

1a)

$$x(t) = 2 \cos(10\pi t + \frac{\pi}{6}) + 5\pi \cos(17\pi t - \frac{\pi}{4})$$

$$\omega_1 = 10\pi, \quad T_1 = \frac{2\pi}{\omega_1}$$

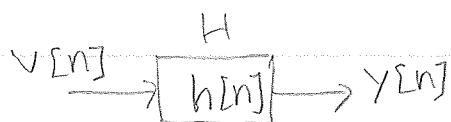
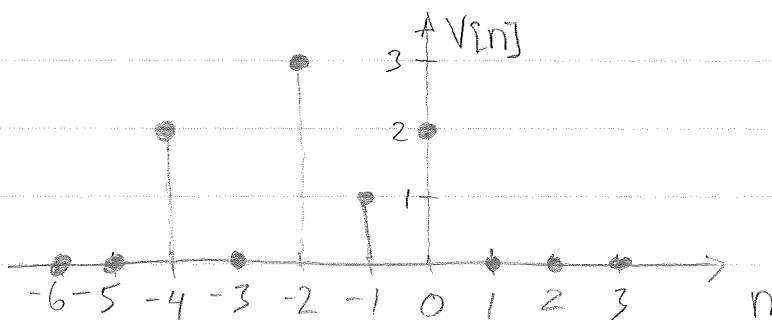
$$\omega_2 = 17\pi, \quad T_2 = \frac{2\pi}{\omega_2}$$

$$T = k_1 T_1 = k_2 T_2$$

$$\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{10}{17}$$

Periodisk? Ja! Med  $T = k_1 T_1 = k_2 T_2 = 10 \cdot \frac{2\pi}{10\pi} = 17 \cdot \frac{2\pi}{17\pi} = 2$  s

$n$	$1-2n$	$v[n] = x[1-2n]$	
-5	11	0	$y[-3]$
-4	9	2	$y[-2]$
-3	7	0	$y[-1]$
-2	5	3	$y[0]$
-1	3	1	$y[1]$
0	1	2	$y[2]$
1	-1	0	$y[3]$



$$h[n] = \delta[n-2]$$

Systemet H fördröjer signalen två steg (tidsenheter)

$$y[n] = v[n-2] = \dots, 0, 0, \underline{2}, 0, \underline{3}, 1, 2, 0, 0, \dots$$

$\uparrow$   
 $n=0$

$$2. \quad H(j\omega) = \frac{\omega_c^2}{\omega_c^2 - \omega^2 + j\sqrt{2}\omega\omega_c}$$

$$a) \quad |H(j\omega)| = \frac{\omega_c^2}{\left[ (\omega_c^2 - \omega^2)^2 + (\sqrt{2}\omega\omega_c)^2 \right]^{1/2}} = \frac{\omega_c^2}{\left[ \omega_c^4 + \omega^4 - 2\omega^2\omega_c^2 + 2\omega^2\omega_c^2 \right]^{1/2}} =$$

$$= \frac{\omega_c^2}{\sqrt{\omega_c^4 + \omega^4}} = \frac{\omega_c^2}{\sqrt{\omega_c^4 \left( 1 + \left( \frac{\omega}{\omega_c} \right)^4 \right)}} = \frac{1}{\sqrt{1 + \left( \frac{\omega}{\omega_c} \right)^4}}$$

$$\arg\{H(j\omega)\} = -\arctan\left\{ \frac{\sqrt{2}\omega\omega_c}{\omega_c^2 - \omega^2} \right\} = -\arctan\left\{ \frac{\sqrt{2}\frac{\omega}{\omega_c}}{1 - \left(\frac{\omega}{\omega_c}\right)^2} \right\} = \phi$$

$$b) \quad x(t) = \frac{4}{\pi} \left( \sin(\omega_0 t) + \frac{1}{3} \sin(3\omega_0 t) \right)$$

$$\omega_c = 600 \text{ rad/s}$$

$$\omega = \omega_0 = 200 \text{ rad/s} \quad |H(j\omega)| = \frac{1}{\sqrt{1 + \left( \frac{200}{600} \right)^4}} = 0,994$$

$$\phi = -\arctan\left\{ \frac{\sqrt{2} \cdot \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2} \right\} = -27,9^\circ$$

$$\omega = 3\omega_0 = 600 \text{ rad/s} \quad |H(j\omega)| = \frac{1}{\sqrt{1 + 1^4}} = \frac{1}{\sqrt{2}} = 0,707$$

$$\phi = -\arctan\left\{ \frac{\sqrt{2} \cdot 1}{1 - 1} \right\} = -90^\circ$$

$\rightarrow \infty$

då  $\omega \rightarrow 600 \text{ rad/s}$

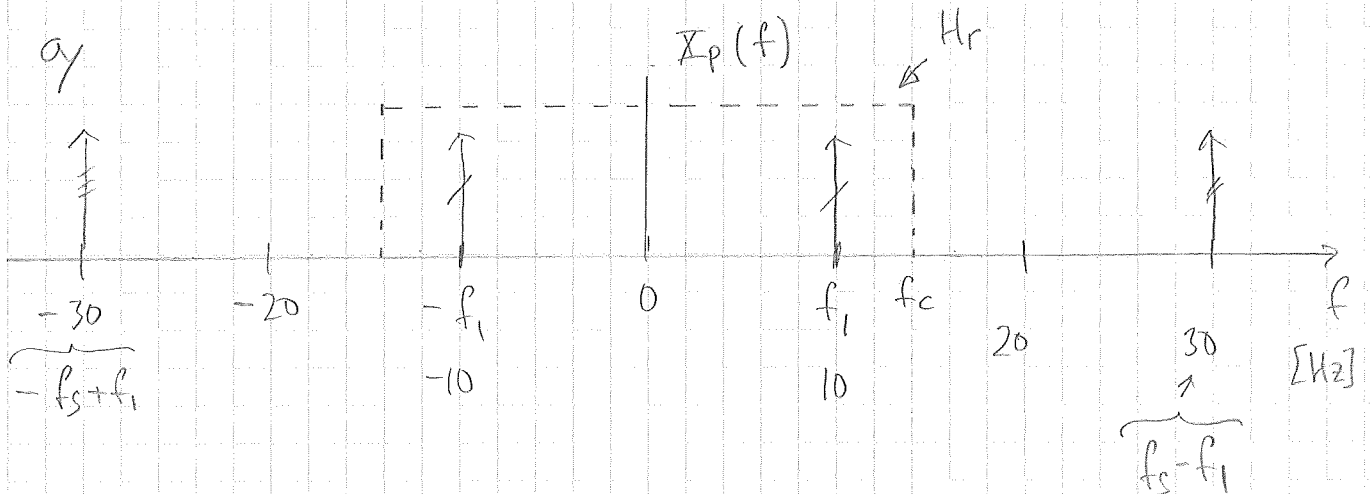
$|H(j\omega)|$ : Ger amplitudpåverkan  
 $\arg\{H(j\omega)\} = \phi$ : Ger fasförskjutning

3.  $x(t) = 4\cos(\omega_1 t)$      $\omega_1 = 20\pi = 2\pi f_1 \Rightarrow f_1 = 10 \text{ Hz}$

Sampleintervall  $T = 25 \text{ ms} \Rightarrow f_s = \frac{1}{T} = 40 \text{ Hz}$  ,  $\omega_s = 2\pi f_s$

$X(j\omega) = \pi [\delta(\omega - \omega_1) + \delta(\omega + \omega_1)]$  , sampling ger

$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$



Studera figur!

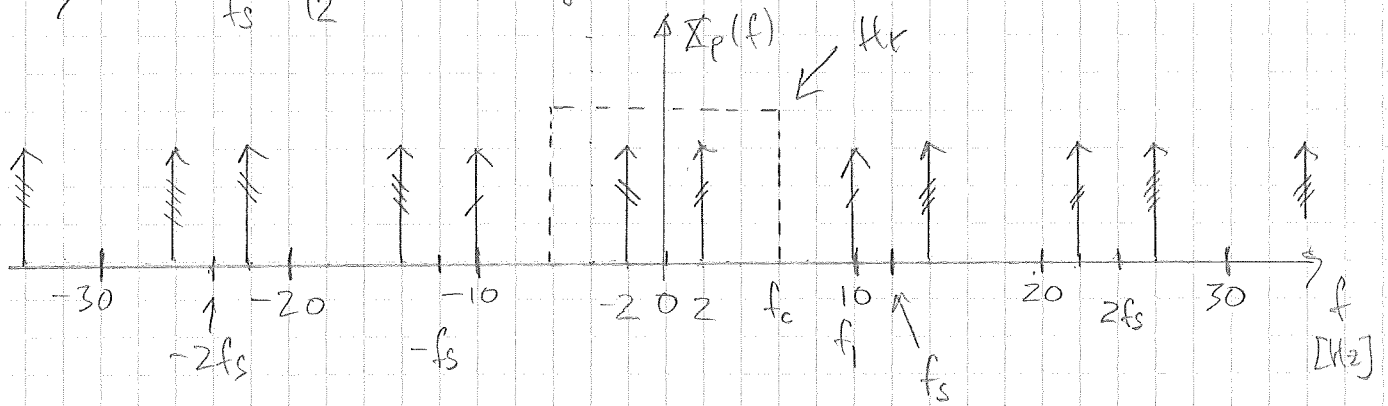
i)  $\omega = 2\pi 30 \Rightarrow f = 30 \text{ Hz}$

ii)  $2\pi\omega_1 < \omega_c < 2\pi \cdot 30$

$10 \text{ Hz} < f_c < 30 \text{ Hz}$

iii)  $f_c < 10 \text{ Hz}$

b)  $T = \frac{1}{f_s} = \frac{1}{12} \text{ s} \Rightarrow f_s = 12 \text{ Hz}$



Studera figur!

i)  $2f_s - f_1 = 24 - 10 = 14 \text{ Hz}$

ii)  $\omega_s - \omega_1 < \omega_c < \omega_1 \Rightarrow 2 \text{ Hz} < f_c < 10 \text{ Hz}$

iii)  $f_c < 2 \text{ Hz} \quad (f_s - f_1)$

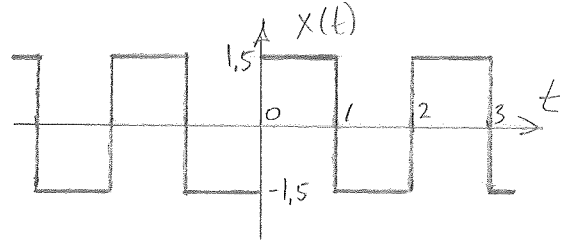
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$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\omega_0 = \pi \Rightarrow T = \frac{2\pi}{\omega_0} = 2 \text{ s}$$

$$x(t) = \begin{cases} 1.5, & 0 \leq t < 1 \\ -1.5, & 1 \leq t < 2 \end{cases}$$



$$c_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{2} \left(-\frac{3}{2}\right) \int_0^1 e^{-jk\omega_0 t} dt + \frac{1}{2} \frac{3}{2} \int_1^2 e^{-jk\omega_0 t} dt =$$

$$= -\frac{3}{4} \left[ \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_0^1 + \frac{3}{4} \left[ \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_1^2 = \left\{ \omega_0 = \pi \right\} =$$

$$= \frac{3}{4 \cdot jk\pi} \left\{ -\left(1 - e^{-jk\pi}\right) - \left(e^{-jk\pi} - 1\right) \right\} =$$

$$= \frac{3}{4jk\pi} \left( -2 + 2e^{-jk\pi} \right) = \frac{3}{2jk\pi} \left( 1 - e^{-jk\pi} \right)$$

Kan skrivas om som

$$c_k = \frac{3}{2\pi k} \cdot \frac{1}{j} e^{-jk\frac{\pi}{2}} \begin{pmatrix} e^{jk\frac{\pi}{2}} & -jk\frac{\pi}{2} \\ e^{-jk\frac{\pi}{2}} & -1 \end{pmatrix} =$$

$$= \frac{3}{\pi k} \cdot e^{-jk\frac{\pi}{2}} \cdot \sin\left(\frac{k\pi}{2}\right)$$

Studera  $c_0$  separat  $\Rightarrow c_0 = 0$  ty medelvärde = 0

5. Total energi  $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

"Parsevals formel" där  $x(t) \xleftrightarrow{FT} X(j\omega)$

$$x_1(t) = e^{-at} u(t)$$

$$E_1 = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_0^{\infty} e^{-2at} dt = \left[ \frac{e^{-2at}}{-2a} \right]_0^{\infty} = 0 - \left( -\frac{1}{2a} \right) = \frac{1}{2a}$$

$$x_2(t) = \text{sinc}(bt) \xleftrightarrow{FT} \frac{\pi}{b} \left[ u(\omega+b) - u(\omega-b) \right]$$



$$E_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X_2(j\omega)|^2 d\omega =$$

$$= \frac{1}{2\pi} \int_{-b}^b \frac{\pi^2}{b^2} d\omega = \frac{1}{2\pi} \cdot \frac{\pi^2}{b^2} \cdot 2b = \frac{\pi}{b}$$

$$E_1 = E_2 \quad ; \quad \frac{1}{2a} = \frac{\pi}{b}$$

$$\boxed{\frac{b}{a} = 2\pi}$$