Transformer, Signaler & System för D3
SSY080 2011-10-19

1a) Icke linjärt

b) $X_3[n]$ släcks ut $\left(\Omega = \pm \frac{3\pi}{4}\right)$

2) $H(s) = \dfrac{1}{(s+2)(s+4)}$, $X(s) = \dfrac{1}{s+2}$

$$Y(s) = \frac{1}{2}\cdot\frac{1}{(s+2)^2} - \frac{1}{4}\cdot\frac{1}{(s+2)} + \frac{1}{4}\cdot\frac{1}{(s+4)}$$

$$y(t) = \frac{1}{2}\left(\left(t-\frac{1}{2}\right)e^{-2t} + \frac{1}{2}e^{-4t} \right) u(t)$$

3) $$Y_s(s) = \frac{z}{z-1}\cdot\frac{1}{2}\left(1 + \frac{z-1}{z-\frac{1}{3}} - \frac{2(z-1)}{z-\frac{1}{2}}\right)$$

$$Y_s(s) = \frac{z}{z-1}\cdot H(z) \Rightarrow H(z) = \frac{1}{6}\cdot\frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \left[\left(\tfrac{1}{2}\right)^n - \left(\tfrac{1}{3}\right)^n\right] u[n]$$

4a)

	k	N−k
E	338	1710
H	253	1795
G	201	1847

b) $\Delta f = \frac{f_s}{N} = 0{,}977\,Hz$

5) (Jämför "Hem-lab")

$$|B_1^Y| = |B_1^X|\cdot|G(j\omega)|_{\omega=\omega_0} = \frac{2}{\pi}\cdot 0{,}164$$

$$|B_2^Y| = |B_2^X|\cdot|G(j\omega)|_{\omega=2\omega_0} = \frac{1}{\pi}\cdot 1$$

$$|B_3^Y| = \;.\;.\;.\;.\;.\;.\;.\; = \frac{2}{3\pi}\cdot 0{,}287$$

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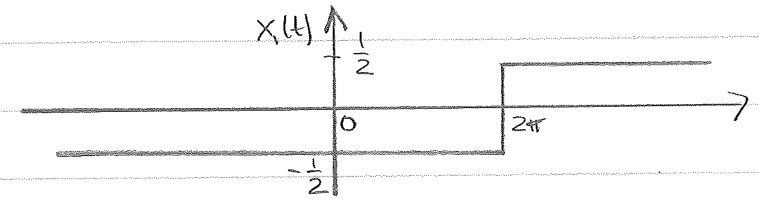
1 a/ $x_1(t) = u(t-2\pi) - \frac{1}{2}$, $\forall t$



Ej periodisk!

b/ $x_2[n+N] = 3\cos\left(\frac{(n+N)\pi}{3} - 7\right) =$

$= 3\cos\left(\frac{n\pi}{3} - 7 + \frac{N\pi}{3}\right)$

Periodisk om $\frac{N\pi}{3} = 2\pi k \Rightarrow N = 6k$ (Låt $k=1$)

Periodisk med $N=6$

c/ $y[n] = x[7n]$

Insignal	Utsignal
$x_1[n]$	$x_1[7n] = y_1[n]$
$a_1 x_1[n]$	$a_1 x_1[7n] = a_1 y_1[n]$
$x_2[n]$	$x_2[7n] = y_2[n]$
$a_2 x_2[n]$	$a_2 x_2[7n] = a_2 y_2[n]$
$x_3[n] =$ $= a_1 x_1[n] + a_2 x_2[n]$	$y_3[n] = x_3[7n] = a_1 x_1[7n] + a_2 x_2[7n] =$ $= a_1 y_1[n] + a_2 y_2[n]$

Linjärt? Ja!

2)
$$\frac{d^2 V_c(t)}{dt^2} + \frac{R}{L}\frac{dV_c(t)}{dt} + \frac{1}{LC}V_c(t) = \frac{1}{LC}V(t)$$

Laplace transf.

$$s^2 V_c(s) + s\frac{R}{L}V_c(s) + \frac{1}{LC}V_c(s) = \frac{1}{LC}V(s)$$

a)
$$H(s) = \frac{V_c(s)}{V(s)} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}} = \frac{9680}{s^2 + s139 + 9680}$$

b) Nollställe: inga

Poler: $s^2 + s139 + 9680 = 0$; $s_{1,2} = -\frac{139}{2} \pm \sqrt{\frac{139}{2} - 9680}$

$$s_{1,2} = -69{,}5 \pm j69{.}64$$

c) Kvadratkomplettera

$$H(s) = \frac{9680}{(s+69{,}5)^2 + 9680 - 69{,}5^2} =$$

$$= \frac{9680}{(s+69{,}5)^2 + 4849{.}75} = \frac{9680 \cdot \frac{69{,}64}{69{,}64}}{(s+69{,}5)^2 + 69{,}64^2}$$

Impulssvar: $h(t) = \mathcal{L}^{-1}\{H(s)\} =$

$$= \mathcal{L}^{-1}\left\{\frac{9680}{69{,}64} \cdot \frac{69{,}64}{(s+69{,}5)^2 + 69{,}64^2}\right\} = 139e^{-69{,}5t}\sin(69{.}64t)$$

för $t \geq 0$

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3/ $h[n] = \left(\frac{1}{3}\right)^n u[n] + 4\left(\frac{1}{2}\right)^n u[n-1] =$

$= \left(\frac{1}{3}\right)^n u[n] + 2\cdot\left(\frac{1}{2}\right)^{-1}\left(\frac{1}{2}\right)^n u[n-1] =$

$= \left(\frac{1}{3}\right)^n u[n] + 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$ Z-transformera!

a/

$\hat{=}$ $H(z) = \frac{z}{z-\frac{1}{3}} + 2\frac{z}{z-\frac{1}{2}}\cdot z^{-1} = \frac{z(z-\frac{1}{2})+2(z-\frac{1}{3})}{(z-\frac{1}{3})(z-\frac{1}{2})} =$

$= \frac{z^2+\frac{3}{2}z-\frac{2}{3}}{z^2-\frac{5}{6}z+\frac{1}{6}} = \frac{1+\frac{3}{2}z^{-1}-\frac{2}{3}z^{-2}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$

b/ $H(z) = Y(z)/X(z)$

$Y(z)\left(1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}\right) = X(z)\left(1+\frac{3}{2}z^{-1}-\frac{2}{3}z^{-2}\right)$

Inv. z-transf.

$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2] = x[n]+\frac{3}{2}x[n-1]-\frac{2}{3}x[n-2]$

c/ Stabilt? Ja!

Ty kausalt system samt poler till $H(z)$ ligger innanför enhetscirkeln $|z_1| = \frac{1}{2} < 1$

$|z_2| = \frac{1}{3} < 1$

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4. Samplingsintervall $T = 6{,}25$ ms

$\Rightarrow$ samplingsfrekvens $f_s = \frac{1}{T} = 160$ Hz

DFT, $N = 64$ punker $X[k]$, $0 \le k \; N-1$

Index k motsvarar frekvensen $f_k = \frac{k}{N} \cdot f_s$

Vilket k-värde motsvarar de angivna frekvenserna

f [Hz]	"k" $\frac{f}{f_s} \cdot N$	"N-k"	I figur?
16	6,4	57,6	Ja
32	12,8	51,2	Ja
48	19,2	44,8	Nej
64	25,6	38,4	Ja
80	32	32 Nej	$= f_s/2$ ei i fig men tre topppar $\neq f_s/2$ finns
96	38,4	25,6	Ja
112	44,8	19,2	Nej
128	51,2	12,8	Ja
144	57,6	6,4	Ja

De frekvenser med ett inringat "k"-värde kan finnas med i insignalen som samplas. Vid närmaste heltalsvärde till "k" har då $|X[k]|$ ett högt värde. (en "topp").

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5. $\hat{x}(t) = A_0 + \sum_{n=1}^{\infty} \left(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right) =$

$= \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin\left(n \frac{\pi}{L} t\right) \Rightarrow$ $A_n = 0, \ \forall n$

$B_n = \frac{2}{\pi} \frac{(-1)^{n+1}}{n}$

$B_1 = \frac{2}{\pi}, \ B_2 = \frac{-1}{\pi}, \ B_3 = \frac{2}{3\pi}$ $\qquad \omega_0 = \frac{\pi}{L} = \frac{2\pi}{2L}$

$T = 2L = \frac{2\pi}{10} \ s$

Systemets frekvenssvar $\qquad \omega_0 = \frac{2\pi}{T} = 10 \ r/s$

$$G(j\omega) = G(s)\big|_{s=j\omega} = \frac{400}{(j\omega + 20)^2}$$

Amplitudkarakteristik: $|G(j\omega)| = \frac{400}{\omega^2 + 20^2}$

Amplitud hos utsignal (tre lägsta frekvenserna)

$\boxed{n=1}$ $\quad \omega = \omega_0 = 10$

$$B_1^Y = |B_1| \cdot |G(j\omega_0)| = \frac{2}{\pi} \cdot \frac{400}{10^2 + 20^2} = \frac{2}{\pi} \cdot 0{,}8 = \frac{8}{5\pi} \approx 0{,}509$$

$\boxed{n=2}$ $\quad \omega = 2\omega_0 = 20$

$$B_2^Y = |B_2| |G(j2\omega_0)| = \frac{1}{\pi} \cdot \frac{400}{20^2 + 20^2} = \frac{1}{\pi} \cdot \frac{1}{2} \approx 0{,}159$$

$\boxed{n=3}$ $\quad \omega = 3\omega_0 = 30$

$$B_3^Y = |B_3| |G(j3\omega_0)| = \frac{2}{3\pi} \cdot \frac{400}{20^2 + 30^2} = \frac{2}{3\pi} \cdot \frac{4}{13} \approx 0{,}065$$

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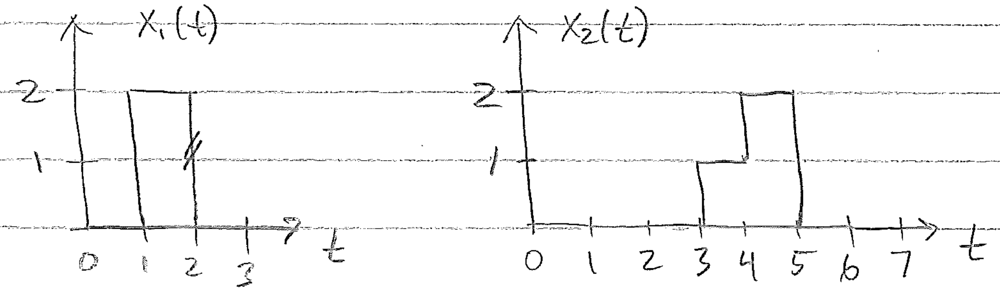
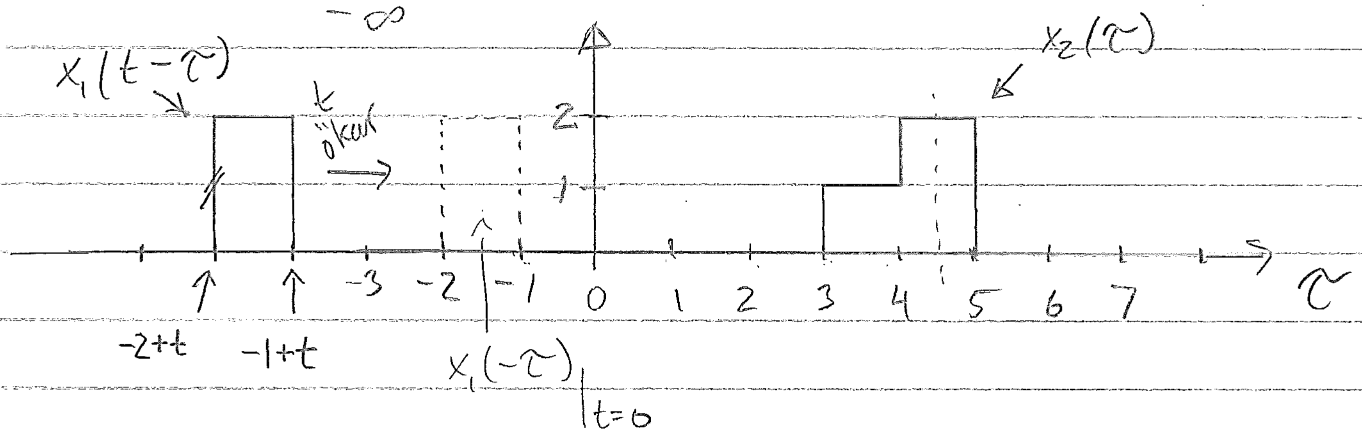
1.



$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau =$$

$$= \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$



Överlap börjar då $\tau = -1+t = 3 \Rightarrow t = 4$

Övelap avslutas då $\tau = -2+t = 5 \Rightarrow t = 7$

Överlap som ger max $x_1(t) * x_2(t)$: $\tau = -1+t = 5 \Rightarrow t = 6$

$$\text{Max} \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau \Big|_{t=6} = 2 \cdot 2 \cdot 1 = 4$$

Svar: a) $4 < t < 7$

b) $t = 6$ c) 4

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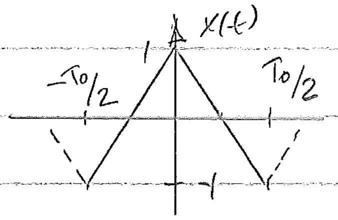
2. $$H(j\omega) = \frac{j\omega \frac{T_0}{2\pi}}{\left(j\omega\frac{T_0}{2\pi}\right)^2 + j\omega\frac{T_0}{2\pi} + 1}$$



Periodtid : $T = T_0$

Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T_0}$

$$H(j\omega) = \frac{j\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0}}$$

$$|H(jk\omega_0)| = \frac{k}{\sqrt{(1-k^2)^2+k^2}} = \frac{k}{\sqrt{1+k^4-k^2}}$$

$$x(t) = \sum_{k=1,3,5,\ldots}^{\infty} \frac{8}{(k\pi)^2}\cos(k\omega_0 t)$$

$$\arg\{H(jk\omega_0)\} = 90^\circ - \arctan\left\{\frac{k}{1-k^2}\right\}$$

Filtret (systemet) H påverkar amplitud och fas hos varje sinusformad signal i x(t) enligt

$$y(t) = \sum_{k=1,3,5,\ldots}^{\infty} \overbrace{\frac{8}{(k\pi)^2}\cdot|H(jk\omega_0)|}^{A_k} \cos\left(k\omega_0 t + \overbrace{\arg\{H(jk\omega_0)\}}^{\theta_k}\right)$$

k	ω	A_k	θ_k
1	ω_0	$\frac{8}{\pi^2}\cdot\frac{1}{1} = \frac{8}{\pi^2}$	$90^\circ - \arctan(\text{"}\infty\text{"}) = 0$
3	$3\omega_0$	$\frac{8}{\pi^2}\cdot\frac{1}{3^2}\cdot\frac{3}{\sqrt{1+3^4-3^2}} \simeq \frac{8}{\pi^2}\cdot 0{,}039$	$90^\circ - \arctan\left(\frac{3}{-8}\right) = -69{,}4^\circ$
5	$5\omega_0$	$\frac{8}{\pi^2}\cdot\frac{1}{5^2}\cdot\frac{5}{\sqrt{1+5^4-5^2}} \simeq \frac{8}{\pi^2}\cdot 8{,}2\cdot10^{-3}$	$90^\circ - \arctan\left(\frac{5}{-24}\right) = -78{,}2^\circ$

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3, $$\frac{d^2y_1(t)}{dt^2}+3\frac{dy_1(t)}{dt}+2y_1(t)=3x(t)+\frac{dx(t)}{dt}$$

Laplacetransformera!

$$s^2Y_1(s)+3sY_1(s)+2Y_1(s)=3X(s)+sX(s)$$

$$H_1(s)=\frac{Y_1(s)}{X(s)}=\frac{s+3}{s^2+3s+2}=\dots=\frac{s+3}{(s+1)(s+2)}$$

$$\mathcal{L}\{\text{"Integrator"}\}=\frac{1}{s}$$

$$Y(s)=X(s)\cdot H_1(s)\cdot\frac{1}{s}$$ Impulssvar $\Rightarrow$ $x(t)=\delta(t)\overset{\mathcal{L}}{\longleftrightarrow}X(s)=1$

$$Y(s)=\frac{s+3}{s(s+1)(s+2)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$ (P.B.U.)

$$s+3=A\underbrace{(s+1)(s+2)}_{s^2+3s+2}+Bs(s+2)+Cs(s+1)$$

s^0: $3=2A\Rightarrow A=\frac{3}{2}$

s^1: $1=3A+2B+C$

s^2: $0=A+B+C\Rightarrow C=-A-B$

$1=3A+2B-A-B$

$1+A-3A=B$; $B=1+\frac{3}{2}-\frac{9}{2}=-2$;

$C=-\frac{3}{2}+2=\frac{1}{2}$

$$Y(s)=\frac{3}{2}\cdot\frac{1}{s}-\frac{2}{s+1}+\frac{1}{2}\cdot\frac{1}{s+2}$$ Inv. Laplace ger

$$h(t)=y(t)=\left(\frac{3}{2}-2e^{-t}+\frac{1}{2}e^{-2t}\right)u(t)$$ [Impulssvar!]

Alt: $h_1(t)=\mathcal{L}^{-1}\{H_1(s)\}$

$$h(t)=\int_0^t h_1(\tau)\,d\tau$$

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4.

$$x[n] \longrightarrow \boxed{H} \longrightarrow y[n]$$

$$y[n] + \frac{1}{3} y[n-1] = 2x[n] + x[n-1]$$, z-transformera!

$$Y(z)\left(1 + \frac{1}{3} z^{-1}\right) = X(z)\left(2 + z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 + z^{-1}}{1 + \frac{1}{3} z^{-1}} = \frac{2z+1}{z + \frac{1}{3}}$$

$$x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1] \overset{Z}{\longleftrightarrow} X(z) = \frac{z}{z - \frac{1}{2}} \cdot z^{-1} = \frac{1}{z - \frac{1}{2}}$$

$$Y(z) = H(z) X(z) = \frac{2z+1}{\left(z + \frac{1}{3}\right)\left(z - \frac{1}{2}\right)} = \frac{2z+1}{\left(z^2 - \frac{z}{6} - \frac{1}{6}\right)}$$

Teckna $\frac{Y(z)}{z} = \frac{2z+1}{z\left(z + \frac{1}{3}\right)\left(z - \frac{1}{2}\right)}$ Partialbråksuppdela

$$\frac{2z+1}{z\left(z + \frac{1}{3}\right)\left(z - \frac{1}{2}\right)} = \frac{A}{z} + \frac{B}{z + \frac{1}{3}} + \frac{C}{z - \frac{1}{2}}$$

$$2z+1 = A\left(z^2 - z \cdot \frac{1}{6} - \frac{1}{6}\right) + Bz\left(z - \frac{1}{2}\right) + Cz\left(z + \frac{1}{3}\right)$$

z^0: $1 = A\left(-\frac{1}{6}\right) \Rightarrow A = -6$		$A = -6$
z^1: $2 = -\frac{1}{6}A - \frac{1}{2}B + \frac{1}{3}C$	$2 = 1 - \frac{1}{2}B + \frac{1}{3}(6 - B)$	$B = 1{,}2$
z^2: $0 = A + B + C$	$C = -A - B = 6 - B$	$C = 4{,}8$

$$Y(z) = -6 + 1{,}2 \frac{z}{z + 1/3} + 4{,}8 \frac{z}{z - \frac{1}{2}}$$

Invers z-transf. $y[n] = -6\delta[n] + \left[1{,}2\left(-\frac{1}{3}\right)^n + 4{,}8\left(\frac{1}{2}\right)^n\right] u[n]$

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5. Samplingsfrekvens $f_s = 8000$ Hz

$\omega_s = 2\pi f_s$ r/s

$T_s = \frac{2\pi}{\omega_s} = \frac{1}{f_s}$ sampelintervall

DFT : $N = 256$ punkter $X[k]$, $k = 0, 1, .., N-1$

Index k motsvarar frekvensen
$f_k = \frac{k}{N} \cdot f_s$

Ur figuren ser vi att två k-värden ger
höga $|X[k]|$

$k = 25 \Rightarrow f_{25} = \frac{25}{256} \cdot 8000 = 781$ Hz

$k = 43 \Rightarrow f_{43} = \frac{43}{256} \cdot 8000 = 1344$ Hz

Frekvensupplösning i DFT: $\Delta f = \frac{f_s}{N} = 31{,}25$ Hz

Från tabellen ser vi att bästa överens-
stämmelse fås för knapp "5".

(781 ligger i intervallet $770 \pm \Delta f$
1344 —"— —"— $1336 \pm \Delta f$)