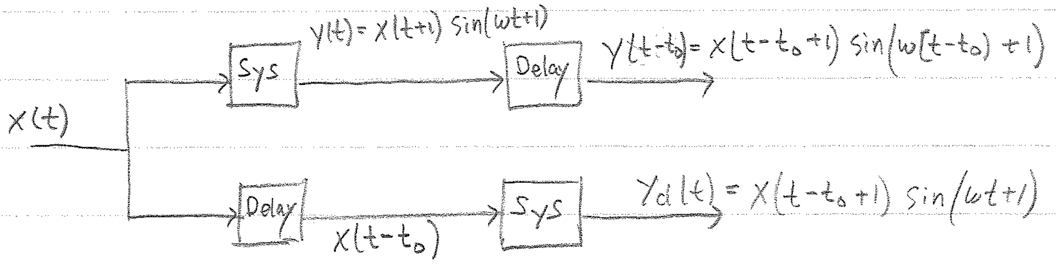
Transformer, Signaler & System
D3 SSY080 091021

1/ $y(t) = x(t+1)\sin(\omega t + 1)$

a/



$y_d(t) \neq y(t-t_0)$ Ej tidsinv.

Insignal	Utsignal
$x_1(t)$	$y_1(t) = x_1(t+1)\sin(\omega t+1)$
$x_2(t)$	$y_2(t) = x_2(t+1)\sin(\omega t+1)$
$x_3(t) = ax_1(t) + bx_2(t)$	$y_3(t) = x_3(t+1)\sin(\omega t+1) =$

b/

$$= [a x_1(t+1) + b x_2(t+1)]\sin(\omega t+1) =$$
$$= a x_1(t+1)\sin(\omega t+1) + b x_2(t+1)\sin(\omega t+1) =$$
$$= a y_1(t) + b y_2(t)$$

Linjärt!

c/ Ej kausalt
$y(t)$ beror av $x(t+1)$

d/ Stabilt!
Begränsad insignal $|x(t)| \leq M_x < \infty$, $\forall t$
$\sin(\omega t+1) \leq 1$

$$|y(t)| = |x(t+1)|\,|\sin(\omega t+1)| \leq M_x < \infty,\ \forall t$$

$|y(t)|$ begränsad då $|x(t)|$ begränsad

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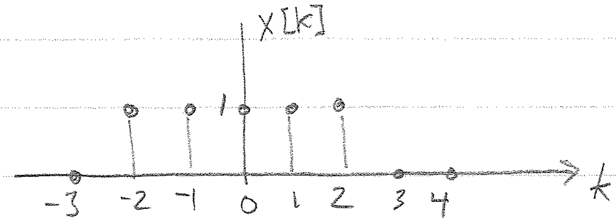
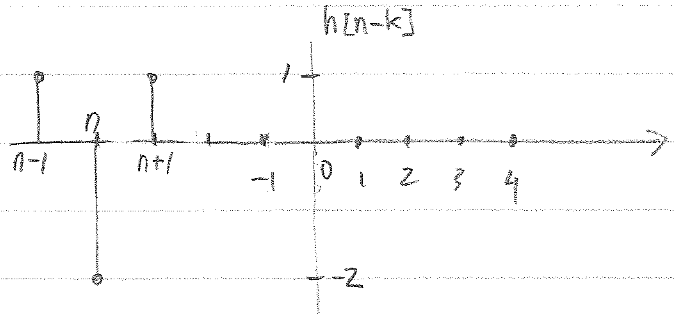
2)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





a)

"Grafisk lösning"

$y[-3] = 1 \cdot 1 = 1$

$y[-2] = -2 \cdot 1 + 1 \cdot 1 = -1$

$y[-1] = 1 \cdot 1 - 2 \cdot 1 + 1 \cdot 1 = 0$

$y[0] = 1 - 2 + 1 = 0$

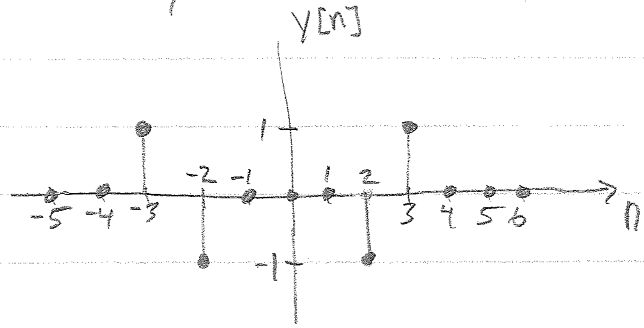
$y[1] = 1 - 2 + 1 = 0$

$y[2] = 1 \cdot 1 - 2 \cdot 1 = -1$

$y[3] = 1 \cdot 1 = 1$

övrika $y[n] = 0$

Svar a)



b) Insignal $e[n] = -\delta[n+1]$ ger utsignal $-h[n+1]$

$$y_e[n] = y[n] - h[n+1]$$

n	-4	-3	-2	-1	0	1	2	3	4
$y[n]$	0	1	-1	0	0	0	-1	1	0
$-h[n+1]$	0	0	-1	2	-1	0	0	0	0
$y_e[n]$	0	1	-2	2	-1	0	-1	1	0

(övriga $y_e[n] = 0$)

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3)

$$H_1: \quad \frac{dw(t)}{dt} + 6w(t) = \frac{dx(t)}{dt} + 5x(t)$$

Laplacetransf. $sW(s) + 6W(s) = sX(s) + 5X(s)$

$$W(s)(s+6) = X(s)(s+5)$$

$$H_1(s) = \frac{W(s)}{X(s)} = \frac{s+5}{s+6}$$

$$H_2: \quad h_2(t) = e^{-10t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} H_2(s) = \frac{1}{s+10} = \frac{Y(s)}{W(s)}$$

$$H(s) = H_1(s)H_2(s) = \frac{W(s)}{X(s)}\frac{Y(s)}{W(s)} = \frac{Y(s)}{X(s)} = \frac{s+5}{(s+6)(s+10)}$$

a)

$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{j\omega+5}{(j\omega+6)(j\omega+10)}$$

b)

$$H(s) = \frac{s+5}{(s+6)(s+10)} = \frac{A}{s+6} + \frac{B}{s+10} = \ldots = -\frac{1}{4}\cdot\frac{1}{s+6} + \frac{5}{4}\cdot\frac{1}{s+10}$$

Inv. Laplace ger $h(t) = \frac{1}{4}\left(5e^{-10t} - e^{-6t}\right)u(t)$

c)

$$H(s) = \frac{s+5}{s^2+16s+60} = \frac{Y(s)}{X(s)}$$

$Y(s)(s^2+16s+60) = X(s)(s+5)$ vilket motsvarar

$$\frac{d^2y(t)}{dt^2} + 16\frac{dy(t)}{dt} + 60y(t) = \frac{dx(t)}{dt} + 5x(t)$$

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4.
a/ $H(z) = \dfrac{1 - az^{-1}}{z^{-1} - a} =$

$$= \frac{1}{a}\left(\frac{1 - az^{-1}}{\frac{z^{-1}}{a} - 1}\right) = -\frac{1}{a}\left(\frac{1 - az^{-1}}{1 - a^{-1}z^{-1}}\right) =$$

$$= -\frac{1}{a}\left(\frac{1}{1 - a^{-1}z^{-1}} - z^{-1}\frac{a}{1 - a^{-1}z^{-1}}\right)$$

Inv. z-transf.

$$h[n] = -\frac{1}{a}(a^{-1})^{n} u[n] + (a^{-1})^{n-1} u[n-1]$$

$n=0$; $h[0] = -\dfrac{1}{a}$

$n>0$; $h[n] = -\dfrac{1}{a}\left(\dfrac{1}{a}\right)^{n} + \left(\dfrac{1}{a}\right)^{n}\left(\dfrac{1}{a}\right)^{-1} =$

$$= -\frac{1}{a^{n+1}} + \frac{1}{a^{n-1}} = -\frac{1}{a^{n+1}} + \frac{a^2}{a^2 a^{n-1}} =$$

$$= \frac{a^2 - 1}{a^{n+1}}$$

b/ Poler till $H(z)$: $z^{-1} - a = 0$

$z^{-1} = a \Rightarrow z = \dfrac{1}{a}$

Stabilt kausalt system - pol innanför enhetscirkeln

$\therefore \left|\dfrac{1}{a}\right| < 1$ alt. $|a| > 1$

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5/ Kretsekv.

$$V_1(t) = V_L(t) + V_2(t) = L\frac{di}{dt} + V_2(t) = \left\{ i = \frac{V_2(t)}{R} \right\} =$$

$$= \frac{L}{R}\frac{dV_2(t)}{dt} + V_2(t)$$

$R = 70\,\Omega$

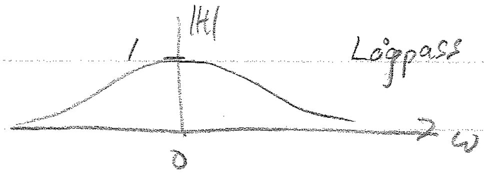
$T = 2\pi \cdot 10^{-3}$ s

$$\therefore \quad \frac{L}{R}\frac{dV_2(t)}{dt} + V_2(t) = V_1(t) \qquad \text{Fouriertransf.}$$

$$j\omega\frac{L}{R}V_2(j\omega) + V_2(j\omega) = V_1(j\omega)$$

Frekv. svar $\quad H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{1}{1 + j\omega\frac{L}{R}} = \left\{ \omega_c = \frac{R}{L} \right\} = \frac{1}{1 + j\frac{\omega}{\omega_c}}$

Ampl. kar. $\quad |H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$



Fyrkantsvåg har endast udda termer i Fourierserien

c_k : $k = 1, 3, 5, 7, \ldots$ osv. $\quad$ Högst frekv. för de fyra första termerna är $k\omega_0$ med $k = 7$ och $\omega_0 = \frac{2\pi}{T}$

Nu skall

$$\frac{1}{\sqrt{1 + \left(\frac{7\omega_0}{\omega_c}\right)^2}} \geq 0{,}8$$

$$\frac{1}{1 + \frac{(7\omega_0)^2}{\omega_c^2}} \geq 0{,}8^2$$

$$\frac{1}{0{,}8^2} \geq 1 + \left(\frac{7\omega_0}{\omega_c}\right)^2$$

$$\frac{1}{0{,}8^2} - 1 \geq \left(\frac{7\omega_0}{\omega_c}\right)^2$$

$$\sqrt{\frac{1}{0{,}8^2} - 1} \geq \frac{7\omega_0}{\omega_c} = \frac{7 \cdot 2\pi}{T \cdot R} \cdot L$$

$$L \leq \frac{T \cdot R}{7 \cdot 2\pi}\sqrt{\frac{1}{0{,}8^2} - 1} = \frac{2\pi \cdot 10^{-3} \cdot 70}{7 \cdot 2\pi}\overbrace{\sqrt{\frac{1}{0{,}8^2} - 1}}^{0.75} =$$

$$= 0{,}75 \cdot 10 \cdot 10^{-3} = 7{,}5 \cdot 10^{-3}$$

Svar $\quad L \leq 7{,}5 \cdot 10^{-3}$ H

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(1)

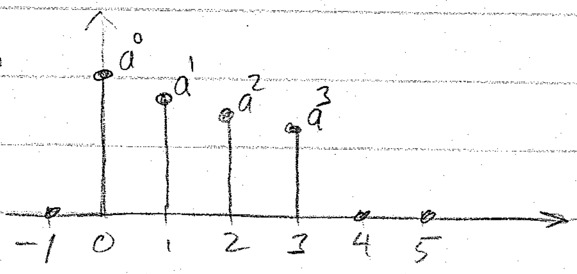
h_1: Fördröjer insignalen: Utsignal: $x[n-2]$

Insignal till h_2 blir $x[n-2] = \delta[n-2] - \delta[n-3] + \delta[n-4]$

Superposition ger

$$y[n] = h_2[n-2] - h_2[n-3] + h_2[n-4]$$

$h_2[n]$



n	-1	0	1	2	3	4	5	6	7
$h_2[n-2]$	0	0	0	a^0	a^1	a^2	a^3	0	0
$-h_2[n-3]$	0	0	0	0	$-a^0$	$-a^1$	$-a^2$	$-a^3$	0
$h_2[n-4]$	0	0	0	0	0	a^0	a^1	a^2	a^3

$\Sigma \Rightarrow y[n]$

$$y[2] = a^0 = 1$$
$$y[3] = a^1 - a^0 = a - 1$$
$$y[4] = a^2 - a^1 + a^0 = 1 + a^2 - a^1$$
$$y[5] = a^3 - a^2 + a^1 = a + a^3 - a^2$$
$$y[6] = -a^3 + a^2 = a^2 - a^3$$
$$y[7] = a^3$$
$$y[n] = 0 \text{ för övriga } n$$

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2)

Stegsvar

$$Y_s(t) = (7 - 8{,}4e^{-2t} + 1{,}4e^{-12t})u(t)$$

Laplace transformera

$$Y_s(s) = \frac{7}{s} - \frac{8.4}{s+2} + \frac{1.4}{s+12} =$$

$$= \frac{7(s+2)(s+12) - 8{,}4s(s+12) + 1{,}4s(s+2)}{s(s+2)(s+12)} =$$

$$= \ldots = \frac{168}{s(s+2)(s+12)}$$

$Y_s(s) = H(s) \cdot \frac{1}{s}$ där $X(s) = \frac{1}{s}$ Insignal ett enhetssteg

$$\therefore \; H(s) = \frac{168}{(s+2)(s+12)} = \ldots = \frac{16{,}8}{(s+2)} - \frac{16{,}8}{(s+12)}$$

a) Impulssvar: $h(t) = \mathcal{L}^{-1}\{H(s)\} = 16{,}8(e^{-2t} - e^{-12t})\, u(t)$

Alt. lösning: $h(t) = \frac{d}{dt}\{Y_s(t)\}$

b) $H(s)$: Poler: $s_1 = -2$ och $s_2 = -12$

Inga nollställen

c) Frekv. svar $H(s)\big|_{s=j\omega} = \frac{168}{(j\omega+2)(j\omega+12)}$

$$|H(j\omega)|_{max} = |H(j\omega)|_{\omega \to 0} = \frac{168}{2 \cdot 12} = 7$$

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(3)

$$H(z) = \frac{1+\frac{7}{6}z^{-1}}{1+\frac{1}{3}z^{-1}}$$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] \quad \overset{\text{z-transf.}}{\Rightarrow} \quad X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad Y(z) = H(z)X(z)$$

$$Y(z) = \frac{1+\frac{7}{6}z^{-1}}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{(1+\frac{1}{3}z^{-1})} + \frac{B}{(1-\frac{1}{2}z^{-1})}$$

$$A = \frac{1+\frac{7}{6}(-3)}{1-\frac{1}{2}(-3)} = \frac{\frac{6-21}{6}}{\frac{2+3}{2}} = -\frac{15}{6}\cdot\frac{2}{5} = -1$$

$$B = \frac{1+\frac{7}{6}\cdot 2}{1+\frac{1}{3}\cdot 2} = \frac{\frac{6+14}{6}}{\frac{3+2}{3}} = \frac{20}{6}\cdot\frac{3}{5} = 2$$

$$Y(z) = -\frac{1}{(1+\frac{1}{3}z^{-1})} + \frac{2}{(1-\frac{1}{2}z^{-1})}$$

Inv. z-transf. ger

$$y[n] = \left[-\left(-\frac{1}{3}\right)^n + 2\left(\frac{1}{2}\right)^n\right] u[n]$$

4

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a) Signalernas (vinkel) frekvenser $\omega < \omega_M = 10\pi$ r/s

Enligt samplingsteoremet
Samplingsfrekvens $\omega_s \geq 2\omega_M = 20\pi$ r/s

Sampelintervall $T = \frac{2\pi}{\omega_s}$ s

b) "Avstånd" mellan ingående frekvenser

$\omega_r = 2\pi(0{,}45 - 0{,}40) = 0{,}1\pi$ r/s

Frekvensupplösning hos DFT ; $\Delta\omega = \frac{\omega_s}{N}$
N = antal sampel

Krav: $|k_1 - k_2| \geq 10 \Rightarrow \omega_r \geq 10 \cdot \Delta\omega = \frac{10 \cdot \omega_s}{N}$

$$N \geq \frac{10 \cdot \omega_s}{\omega_r} = \left\{ \text{Välj } \omega_s = 2\omega_M = 20\pi \text{ r/s} \right\} =$$

$$= \frac{10 \cdot 20\pi}{0{,}1\pi} = 2000$$

Tid signalen samplas $t_{tot} = N \cdot T$

$$t_{tot} = N \cdot T = \frac{10\,\omega_s}{\omega_r} \cdot \frac{2\pi}{\omega_s} = \frac{2\pi}{\frac{\omega_r}{10}} = \frac{2\pi}{\Delta\omega} =$$

$$= \frac{2\pi}{0{,}1\pi} \cdot 10 = 200 \text{ s}$$

(5)

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Denna lösning använder Beta för att bestämma Fourierkoeff. till x(t).

x(t): Medelvärde = 0, topp-topp värde |h| = 2 (2L = T)

$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = \frac{2\pi}{T}$ Från Beta fås

$$x(t) = -\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left\{\frac{(2n-1)\,2\pi}{\underbrace{2L}_{T}}\cdot t\right\} =$$

$$= -\frac{8}{\pi}\sum_{k=1,3,5,\ldots}^{\infty}\frac{1}{k^2}\cos(k\omega_0 t) \quad ; \quad A'_k = \frac{1}{k^2}, \; k = 1,3,5,\ldots$$

$$H(j\omega) = \frac{j\frac{\omega}{\omega_0}}{\left(j\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0} + 1}$$

Studera frekvenserna $\omega = \omega_0, 3\omega_0$ och $5\omega_0$

$$H(j\omega)\Big|_{\omega=\omega_0} = \frac{j}{-1+j+1} = 1$$

$|H(j\omega_0)| = 1$

$\arg\{H(j\omega_0)\} = 0$

$$H(j\omega)\Big|_{\omega=3\omega_0} = \frac{3j}{-9+3j+1} = \frac{j3}{-8+j3}$$

$|H(j3\omega_0)| = \frac{3}{\sqrt{64+9}} = \frac{3}{\sqrt{73}}$

$\arg\{H(j3\omega_0)\} = 90^\circ - 159.4^\circ = -69.4^\circ$

$$H(j\omega)\Big|_{\omega=5\omega_0} = \frac{5j}{-25+j5+1} = \frac{j5}{-24+j5}$$

$|H(j5\omega_0)| = \frac{5}{\sqrt{601}}$

$\arg\{H(j5\omega_0)\} = 90^\circ - 168.2^\circ = -78.2^\circ$

$$A_k = -\frac{8}{\pi^2}\cdot A'_k\cdot|H(jk\omega_0)| \qquad \varphi_k = \arg\{H(jk\omega_0)\}$$

$A_1 = -\frac{8}{\pi^2}\cdot 1\cdot 1 = -\frac{8}{\pi^2}$ $\qquad \varphi_1 = 0^\circ$

$A_3 = -\frac{8}{\pi^2}\cdot\frac{1}{9}\cdot\frac{3}{\sqrt{73}}$ $\qquad \varphi_3 = -69.4^\circ$

$A_5 = -\frac{8}{\pi^2}\cdot\frac{1}{25}\cdot\frac{5}{\sqrt{601}}$ $\qquad \varphi_5 = -78.2^\circ$

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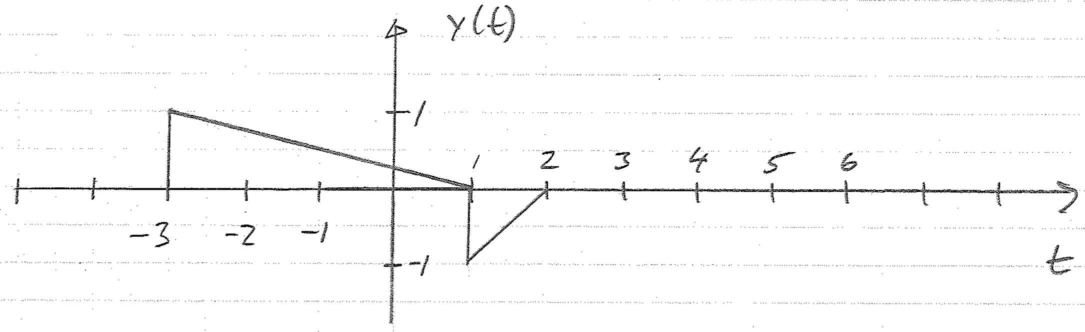
2010-08-25

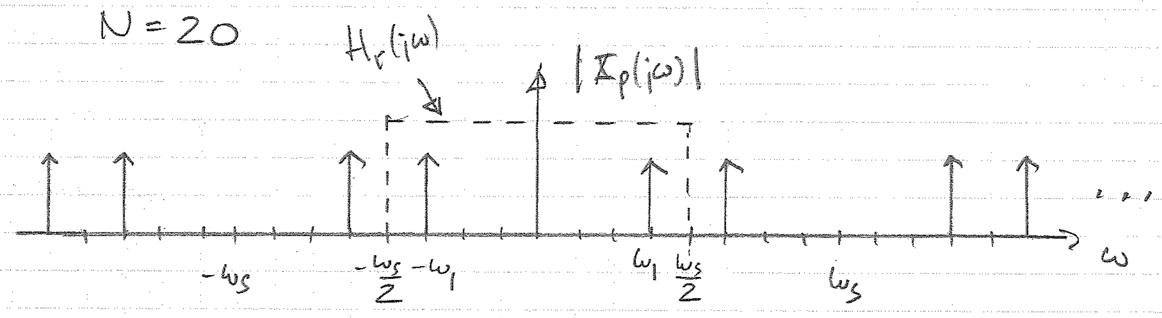
1a)



b)

$N = 20$

2a)



b) $y(t) = x(t) = \sin(\omega_1 t)$ $\quad \omega_1 < \frac{\omega_s}{2}$ $\quad$ No aliasing

3) $Y(s) = \frac{4}{(s+2)} \cdot \frac{5}{(s+5)} \cdot \frac{6}{(s+3)} = \ldots = \frac{40}{s+2} + \frac{20}{s+5} - \frac{60}{s+3}$

$y(t) = 20(2e^{-2t} + e^{-5t} - 3e^{-3t})\, u(t)$

4) $H(z) = \dfrac{1 + \frac{3}{2} z^{-1} - \frac{2}{3} z^{-2}}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$

$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n] + \frac{3}{2} x[n-1] - \frac{2}{3} x[n-2]$

5) a) $X_a[3] = 8$, $\quad X_a[k] = 0$ för $k = 0,1,2,4,5,6,7$

b) $X_b[6] = 8$, $\quad X_b[k] = 0$ för $k = 0,1,2,3,4,5,7$

c) $\Delta\omega = \frac{200\pi}{256} \approx 2{,}45$ r/s