

i) $y = g(t - t_0)$

ii) $y = g(t_0) \delta(t - t_0)$

iii) $y = g(-t_0)$

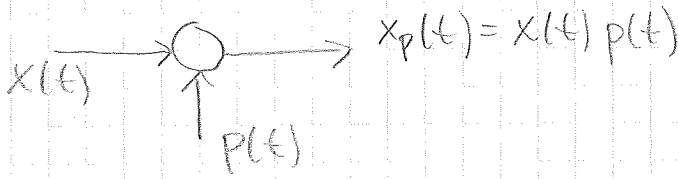
b) $u(t) = L \frac{di(t)}{dt} + Ri(t)$

Insignal	Utsignal
$i_1(t)$	$L \frac{di_1(t)}{dt} + Ri_1(t) = u_1(t)$
$i_2(t)$	$L \frac{di_2(t)}{dt} + Ri_2(t) = u_2(t)$
$ai_1(t)$	$L \frac{d}{dt}(ai_1(t)) + R(ai_1(t)) = au_1(t)$
$ai_1(t) + bi_2(t)$	$L \frac{d}{dt}(ai_1(t)) + R(ai_1(t)) +$ $+ L \frac{d}{dt}(bi_2(t)) + R(bi_2(t)) =$ $= au_1(t) + bu_2(t)$

Superposition!

Systemet är linjärt

2/

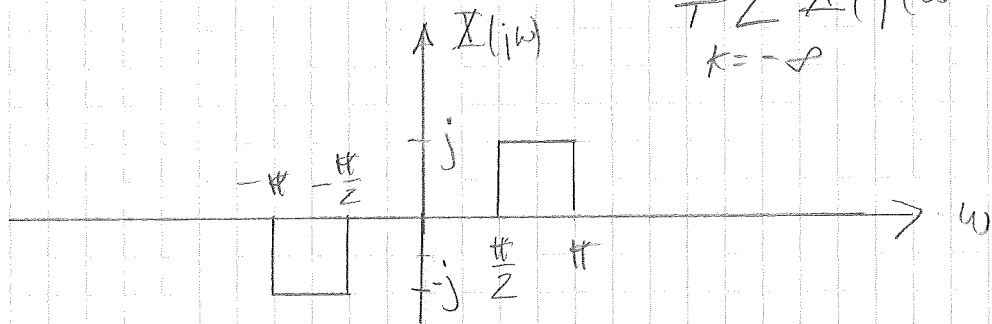


$x(t) \xleftrightarrow{FT} X(j\omega)$

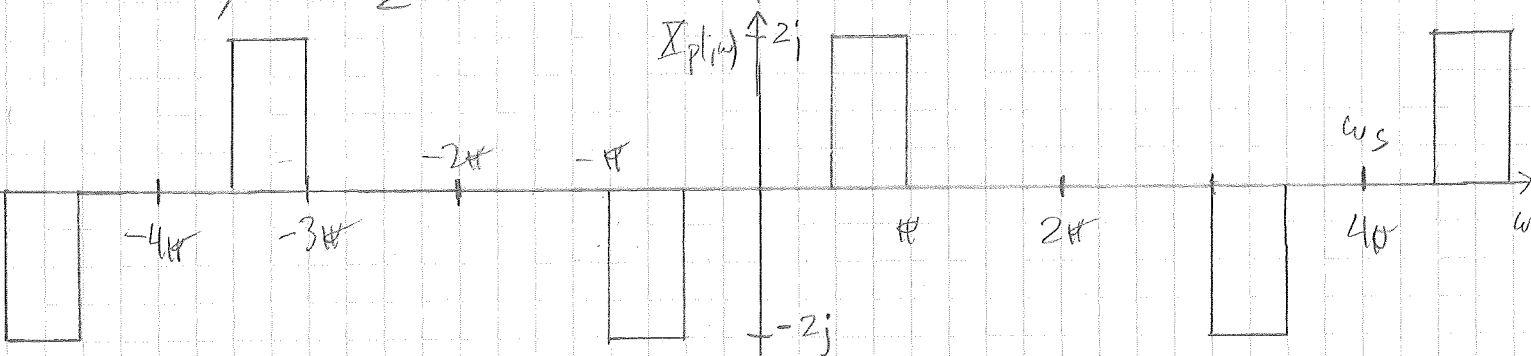
$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) \xleftrightarrow{FT} P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$

med $\omega_s = \frac{2\pi}{T}$

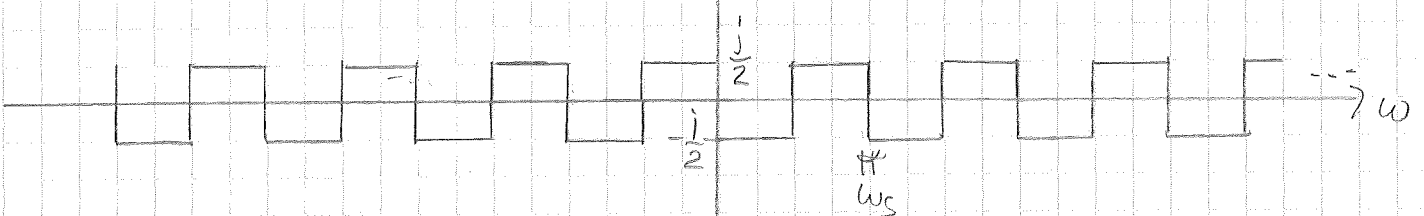
$x_p(t) = x(t) \cdot p(t) \xleftrightarrow{FT} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$



a) $T = \frac{1}{2} \text{ s} \Rightarrow \omega_s = \frac{2\pi}{T} = 4\pi$



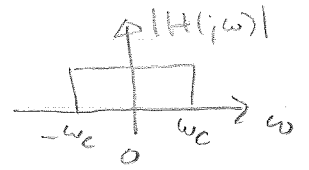
b) $T = 2 \text{ s} \Rightarrow \omega_s = \frac{2\pi}{T} = \pi$



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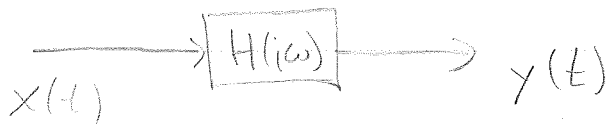
3/

$$H(j\omega) = \begin{cases} e^{-j\omega t_0} & , |\omega| \leq \omega_c \\ 0 & , |\omega| > \omega_c \end{cases}$$



$$|H(j\omega)| = 1, \quad \arg\{H(j\omega)\} = -\omega t_0 \quad \text{for } |\omega| \leq \omega_c$$

$$\omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = 3.5\omega_0$$



a/

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \begin{matrix} c_0 = 0 \\ c_k = \frac{2}{k^2}, \quad k \neq 0 \end{matrix}$$

$$x(t) \xrightarrow{FT} X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\omega - k\omega_0)$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) = \sum_{k=-3}^3 2\pi e^{-j\omega t_0} c_k \delta(\omega - k\omega_0) = 2\pi \sum_{k=-3}^3 c_k e^{-jk\omega_0 t_0} \delta(\omega - k\omega_0)$$

$$y(t) = \sum_{k=-3}^3 c_k^y e^{jk\omega_0 t} \quad \text{med } c_k^y = c_k \cdot e^{-jk\omega_0 t_0}$$

b/ Medeleffekt $\bar{P}_y = \frac{1}{T} \int_T |y(t)|^2 dt = \sum_{k=-3}^3 |c_k^y|^2 =$

$$= \left\{ \omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = 3.5\omega_0 \right\} = \sum_{k=-3}^3 |c_k^y|^2 = \{c_0 = 0\}$$

$$= 2 \cdot \left(\frac{2}{1}\right)^2 + 2 \cdot \left(\frac{2}{4}\right)^2 + 2 \cdot \left(\frac{2}{9}\right)^2 = 8 + \frac{1}{2} + \frac{8}{81} = 8.56$$

4/



$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$$a) \quad x[n] = u[n]$$

$$h[n] = \beta^n u[n] \quad |\beta| < 1$$

$$y[n] = \sum_{k=0}^{\infty} \beta^k u[n-k] =$$

$$= \sum_{k=0}^n \beta^k = \left\{ \text{Ändlig geom. Summe} \right\} =$$

$$= \frac{1 - \beta^{n+1}}{1 - \beta}, \quad n \geq 0$$

$$b) \quad x[n] = u[n-3]$$

$$y[n] = 0 \text{ f\"ur } n-3 < 0 \Rightarrow \underline{\underline{n < 3}}$$

$$\underline{\underline{n \geq 3}}, \quad y[n] = \sum_{k=0}^{n-3} \beta^k = \left\{ \text{Ändlig geom. Summe} \right\} =$$

$$= \frac{1 - \beta^{n-2}}{1 - \beta}$$

$$\text{Alt.} \quad x[n] \rightarrow y[n]$$

$$x[n-3] \rightarrow y[n-3]$$

5/

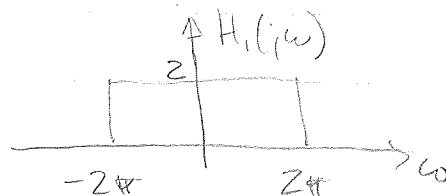
$$h(t) = 2 \frac{\sin(2\pi t)}{\pi t} \cdot \cos(7\pi t)$$

$$\underbrace{2 \frac{\sin(2\pi t)}{\pi t}}_{h_1(t) \xleftrightarrow{FT} H_1(j\omega)} \cdot \underbrace{\cos(7\pi t)}_{h_2(t) \xleftrightarrow{FT} H_2(j\omega)}$$

$$h(t) = h_1(t) \cdot h_2(t) \xleftrightarrow{FT} \frac{1}{2\pi} H_1(j\omega) * H_2(j\omega)$$

H_1 :

$$2 \frac{\sin(2\pi t)}{\pi t} \xleftrightarrow{FT} 2 [u(\omega + 2\pi) - u(\omega - 2\pi)] = H_1(j\omega)$$



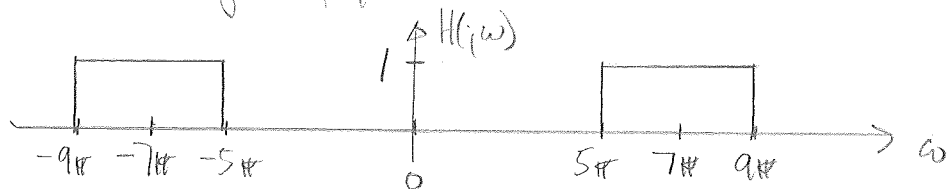
Se beta!

H_2 :

$$\cos(7\pi t) \xleftrightarrow{FT} \pi [\delta(\omega + 7\pi) + \delta(\omega - 7\pi)]$$

$$\begin{aligned} \frac{1}{2\pi} H_1(j\omega) * H_2(j\omega) &= \frac{1}{2\pi} H_1(j\omega) * \pi [\delta(\omega + 7\pi) + \delta(\omega - 7\pi)] = \\ &= \frac{\pi}{2\pi} [H_1(j(\omega + 7\pi)) + H_1(j(\omega - 7\pi))] = H(j\omega) \end{aligned}$$

Resultat enligt figur



b/

$$x(t) = \cos(2\pi t) + \sin(6\pi t)$$

$$X(j\omega) = \pi [\delta(\omega + 2\pi) + \delta(\omega - 2\pi)] + \frac{\pi}{j} [\delta(\omega + 6\pi) - \delta(\omega - 6\pi)]$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) \Rightarrow \begin{array}{l} \cos(2\pi t) \text{ filteras bort} \\ \sin(6\pi t) \text{ passerar} \end{array}$$

$$y(t) = \sin(6\pi t)$$