554080 081025

1a) $y(t) = x(t-2) = u(t-2) - u(t-6)$

1b) i) $y(t) = e^{-t} - e^{-2t}$, för $t \geq 0$

$y(t) = 0$, för $t < 0$

ii) $t = \ln 2$ [s]

2) $y(t) = 10\left[1 - e^{-50t}\left(\cos 800t + \frac{1}{16}\sin 800t\right)\right] u(t)$

$y'(t) = -y(t)$ också en möjlighet

3) $y[n] = \left(2 \cdot 3^n + 0{,}5^n\right) u[n]$

Instabilt. Pol utanför enhetscirkeln.

4) i) D ii) C iii) A iv) B

5) Insignalens medeleffekt $\bar{P}_x = 1$

Utsignalens —"— $\bar{P}_y = 0{,}9006$

Utsignalens $\underbrace{\text{effekt}}_{\text{medel}}$ är $\frac{\bar{P}_y}{\bar{P}_x} \cdot 100\% = 90{,}06\,\%$ av

insignalens medeleffekt

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1/ a)

Superposition ger $y[n] = h[n+3] + 2h[n+2] -$
$- h[n] - 2h[n-1] + h[n-2]$

	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$h[n+3]$		1	0,5	0,25								
$2h[n+2]$			2	1	0,5							
$-h[n]$					-1	-0,5	-0,25					
$-2h[n-1]$						-2	-1	-0,5				
$h[n-2]$							1	0,5	0,25			
$\Sigma \Rightarrow y[n]$		1	2,5	1,25	-0,5	-2,5	-0,25	0	0,25			

Svar: $y[n] = \delta[n+3] +$
$+ 2{,}5\,\delta[n+2] +$
$+ 1{,}25\,\delta[n+1]$
$- 0{,}5\,\delta[n] -$
$- 2{,}5\,\delta[n-1] -$
$- 0{,}25\,\delta[n-2] +$
$+ 0{,}25\,\delta[n-4]$

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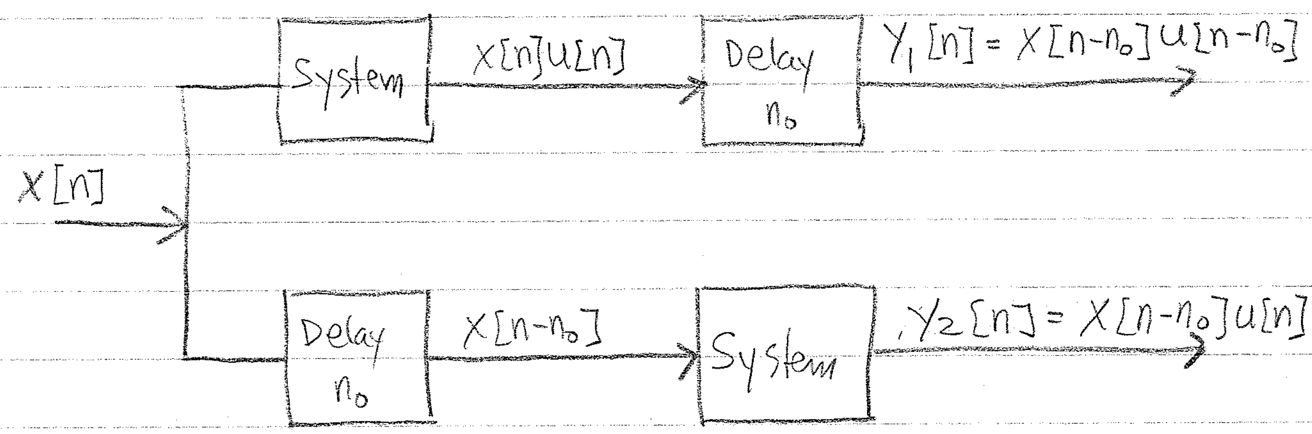
1b/

$$y[n] = x[n]\,u[n]$$

Test



$$y_1[n] \neq y_2[n]$$

Ej tidsinvariant

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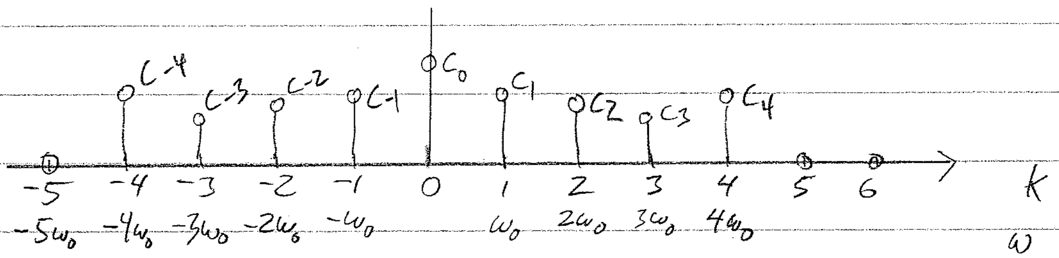
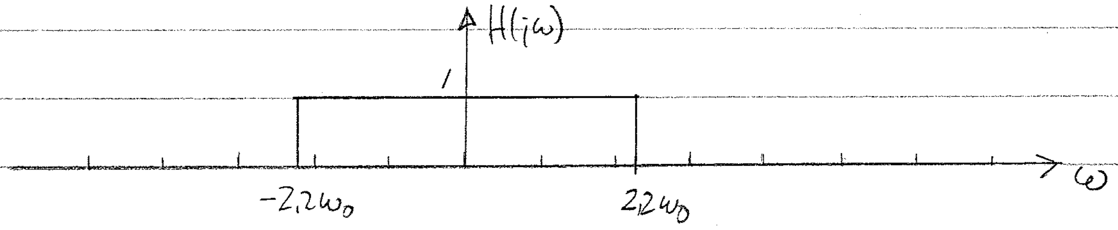
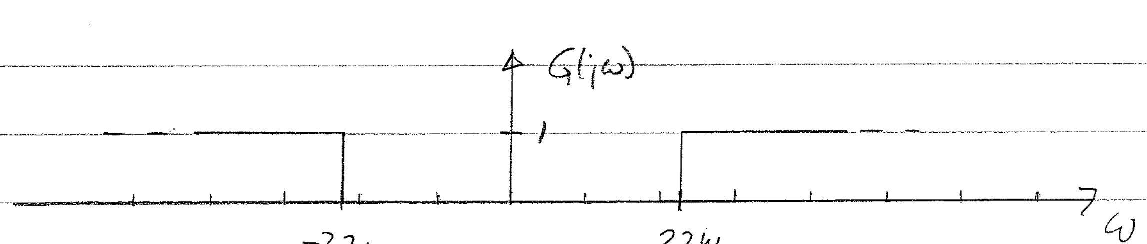
FS

2)







$$x(t) \longrightarrow \boxed{G(j\omega)} \longrightarrow y(t)$$

$G(j\omega)$ släpper endast igenom frekvenser $|\omega| > 2{,}2\,\omega_0$

FS-koeff till $y(t)$ blir då $\begin{cases} c_3,\ c_{-3},\ c_4 \text{ och } c_{-4} \\ \text{övriga } c_k = 0 \end{cases}$

Medeleffekt $P = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$P_x = |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 + 2|c_3|^2 + 2|c_4|^2 =$$
$$= 4 + 2\cdot 1 + 2\cdot 0{,}5^2 + 2\cdot 0{,}2^2 + 2\cdot 0{,}4^2 = 6{,}9$$

$$P_y = 2|c_3|^2 + 2|c_4|^2 = 2\cdot 0{,}2^2 + 2\cdot 0{,}4^2 = 0{,}4$$

$$\frac{P_y}{P_x} = \frac{0{,}4}{6{,}9} \approx 0{,}058$$

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3/ $H_1(s) = \dfrac{K}{s+3}$ $\quad H(s)\Big|_{\substack{s=j\omega \\ \omega \to 0}} = \dfrac{K}{3} = \dfrac{2}{3} \Rightarrow K=2$

$H_1(s) = \dfrac{2}{s+3}$

Stegsvar: $\dfrac{1}{s} \cdot H_2(s) = \mathcal{L}\{(6-5e^{-t})u(t)\} = \dfrac{6}{s} - \dfrac{5}{s+1} =$

$= \dfrac{6(s+1)-5s}{s(s+1)} = \dfrac{1}{s} \cdot \dfrac{s+6}{s+1} \Rightarrow H_2(s) = \dfrac{s+6}{s+1}$

Hela systemet $Y(s) = H_1(s)H_2(s)X(s)$; $X(s) = \mathcal{L}\{\delta(t)\} = 1$

$Y(s) = \dfrac{2}{(s+3)} \dfrac{(s+6)}{(s+1)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$

$2(s+6) = A(s+3) + B(s+1)$ | s^1: $2 = A+B$ | $10 = 2A$
| s^0: $12 = 3A+B$ | $A=5$, $B=-3$

$Y(s) = \dfrac{5}{s+1} - \dfrac{3}{s+3}$

Inv. Laplacetransf. ger

$y(t) = (5e^{-t} - 3e^{-3t})u(t)$

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4/ $y[n] - 1{,}2\,y[n-1] - 0{,}28\,y[n-2] = x[n] - 3x[n-1]$

z-transformera

a/ $Y(z) - 1{,}2\,Y(z)z^{-1} - 0{,}28\,Y(z)z^{-2} = X(z) - 3X(z)z^{-1}$

$Y(z)(1 - 1{,}2z^{-1} - 0{,}28z^{-2}) = X(z)(1 - 3z^{-1})$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-3z^{-1}}{1-1{,}2z^{-1}-0{,}28z^{-2}} = \frac{z(z-3)}{z^2-1{,}2z-0{,}28}$$

b/ Impulssvar $h[n] = \mathcal{Z}^{-1}\{H(z)\} = y[n]$

Sök poler: $z_{1,2} = 0{,}6 \pm \sqrt{0{,}6^2+0{,}28} = 0{,}6 \pm 0{,}8 = \begin{cases} 1{,}4 \\ -0{,}2 \end{cases}$

$$\frac{H(z)}{z} = \frac{z-3}{(z-1{,}4)(z+0{,}2)} = \frac{A}{z-1{,}4} + \frac{B}{z+0{,}2}$$

$z-3 = A(z+0{,}2) + B(z-1{,}4)$

$z^1:\; 1 = A + B$

$z^0:\; -3 = 0{,}2A - 1{,}4B$

$$\begin{aligned} -0{,}2 &= -0{,}2A - 0{,}2B \\ -3 &= 0{,}2A - 1{,}4B \\ \hline -3{,}2 &= -1{,}6B \end{aligned}$$

$\Rightarrow B = 2,\; A = -1$

$$H(z) = 2\frac{z}{z+0{,}2} - \frac{z}{z-1{,}4} = 2\cdot\frac{1}{1+0{,}2z^{-1}} - \frac{1}{1-1{,}4z^{-1}}$$

Inv. z-transf.

$$y[n] = h[n] = \left[2\cdot(-0{,}2)^n - (1{,}4)^n\right]u[n]$$

c/ Pol utanför enhetscirkeln $(z=1{,}4)$ $\Rightarrow$ Instabilt

Se info på sidan innan

5/ Notera

i) $N=4$ för alla $x_i[n]$, Då måste även $X[k]$ ha $N=4$

Uteslut: X_h och X_e med $N=5$

ii) $X[0] = \sum_{n=0}^{N-1} x[n]$

$x_1[n]$: $\sum_{n=0}^{3} x_1[n] = 1 \Rightarrow X[0]=1$ c, d, g OK

$$X[1] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}\cdot 1\cdot n} = 1+0+0+0 = 1$$

$$X[2] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}\cdot 2\cdot n} = 1+0+0+0 = 1$$

$\therefore$ $x_1[n] \xleftrightarrow{DFT} X_d[k]$

(Impuls ger bidrag vid alla frekvenser)

$x_2[n]$: $\sum_{n=0}^{3} x_2[n] = 0 = X[0]$

$\therefore$ $x_2[n] \xleftrightarrow{DFT} X_a[k]$

$x_3[n]$: $\sum_{n=0}^{3} x_3[n] = 2 = X[0]$ $\therefore$ $x_3[n] \longleftrightarrow X_f[k]$

$x_4[n]$: $\sum_{n=0}^{3} x_4[n] = 1 = X[0]$ c, g möjliga

OBS! $x_4[n]$ en konstant signal - (DC)

$X[k] \neq 0$ endast för $k=0$

$x_4[n] \xleftrightarrow{DFT} X_c[k]$

/forts 5

$x_5[n]$: $\sum_{n=0}^{3} x_5[n] = 1$ c, d, g möjliga men endast g kvar

Dessutom

$x_5[n]$ en fördröjd impuls ($x_1[n]$)

Då borde $|DFT\{x_1[n]\}| = |DFT\{x_5[n]\}|$

vilket också stämmer på $X_g[k]$

Svar:

Signal	DFT
$x_1[n]$	$X_d[k]$
$x_2[n]$	$X_a[k]$
$x_3[n]$	$X_f[k]$
$x_4[n]$	$X_c[k]$
$x_5[n]$	$X_g[k]$

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1. $y(t) = \cos(0{,}5\pi t)\, x(t)$

a) $x(t) = \delta(t)$

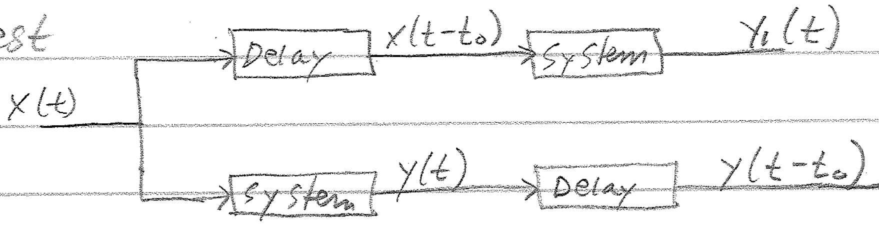
$y(t) = \cos(0{,}5\pi t)\,\delta(t) = \cos(0)\,\delta(t) = \delta(t)$

b) $x(t) = \delta(t-1)$

$y(t) = \cos(0{,}5\pi t)\,\delta(t-1) = \cos(0{,}5\pi \cdot 1)\,\delta(t-1) = 0$

c)

Test



Tidsinvariant om $y_1(t) = y(t-t_0)$

Enligt a) och b) är så inte fallet

Systemet ej tidsinvariant

d)

Insignal	Utsignal
$x_1(t)$	$y_1(t) = \cos(0{,}5\pi t)\, x_1(t)$
$x_2(t)$	$y_2(t) = \cos(0{,}5\pi t)\, x_2(t)$
$a\,x_1(t)$	$\cos(0{,}5\pi t)\, a\, x_1(t) = a\, y_1(t)$
$a x_1(t) + b x_2(t)$	$\cos(0{,}5\pi t)\left(a x_1(t) + b x_2(t)\right) =$
	$a\cos(0{,}5\pi t)\, x_1(t) + b\cos(0{,}5\pi t)\, x_2(t) =$
	$= a\, y_1(t) + b\, y_2(t)$

Superposition gäller!

Systemet är linjärt.

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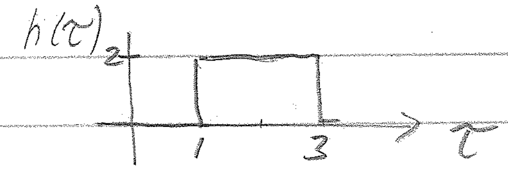
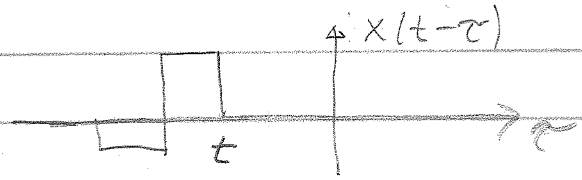
2) $y(t) = \int_{-\infty}^{t} h(\tau)\, x(t-\tau)\, d\tau$ Faltning!

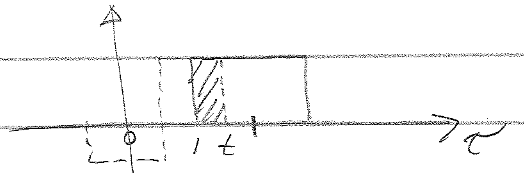
"Grafisk" lösn.





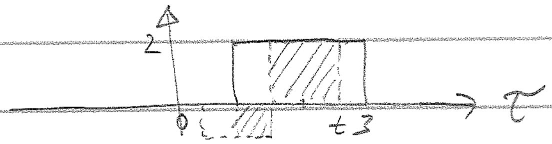
$\boxed{t<1}$ $h(\tau)x(t-\tau) = 0 \Rightarrow y(t) = 0$

$\boxed{1<t<2}$



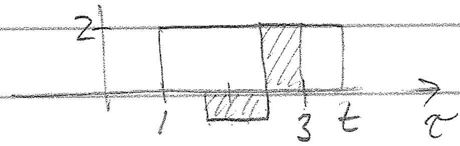
$y(t) = \int_{-\infty}^{t} h(\tau)\, x(t-\tau)\, d\tau$ växer linjärt från 0 till 4 | $y(t) = 2 \cdot 2(t-1)$

$\boxed{2<t<3}$



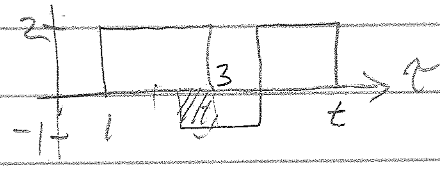
$y(t) = \int_{-\infty}^{t} h(\tau)\, x(t-\tau)\, d\tau$ Linjär minskning från 4 till $4-2=2$ | $y(t) = 4-2(t-1-1)$
$= 4-2(t-2)$

$\boxed{3<t<4}$



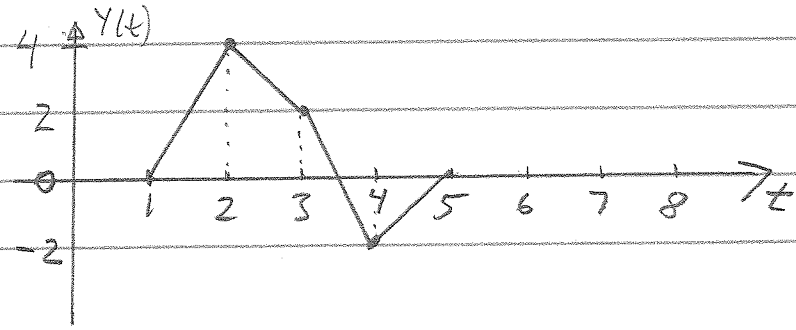
$y(t) = -2+4(3-(t-1)) =$
$= 4(4-t)-2$

$\boxed{4<t<5}$



$y(t) = -2(3-(t-2)) =$
$= -2(5-t)$

$t>5$ $y(t) = 0$



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3/ , sätt $K=1$.

$$H(s) = \frac{1}{s(s+1)}$$

$$\begin{matrix} x(t) \\ X(s) \end{matrix} \longrightarrow \boxed{\begin{matrix} h(t) \\ H(s) \end{matrix}} \longrightarrow \begin{matrix} y(t) \\ Y(s) \end{matrix}$$

$$x(t) = e^{-t}u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s+1}$$

$$Y(s) = H(s)X(s) = \frac{1}{s(s+1)} \cdot \frac{1}{s+1} = \frac{1}{s(s+1)^2}$$

Inverstransformera Partialbråksuppdela

$$\frac{1}{s\,(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$1 = A(s+1)^2 + Bs(s+1) + sC = A(s^2+2s+1) + B(s^2+s) + sC$$

$s^2:\quad 0 = A+B$	$A=1,\ B=-1$
$s^1:\quad 0 = 2A+B+C$	$C = -2A-B = -2+1 = -1$
$s^0:\quad 1 = A$	

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$y(t) = u(t) - e^{-t}u(t) - te^{-t}u(t) =$$

$$= \left[1 - e^{-t}(1+t)\right]u(t)$$

4 $y[n] - 1,5\,y[n-1] + 0,5\,y[n-2] = x[n]$

a/

z-transformera: $Y(z) - 1,5\,z^{-1}Y(z) + 0,5\,z^{-2}Y(z) = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 1,5z^{-1} + 0,5z^{-2}} = \frac{z^2}{z^2 - 1,5z + 0,5}$$

Rötter till nämnaren $z_{1,2} = 0,75 \pm \sqrt{0,75^2 - 0,5} = 0,75 \pm 0,25 = \begin{cases} 1 \\ 0,5 \end{cases}$

$$H(z) = \frac{z^2}{(z-1)(z-0,5)}$$

Insignal $x(t) = \delta(t) \xleftrightarrow{Z} X(z) = 1$

Utsignal (impulssvar) $Y(z) = H(z)X(z) = H(z)$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)(z-0,5)} = \left\{\begin{array}{l}\text{Partialbråks-}\\ \text{uppdela}\end{array}\right\} = \frac{A}{z-1} + \frac{B}{z-0,5}$$

$z = A(z-0,5) + B(z-1)$ | z^1: $1 = A + B$ (1)
| z^0: $0 = -0,5A - B$ (2)

(1)+(2): $1 = 0,5A \Rightarrow A = 2$, $B = -1$

$$Y(z) = 2\frac{z}{z-1} - \frac{1}{z-0,5} = 2\cdot\frac{1}{1-z^{-1}} - \frac{1}{1-0,5z^{-1}}$$

Inv. transf. $y[n] = h[n] = 2u[n] - 0,5^n u[n] = (2 - 0,5^n)\,u[n]$

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Differens ekv.

4 b) $y[n] = x[n] + 1{,}5\,y[n-1] - 0{,}5\,y[n-2]$

$x[n] = \delta[n]$ $(y[-1] = y[-2] = 0)$

n	y[n]	
0	$1+0+0 =$	1
1	$0+1{,}5\cdot 1+0 =$	$1{,}5$
2	$0+1{,}5\cdot 1{,}5-0{,}5\cdot 1 =$	$1{,}75$
3	$0+1{,}5\cdot 1{,}75-0{,}5\cdot 1{,}5 =$	$1{,}875$

y[n] på sluten form

n	$y[n] = (2-0{,}5^n)\,u[n]$	
0	$2-1 =$	1
1	$2-0{,}5^1 =$	$1{,}5$
2	$2-0{,}5^2 =$	$1{,}75$
3	$2-0{,}5^3 =$	$1{,}875$

OK!

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5 a/ Studera signalerna $x_1[n]$, $x_2[n]$ och $x_3[n]$ i figuren. De är periodiska.

Signal	Antal hela perioder i intervallet
$x_1[n]$	5
$x_2[n]$	6
$x_3[n]$	4

Jämför uppbyggnad av en Fourierserie

$X[k]$ ger högt värde vid anpassning till komplex sinusformad signal $e^{j\frac{2\pi}{N}n \cdot k}$. Här svarar k mot antal perioder i intervallet med N värden ($n = 0, 1, \ldots, N-1$)

Alltså $x_1[n] \triangleq X_B[k]$, $x_2[n] \triangleq X_C[k]$, $x_3[n] \triangleq X_A[k]$

b/ Dominerande bidrag hos $|X[k]|$ för $k = 2$ och $k = N-2 = 6$
$N = 8$

Möjliga "graderingar" av frekvensaxeln till $X[k]$

0	1	2	3	$N-1$	(N)	k
0	$\Delta\omega$	$2\Delta\omega$	$3\Delta\omega$	$(N-1)\Delta\omega$	ω_s	ω r/s
0	Δf	$2\Delta f$	$3\Delta f$	$(N-1)\Delta f$	f_s	f Hz

$k = 2$: $2\Delta\omega = 480\pi$ r/s $\Rightarrow \omega_s = N\Delta\omega_0 = 8 \cdot \frac{480\pi}{2} = 1920\pi$ r/s $=$
$= 960 \cdot 2\pi$ r/s $\Rightarrow f_s = 960$ Hz

(Andra lösningsvägar finns!)