Transformer, Signaler o System D3 SSY080

071027

1/ $h[n] = 2\delta[n] + \delta[n-1] + \delta[n-3] + 2\delta[n-4]$

a/ $x[n] = u[n-3] - u[n-6] = \delta[n-3] + \delta[n-4] + \delta[n-5]$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Tre alternativ: i) Faltning $\quad y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

ii) Superposition $\quad y[n] = h[n-3] + h[n-4] + h[n-5]$

iii) z-transformera

$H(z) = 2 + z^{-1} + z^{-3} + 2z^{-4}$, $X(z) = z^{-3} + z^{-4} + z^{-5}$

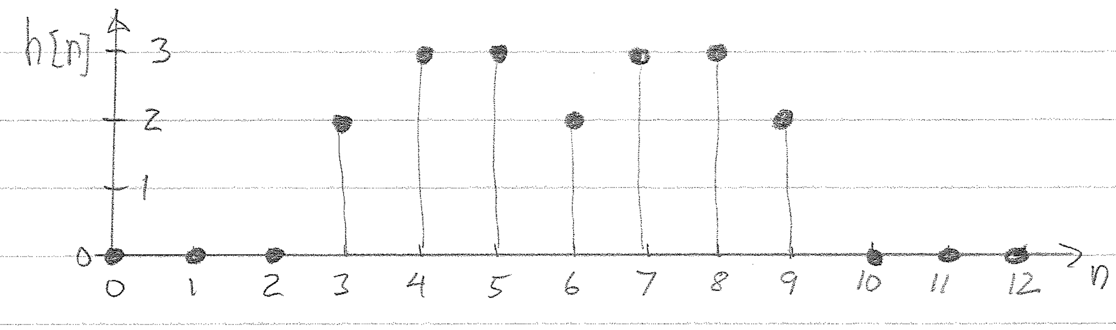
$$Y(z) = H(z)X(z) = (2 + z^{-1} + z^{-3} + 2z^{-4})(z^{-3} + z^{-4} + z^{-5}) =$$

$$= 2z^{-3} + 2z^{-4} + 2z^{-5} + z^{-4} + z^{-5} + z^{-6} + z^{-6} + z^{-7} + z^{-8} + 2z^{-7} + 2z^{-8} + 2z^{-9} =$$

$$= z^{-3}(2) + z^{-4}(2+1) + z^{-5}(2+1) + z^{-6}(1+1) + z^{-7}(1+2) + z^{-8}(1+2) + z^{-9}\cdot 2 =$$

$$= 2z^{-3} + 3z^{-4} + 3z^{-5} + 2z^{-6} + 3z^{-7} + 3z^{-8} + 2z^{-9}$$

$$\Rightarrow h[n] = 2\delta[n-3] + 3\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + 3\delta[n-7] + 3\delta[n-8] + 2\delta[n-9]$$



b/ För att begränsa bandbredden hos en kontinuerlig signal. Görs innan sampling för att undvika aliasing (vikning).

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2) a) $X[k]$ har $N = 32$ st värden

b) $\Delta\Omega = \frac{2\pi}{N}$ $\quad \Omega = \omega T_s$

$$\Delta\omega = \frac{\Delta\Omega}{T_s} = \frac{2\pi}{N \cdot T_s} = \frac{2\pi \cdot 112}{32 \cdot 1} = 7\pi \text{ r/s}$$

c) $x(t)$ består av 3 reella sinusformade signaler med olika frekvenser som är jämna multiplar av $\Delta\omega$. Varje reell sinusformad signal ger två distinkta toppar

Totalt: $2 \cdot 3 = 6$ st distinkta toppar

d) Sampling $x(t) = x(nT_s) = x[n]$

I DFT representeras de diskreta frekvenserna $\Omega_k = \frac{2\pi}{N} \cdot k = \Delta\Omega \cdot k$

i) $\cos(14\pi t) = \cos(14\pi \cdot nT_s) = \cos\left(\frac{14\pi}{112} \cdot n\right) =$

$= \cos\left(\frac{2\pi \cdot 7}{32 \cdot 3{,}5} n\right) = \cos\left(\frac{2\pi}{\underset{N}{32}} \cdot \underset{k}{2} \cdot n\right) \quad \therefore \ k = 2$

ii) $\sin(28\pi t) = \sin(28\pi \cdot nT_s) = \sin\left(\frac{28\pi}{112} \cdot n\right) =$

$= \sin\left(\frac{2\pi}{32} \cdot \frac{14}{3{,}5} n\right) = \sin\left(\frac{2\pi}{32} \cdot 4 \cdot n\right) \quad \therefore \ k = 4$

iii) $\cos(70\pi t) = \cos(70\pi nT_s) = \cos\left(\frac{2\pi \cdot 35}{32 \cdot 3{,}5} n\right) =$

$= \cos\left(\frac{2\pi}{32} \cdot 10 \cdot n\right) \quad \therefore \ k = 10$

Topp vid k' ger även topp vid $N - k'$

Svar: $k = 2, 4, 10$ och $30, 28, 22$

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3. $$\frac{d^2 v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC} v_c(t) = \frac{1}{LC} v(t)$$

Laplacetransformera

$\frac{1}{LC} = 2{,}0 \cdot 10^6$

$\frac{R}{L} = 2{,}0 \cdot 10^3$

$$s^2 V_c(s) + \frac{R}{L} s V_c(s) + \frac{1}{LC} V_c(s) = \frac{1}{LC} V(s)$$

$$V_c(s)\left(s^2 + s\,2000 + 2{,}0 \cdot 10^6\right) = 2{,}0 \cdot 10^6\, V(s)$$

$$H(s) = \frac{V_c(s)}{V(s)} = \frac{2{,}0 \cdot 10^6}{s^2 + s\,2000 + 2{,}0 \cdot 10^6}$$

Komplexa rötter! Kvadratkomplettera

$$H(s) = \frac{2{,}0 \cdot 10^6}{(s+1000)^2 \underbrace{- 1000^2 + 2{,}0 \cdot 10^6}_{10^6 = (10^3)^2}} = \frac{2000 \cdot 1000}{(s+1000)^2 + 1000^2}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \left[2000\, e^{-1000t} \sin(1000t)\right] u(t)$$

Icke oscillatoriska inslag i $h(t)$ $\Rightarrow$ reella poler till $H(s)$

$$H(s) = \frac{2{,}0 \cdot 10^6}{s^2 + s\frac{R}{L} + 2 \cdot 10^6}$$

$$s^2 + s\frac{R}{L} + 2 \cdot 10^6 = 0 \quad ; \quad s_{1,2} = -\frac{R}{2L} \pm \sqrt{\underbrace{\left(\frac{R}{2L}\right)^2 - 2 \cdot 10^6}_{>0}}$$

$$\frac{R}{2L} > \sqrt{2} \cdot 10^3$$

$$R > 2\sqrt{2}\, L \cdot 10^3 = \sqrt{2} \cdot 2 \cdot 0{,}050 \cdot 10^3 = \sqrt{2} \cdot 100$$

Svar: $R > \sqrt{2} \cdot 100\ \Omega$

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4. $$y[n] = x[n-1] + 0{,}7\,y[n-1]$$

$$y[n] - 0{,}7\,y[n-1] = x[n-1] \qquad \text{z-transf.}$$

$$Y(z) - 0{,}7z^{-1}Y(z) = z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-0{,}7z^{-1}} = \frac{1}{z-0{,}7}$$

Insignal: $x[n] = (-0{,}8)^n u[n] \overset{z}{\rightleftarrows} X(z) = \frac{1}{1+0{,}8z^{-1}} = \frac{z}{z+0{,}8}$

$$Y(z) = H(z)X(z) = \frac{1}{(z-0{,}7)} \cdot \frac{z}{(z+0{,}8)}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-0{,}7)(z+0{,}8)} = \frac{A}{z-0{,}7} + \frac{B}{z+0{,}8}$$

$$1 = A(z+0{,}8) + B(z-0{,}7)$$

z^1: $0 = A + B$

z^0: $1 = 0{,}8A - 0{,}7B$

$1 = 1{,}5A$

$A = \frac{1}{1{,}5} = \frac{2}{3} \qquad B = -\frac{2}{3}$

$$Y(z) = \frac{2}{3}\frac{z}{z-0{,}7} - \frac{2}{3}\frac{z}{z+0{,}8}$$

Inv. z-transform

$$y[n] = \frac{2}{3}\left(0{,}7^n - (-0{,}8)^n\right)u[n]$$

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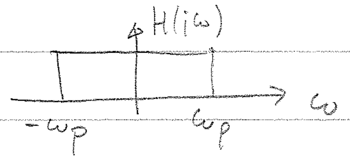
5/ Fyrkantssignalens medeleffekt (, $T = 2\pi$)

$$E = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{2\pi}\int_0^{2\pi} 1 \cdot dt = \frac{1}{2\pi}\Big[t\Big]_0^{2\pi} = 1$$

Impulssvar : $h(t) = \frac{1}{\pi t}\sin(\omega_p t)$

Fouriertransf. (Frekv. svar) enligt Beta

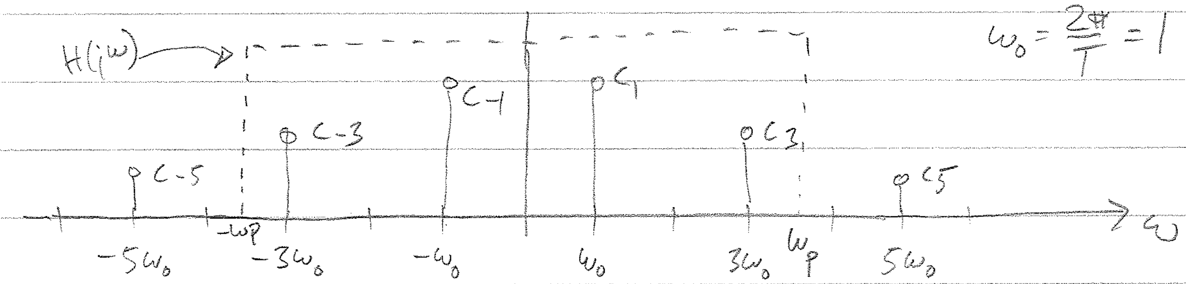
$$H(j\omega) = \begin{cases} 1 & , |\omega| < \omega_p \\ 0 & , |\omega| > \omega_p \end{cases}$$



Fyrkantssignalens c_k från lab

$c_k = -j\frac{2}{\pi k}$ $\quad k = \pm 1, \pm 3, \pm 5, \pm 7$, övriga $c_k = 0$

$\omega_0 = \frac{2\pi}{T} = 1$ r/s



För vilket $k = m$ är $\sum_{k=-m}^{m} |c_k|^2 > 0{,}92 \cdot E = 0{,}92$?

$m = 1 \Rightarrow E_1 = 2\left|\frac{2}{\pi}\right|^2 \approx 0{,}81$

$m = 3 \Rightarrow E_2 = E_1 + 2\left|\frac{2}{3\pi}\right|^2 \approx 0{,}90 < 0{,}92$

$m = 5 \Rightarrow E_3 = E_1 + E_2 + 2\left|\frac{2}{5\pi}\right|^2 \approx 0{,}93 > 0{,}92$ OK

Alltså måste $\omega_p > 5\omega_0$ $\qquad$ Svar $\omega_p > 5$ r/s

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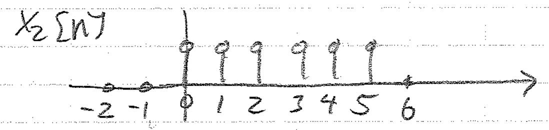
1a) Låt $x_1[n]$ vara insignal till ett diskret LTI-system med impulssvar $x_2[n]$

För varje impuls i $x_1[n]$ genereras ett impulssvar med längden M

$\delta[n-5]$: Svar börjar i $n=5$ och håller på till $n=10$

Längd: $M = 10-5+1 = 6$

Alltså skall $x_2[n] = u[n] - u[n-N]$ ha M st värden $= 1$



Svar: $N = 6$

b)

i) Enligt def: $X[k]$ har $N = 64$ st värden.

ii) Varje reell sinusformad signal ger två distinkta toppar i DFT (om de har ett jämt antal perioder i intervallet).

$$\Omega = \omega \cdot T_s = \frac{2\pi}{N} \cdot k \Rightarrow k = \frac{\omega T_s \cdot N}{2\pi}$$

$$\omega = 9\pi \Rightarrow k = \frac{9\pi \cdot 64}{96 \cdot 2\pi} = 3 \quad (\text{Även } N-k = 61)$$

$$\omega = 33\pi \Rightarrow k = \frac{33\pi \cdot 64}{96 \cdot 2\pi} = 11 \quad (\text{Även } N-k = 53)$$

Svar: 4 st distinkta toppar

iii) Distinkta toppar hos $|X[k]|$ vid

$k = 3, 11, 53$ och 61

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2)

$$\begin{cases} v_i(t) = i(t)\cdot R + v_c(t) \\ i(t) = C\,\dfrac{dv_c(t)}{dt} \end{cases}$$

$$v_i(t) = RC\cdot \frac{dv_c(t)}{dt} + v_c(t)$$

Fouriertransformera!

$$V_i(j\omega) = RC\cdot j\omega V_c(j\omega) + V_c(j\omega)$$

$$H(j\omega) = \frac{V_c(j\omega)}{V_i(j\omega)} = \frac{1}{1+j\omega RC} = \left\{\omega_0 = \frac{1}{RC}\right\} = \frac{1}{1+j\frac{\omega}{\omega_0}}$$

Utsignalens amplitud dämpas med faktorn

$$|H(j\omega)| = \left|\frac{1}{1+j\frac{\omega}{\omega_0}}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} = 0{,}6 \quad \text{enl. fig.}$$

$$1+\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{0{,}6^2}$$

$$1+(\omega RC)^2 = \frac{1}{0{,}36}$$

$$(\omega RC)^2 = \frac{1}{0{,}36} - 1 = \frac{1-0{,}36}{0{,}36} = \frac{0{,}64}{0{,}36}$$

$$\omega RC = (\pm)\,\frac{0{,}8}{0{,}6} = \frac{4}{3}$$

$R = 100\ \Omega$, $\omega = 2\pi\cdot 600$ r/s

$$C = \frac{4}{3}\cdot\frac{1}{\omega R} = \frac{4}{3\cdot 2\pi\, 600\cdot 100} = 3{,}54\cdot 10^{-6}$$

Svar: $C = 3{,}54\ \mu F$

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$$x[n] \longrightarrow \boxed{H(z)} \longrightarrow y[n]$$

$$x[n] = (-0{,}6)^n u[n] \quad \overset{Z}{\longleftrightarrow} \quad X(z) = \frac{1}{1+0{,}6z^{-1}} = \frac{z}{z+0{,}6}$$

$$y[n] = 0{,}2\, y[n-1] + 1{,}6\, x[n-1]$$

z-transf.

$$Y(z) = 0{,}2 z^{-1} Y(z) + 1{,}6 z^{-1} X(z)$$

$$Y(z)(1-0{,}2z^{-1}) = 1{,}6 \cdot z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1.6 \cdot z^{-1}}{1-0{,}2z^{-1}} = \frac{1.6}{z-0{,}2}$$

$$Y(z) = H(z) \cdot X(z) = \frac{1{,}6 \cdot z}{(z-0{,}2)(z+0{,}6)}$$

$$\frac{Y(z)}{z} = \frac{1{,}6}{(z-0{,}2)(z+0{,}6)} = \frac{A}{z-0{,}2} + \frac{B}{z+0{,}6}$$

$$1{,}6 = A(z+0{,}6) + B(z-0{,}2) \qquad \begin{cases} 1{,}6 = 0{,}6A - 0{,}2B \\ 0 = A + B \end{cases}$$

$$\Rightarrow A = 2, \; B = -2$$

$$Y(z) = \frac{2z}{z-0{,}2} - \frac{2z}{z+0{,}6} = 2\left(\frac{1}{1-0{,}2z^{-1}} - \frac{1}{1+0{,}6z^{-1}}\right)$$

Inv. z-transf.

$$y[n] = 2\left(0{,}2^n - (-0{,}6)^n\right) u[n]$$

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4. $$H(s) = \frac{25}{s^2 + 10s + 125}$$

$$x(t) \longrightarrow \boxed{H(s)} \longrightarrow y(t)$$

$$x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$$

$$Y(s) = X(s)H(s) = \frac{1}{s} \cdot \frac{25}{(s^2+10s+125)}$$

Partialbråksuppdela, behöll andragradsuttrycket ty komplexa rötter

$$Y(s) = \frac{25}{s(s^2+10s+125)} = \frac{A}{s} + \frac{Bs+C}{s^2+10s+125}$$

$$25 = A(s^2+10s+125) + (Bs+C)s$$

$$\left.\begin{aligned} s^2:&\quad 0 = A+B \\ s^1:&\quad 0 = 10A + C \\ s^0:&\quad 25 = 125A \end{aligned}\right\} \quad \begin{aligned} &A = 0{,}2, \quad B = -0{,}2 \\ &C = -10A = -2 \end{aligned}$$

$$Y(s) = \frac{0{,}2}{s} - \left(\frac{0{,}2s+2}{\underbrace{s^2+10s+125}_{(s+5)^2-25+125}}\right) = \frac{0{,}2}{s} - 0{,}2\left(\frac{s+\overbrace{10}^{5+5}}{(s+5)^2+10^2}\right) =$$

$$= \frac{0{,}2}{s} - 0{,}2\,\frac{s+5}{(s+5)^2+10^2} - \frac{0{,}2}{2} \cdot \frac{\overbrace{5 \cdot 2}^{10}}{(s+5)^2+10^2}$$

Inv. Laplacetransf.

$$y(t) = 0{,}2\,u(t) - 0{,}2\,e^{-5t}\cos 10t - 0{,}1\,e^{-5t}\sin 10t\,)\,u(t) =$$

$$= 0{,}1\left(2 - e^{-5t}\left(2\cos 10t + \sin 10t\right)\right)u(t)$$

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5.

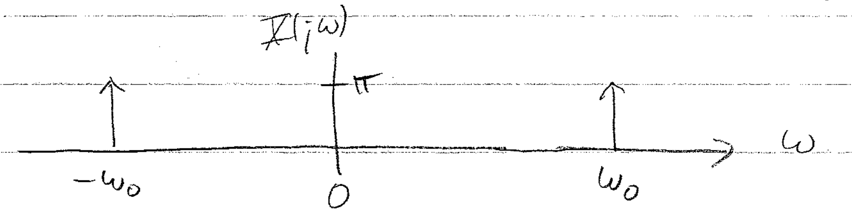
$$w(t) = \text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} T\,\text{sinc}\left(\frac{\omega T}{2}\right) = \frac{2}{\omega}\sin\left(\frac{\omega T}{2}\right) = W(j\omega)$$

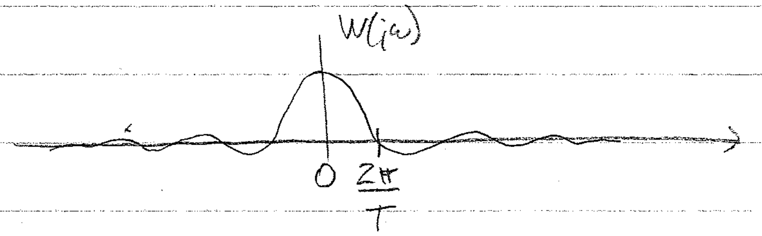
$$x(t) = \cos(\omega_0 t) \xleftrightarrow{FT} \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right] = X(j\omega)$$

$$w(t)\cdot x(t) \xleftrightarrow{FT} \frac{1}{2\pi} W(j\omega) * X(j\omega) =$$

$$= \frac{\pi}{2\pi}\left[W(j(\omega-\omega_0)) + W(j(\omega+\omega_0))\right] =$$

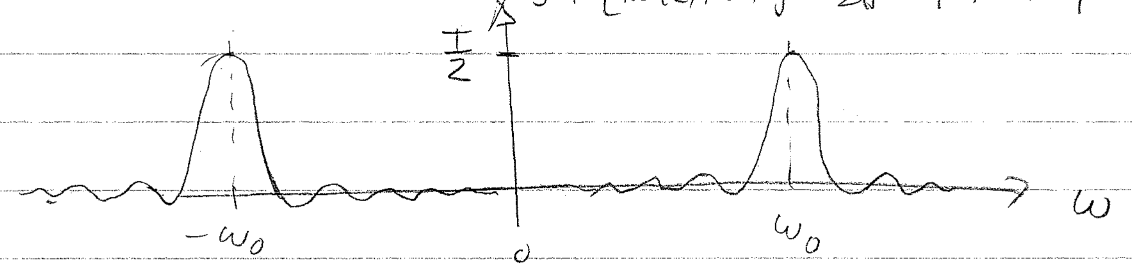
$$= \frac{T}{2}\left[\text{sinc}\left(\frac{(\omega-\omega_0)T}{2}\right) + \text{sinc}\left(\frac{(\omega+\omega_0)T}{2}\right)\right]$$





$\omega_0 \gg \frac{2\pi}{T}$

$$FT\{w(t)x(t)\} = \frac{1}{2\pi} W(j\omega) * X(j\omega$$



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1. a/ $\omega_0 = 2\pi \cdot 10^3$ r/s

b/ $c_{a0} \approx 0{,}35$

c/ $c_{bk} = e^{-jk\pi} \cdot c_{ak}$

d/ $c_{ck} = c_{ak}$ men med annan periodtid

2. a/ Ej periodisk

b/ Periodisk med $N=2$.

c/ Systemet är linjärt (Visas med "superposition")

3. $Y(s) = H_1(s)\,H_2(s)\,X(s) = \dfrac{10(s+6)}{(s+1)(s+3)} = \ldots = \dfrac{25}{s+1} - \dfrac{15}{s+3}$

$y(t) = 5\left(5e^{-t} - 3e^{-3t}\right)u(t)$

4. $Y(z) = H(z)\,X(z) = \dfrac{1}{1-az^{-1}} \cdot \dfrac{1-z^{-N}}{1-z^{-1}}$

$y[n] = \dfrac{1}{1-a}\left\{u[n] - u[n-N]\right\} - \dfrac{a}{1-a}\left\{a^n u[n] - a^{n-N} u[n-N]\right\}$

5. $f = 50$ Hz, $f_s = 200$ Hz $\Rightarrow k = 2$

$X[2] = \sum_{n=0}^{7} x[n]\, e^{-j\frac{2\pi}{8}\cdot 2\cdot n} = \ldots = 0$