)$ svarar mot en reell sinusformad signal med frekvensen

$$f_8 = \frac{8}{32} \cdot 200 = 50 \text{ Hz}$$ (vår brum-signal)

$k = 13$ och 19 $(N-13=19)$ svarar då mot den sinusformade signalen $g(t)$ med

$$f_g = \frac{13}{32} \cdot 200 \approx 81{,}3 \text{ Hz} \quad \left(< \frac{f_s}{2}\right)$$

Svar: Den sinusformade signalen $g(t)$ har frekvensen 81 Hz

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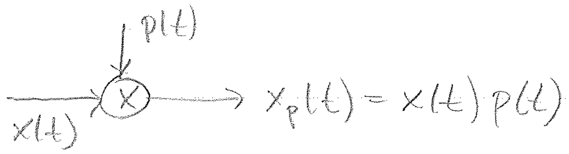
5.



Fouriertransformera

$$x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$$
$$p(t) \xleftrightarrow{\mathcal{F}} P(j\omega)$$
$$x_p(t) \xleftrightarrow{\mathcal{F}} X_p(j\omega)$$

Egenskap $\quad x_p(t) = x(t)\,p(t) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

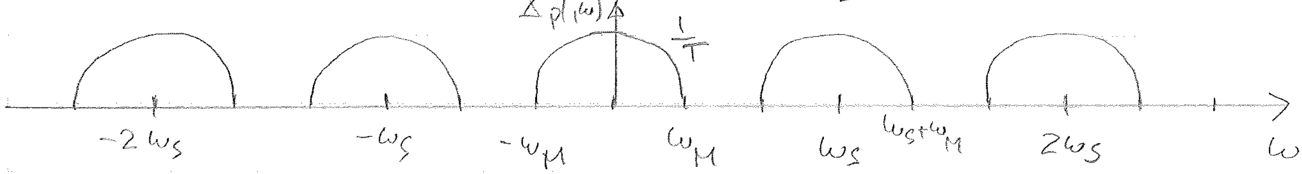
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \xleftrightarrow{\mathcal{F}} P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

där $\omega_s = \frac{2\pi}{T}$

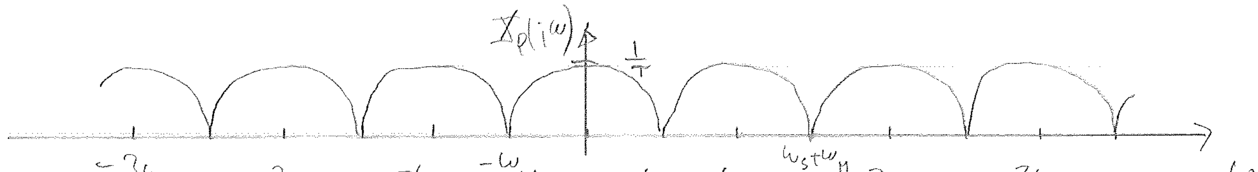
Alltså $X(j\omega)$ faltas med alla impulser i $P(j\omega)$

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

a/ $\quad \omega_s = \frac{2\pi}{T} = 2\pi \cdot 10^3 \qquad \frac{\omega_s}{\omega_M} = \frac{2\pi \cdot 10^3}{\frac{2\pi \cdot 10^3}{3}} = 3 \quad ; \ \omega_M = \frac{1}{3}\omega_s$



b/ $\quad \omega_s = \frac{2\pi}{T} = \frac{2\pi}{\frac{3}{2} \cdot 10^{-3}} = \frac{2\pi \cdot 10^3 \cdot 2}{3} \ ; \ \frac{\omega_s}{\omega_M} = \frac{\frac{2\pi \cdot 10^3 \cdot 2}{3}}{\frac{2\pi \cdot 10^3}{3}} = 2 \quad \omega_M = \frac{1}{2}\omega_s$



$\omega_M = \frac{2000\pi}{3}$ rad/s i både a/ och b/ | ω_s ej lika i a/ och b/.

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1 a/ $\omega_0 = 100\pi$ r/s

b/ $c_0 = 1$, $c_3 = c_{-3}^* = e^{j\frac{\pi}{4}}$, $c_5 = c_{-5}^* = -\frac{j}{2}$, övriga $c_k = 0$

c/ Parsevals formel $\sum_{k=-\infty}^{\infty} |c_k|^2 = \ldots = 3{,}5$

2/ a/ $H(z) = \frac{3(z-1)}{z+0{,}9}$

b/ Nollställe: $z = 1$, Pol $z = -0{,}9$

c/ $h[n] = \mathcal{Z}^{-1}\{H(z)\} = 3\delta[n] - 1{,}9(-0{,}9)^{n-1}u[n-1] =$
$= \{\text{alternativt}\} = 3(-0{,}9)^n u[n] - 3(-0{,}9)^{n-1} u[n-1]$

3/ $H(s) = \frac{2(s+3)}{(s+1)(s+6)} = \ldots = \frac{4}{5}\cdot\frac{1}{s+1} + \frac{6}{5}\cdot\frac{1}{s+6}$

$h(t) = \mathcal{L}^{-1}\{H(s)\} = \left(\frac{4}{5}e^{-t} + \frac{6}{5}e^{-6t}\right)u(t)$

4/ a/ 16st b/ $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} = \frac{125\pi}{8}$ r/s

c/ $\Omega_0 = \omega_0 \cdot T_s = \pi/32$ radianer/sampel

d/ $k = 16$ e/ $k = 16 \Rightarrow \omega = \omega_0$

5/ $H(j\omega) = \frac{10+j\omega}{5+j\omega}$

$x_1(t) = 1 \Rightarrow y_1(t) = 1\cdot|H(j\omega)|_{\omega=0} = 2$

$x_2(t) = 2\cos(100t) \Rightarrow y_2(t) = 2\cdot|H(j\omega)|_{\omega=100}\cos(100t + \arg\{H(j\omega)|_{\omega=100}\})$

$x_3(t) = \delta(t-1) \Rightarrow y_3(t) = h(t-1)$

$y = y_1 + y_2 + y_3 = 2 + 2{,}008\cos(100t - 0{,}05) + \delta(t-1) + 5e^{-5(t-1)}u(t-1)$