SSY041 A 101216

1/ a) A $\longleftrightarrow$ I

B $\longleftrightarrow$ II

C $\longleftrightarrow$ III

b) ej tidsinvariant

2/ a) $H(j\omega) = \dfrac{4}{2+j\omega}$

b) $y(t) = 1 + \sqrt{2}\cos\left(2t - \frac{\pi}{4}\right) - \frac{1}{\sqrt{5}}\cos(4t - 0{,}35\pi)$

c) $\overline{P} = 1^2 + \frac{1}{2}\left(\sqrt{2}^2 + \left(\frac{1}{\sqrt{5}}\right)^2\right) = 2{,}1$ (Parseval)

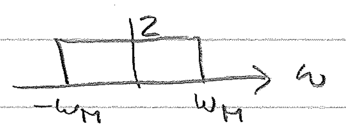
3/ $y[3] + y[4] = 1 + 1 = 2$

4/ $X_s(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$

$X_r(j\omega) = \frac{1}{2}X_s(j(\omega-\omega_c)) + \frac{1}{2}X_s(j(\omega+\omega_c)) =$

$= \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega-2\omega_c)) + \frac{1}{4}X(j(\omega+2\omega_c))$

Rekonstruktionsfilter




5/ $\hat{y}(t) = \frac{1}{2}\sin\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{4\pi}{10}t\right)$

1a)

$T_s = 3s \Rightarrow \omega_s = \frac{2\pi}{T_s}$

$x(t) = \cos\left(t + \frac{\pi}{4}\right)$ med $\omega = 1$ samplas

$x[n] = \cos\left(nT_s + \frac{\pi}{4}\right)$

Diskreta signalers Fouriertransform periodiska i $\omega$ med $\omega_s$. Samma sekvenser fås efter sampling om $\omega = 1 + \omega_s$ och $\omega = 1 + 2\omega_s$ (och allmänt $\omega = 1 + k\omega_s$)

$x_1(t) = \cos\left((1+\omega_s)t + \frac{\pi}{4}\right)$ och $x_2(t) = \cos\left((1+2\omega_s)t + \frac{\pi}{4}\right)$

$x[n] = \cos\left(n \cdot 3 + \frac{\pi}{4}\right)$

$x_1[n] = \cos\left(\left(1 + \frac{2\pi}{3}\right) 3n + \frac{\pi}{4}\right) = \cos\left((3+2\pi)n + \frac{\pi}{4}\right)$

$x_2[n] = \cos\left(\left(1 + \frac{4\pi}{3}\right) 3 \cdot n + \frac{\pi}{4}\right) = \cos\left((3+4\pi)n + \frac{\pi}{4}\right)$

Kontroll:

| $n$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x[n]$ | 0,707 | -0,800 | 0,877 | -0,936 |
| $x_1[n]$ | 0,707 | -0,800 | 0,877 | -0,936 |
| $x_2[n]$ | 0,707 | -0,800 | 0,877 | -0,936 |

1 b) $x(t) = \cos(\omega_0 t) + \cos(3\omega_0 t)$ Fouriertransformera!

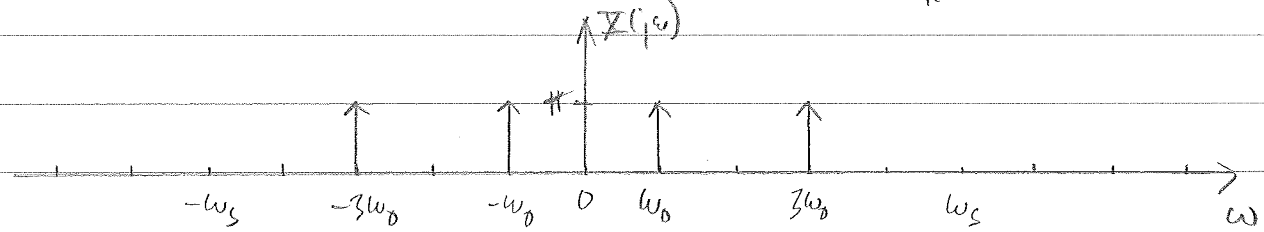
$$X(j\omega) = \pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] + \pi\left[\delta(\omega-3\omega_0)+\delta(\omega+3\omega_0)\right]$$

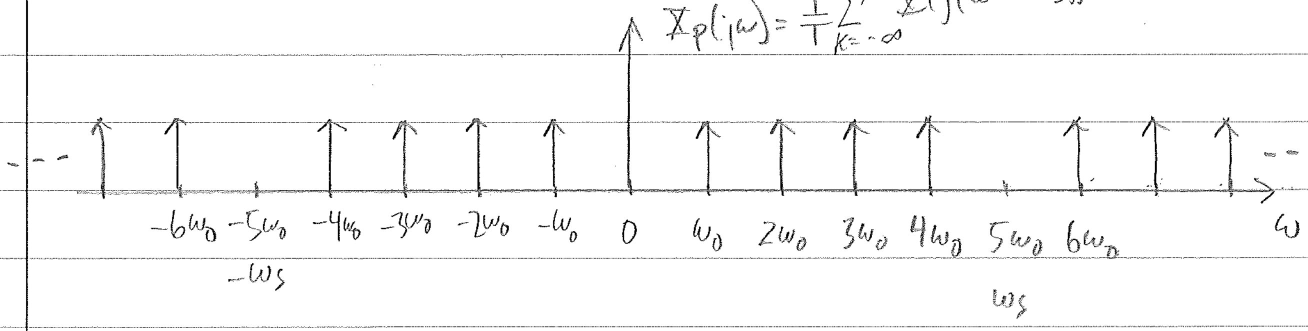
$$T_s = \frac{0{,}4\pi}{\omega_0} \Rightarrow \omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{0{,}4\pi}\omega_0 = 5\omega_0$$

$$x_p(t) = x(t)\cdot p(t) \quad \text{där} \quad p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$P(j\omega) = FT\{p(t)\} = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

$$x(t)\cdot p(t) \xleftrightarrow{FT} \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega-k\omega_s))$$



2. $y[n] = \frac{1}{4}\sum_{k=0}^{3} x[n-k]$

$x[n] = \left(1+\sqrt{2}\cos\left(\frac{n\pi}{4}\right)\right)\left(u[n]-u[n-4]\right)$

$x_1[n] = 1\cdot\left(u[n]-u[n-4]\right) = \ldots,0,0,\underset{n=0}{\underline{1}},1,1,1,0,0,\ldots$

$x_2[n] = \sqrt{2}\cos\left(\frac{n\pi}{4}\right)\left(u[n]-u[n-4]\right) = \ldots,0,0,\underset{n=0}{\underline{\sqrt{2}}},1,0,-1,0,0,\ldots$

Systemets impulssvar ("$x[n]=\delta[n]$") $\Rightarrow$ $h[n] = \frac{1}{4}\sum_{k=0}^{3}\delta[n-k]$

Superposition ger

| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| $x_1$: | $h[n]$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | | | | |
| | $h[n-1]$ | | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | | | |
| | $h[n-2]$ | | | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | | |
| | $h[n-3]$ | | | | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | |
| $x_2$: | $\sqrt{2}\,h[n]$ | $\frac{\sqrt{2}}{4}$ | $\frac{\sqrt{2}}{4}$ | $\frac{\sqrt{2}}{4}$ | $\frac{\sqrt{2}}{4}$ | | | | |
| | $1\cdot h[n-1]$ | | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | | | |
| | $0\cdot h[n-2]$ | | | 0 | 0 | 0 | 0 | | |
| | $-1\cdot h[n-3]$ | | | | $-\frac{1}{4}$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | |
| | $\sum \Rightarrow y[n]$ | $\frac{\sqrt{2}+1}{4}$ | $\frac{\sqrt{2}+3}{4}$ | $\frac{\sqrt{2}}{4}+1$ | $\frac{\sqrt{2}}{4}+1$ | $\frac{3}{4}$ | $\frac{1}{4}$ | 0 | |

Svar: $y[n] = 0,0,\underset{n=0}{\underline{\frac{\sqrt{2}+1}{4}}},\frac{\sqrt{2}+3}{4},\frac{\sqrt{2}}{4}+1,\frac{\sqrt{2}}{4}+1,\frac{3}{4},\frac{1}{4},0,0,\ldots$

3. $$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 10x(t)$$

Fouriertransformera.

$$(j\omega)^2 Y(j\omega) + 3j\omega Y(j\omega) + 2Y(j\omega) = 10X(j\omega)$$

Frekvenssvar

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{10}{2-\omega^2+3j\omega}$$

Insignal: $x(t) = \sqrt{\frac{10}{25}}\cos\left(2t + \frac{\pi}{3}\right)$

Sinusformad signal genom LTI-system

$$y(t) = \sqrt{\frac{10}{25}}\,|H(j\omega)|\cos\left(2t + \frac{\pi}{3} + \arg\{H(j\omega)\}\right)$$

och $\omega = 2$ r/s

$$|H(j\omega)|\Big|_{\omega=2} = \frac{10}{\sqrt{(2-4)^2+6^2}} = \frac{10}{\sqrt{40}} = \frac{10}{2\sqrt{10}} = \frac{5}{\sqrt{10}}$$

$$H(j\omega)\Big|_{\omega=2} = \frac{10}{2-4+j6} = \frac{10}{-2+j6} = \frac{5}{-1+j3}$$


$$\arg\{H(j\omega)\}\Big|_{\omega=2} = -\arg\{-1+j3\} = -108{,}4^\circ \mathrel{\hat{=}} -1{,}89 \text{ rad}$$

$$y(t) = \sqrt{\frac{10}{25}}\cdot\frac{5}{\sqrt{10}}\cos\left(2t + \frac{\pi}{3} - 1{,}89\right) =$$

$$= \cos(2t - 0{,}84) \mathrel{\hat{=}} \cos(2t - 48{,}4^\circ)$$

4.

a) $x(t) \rightarrow x(2t)$ Signalen "komprimeras" med faktor 2 längs tidsaxeln

$\omega_0 \rightarrow 2\omega_0$ (Grundvinkelfrekv.)

Samma signalform $\Rightarrow c_k$ ändras ej

Svar: Fourierseriekoeff. $= c_k$

b) $$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$x(t-t_0) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0(t-t_0)} =$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{-jk\omega_0 t_0} \cdot e^{jk\omega_0 t}$$

Svar: $c_{k_b} = c_k e^{-jk\omega_0 t_0}$

c) $$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t)$$

Om $x(t) = e^{j\omega t} \Rightarrow y(t) = H(j\omega) e^{j\omega t}$

Och för Fourierserien (superposition)

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \Rightarrow y(t) = \sum_{k=-\infty}^{\infty} c_k H(jk\omega_0) e^{jk\omega_0 t}$$

Medeleffekt (Parsevals formel)

$$\bar{P}_y = \frac{1}{T} \int_T y^2(t)\,dt = \sum_{k=-\infty}^{\infty} |c_k H(jk\omega_0)|^2$$

5 $y(t) = \frac{d}{dt}\{x(t)\} = \frac{d}{dt}\{e^{-2t} u(t-3)\}$

a) $x(t) = e^{-2t} u(t-3)$

$$y(t) = \frac{dx(t)}{dt} = -2e^{-2t} u(t-3) + e^{-2t}\delta(t-3) =$$

$$= -2e^{-2(t-3)} e^{-6} u(t-3) + \delta(t-3) \cdot e^{-2\cdot 3}$$

$$= e^{-6}\left(\delta(t-3) - 2e^{-2(t-3)} u(t-3)\right)$$

$$Y(j\omega) = FT\{y(t)\} = e^{-6}\left(e^{-j3\omega} - 2e^{-j3\omega}\frac{1}{2+j\omega}\right) =$$

$$= e^{-6} \cdot e^{-j3\omega}\left(1 - \frac{2}{2+j\omega}\right) = e^{-(6+j3\omega)} \cdot \frac{2+j\omega-2}{2+j\omega} =$$

$$= e^{-(6+j3\omega)} \cdot \frac{j\omega}{2+j\omega}$$

b) $X(j\omega) = FT\{x(t)\} = FT\{e^{-2(t-3)} u(t-3) \cdot e^{-6}\} =$

$$= e^{-6} \cdot \frac{e^{-j3\omega}}{2+j\omega}$$

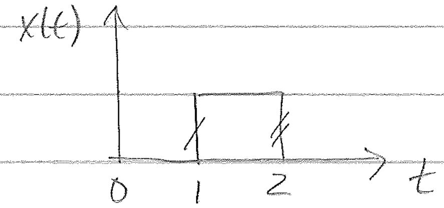
$$Y(j\omega) = j\omega \cdot X(j\omega) = e^{-6} \cdot e^{-j3\omega} \cdot \frac{j\omega}{2+j\omega} =$$

$$= e^{-(6+j3\omega)} \cdot \frac{j\omega}{2+j\omega}$$

V.S.V.

Sensorer, Signaler & System
SSY041 A 110822

1a) $x(t) = u(t-1) - u(t-2)$



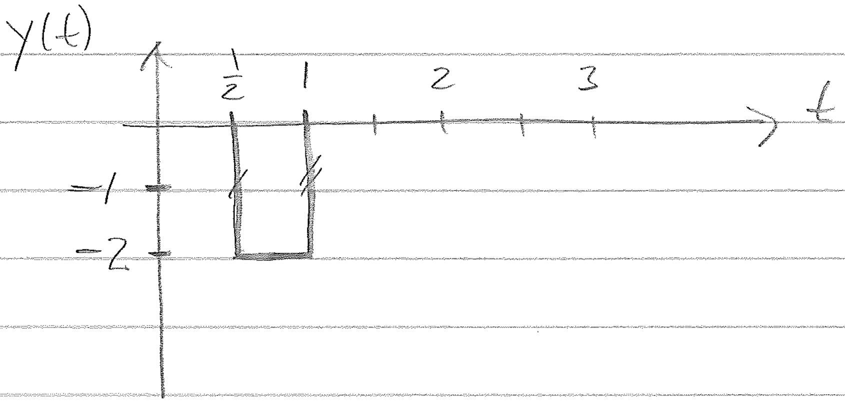
$$y(t) = -2\,x(3-2t) = -2\,x\left(2\left(\tfrac{3}{2}-t\right)\right) =$$
$$= -2\,x\left(2\left(-t+\tfrac{3}{2}\right)\right)$$

$x(1)$: $\quad 2\left(-t+\frac{3}{2}\right) = 1 \quad \Rightarrow \quad t = 1$

$x(1)$ mappas mot $y(1)$

$x(2)$: $\quad 2\left(-t+\frac{3}{2}\right) = 2 \quad \Rightarrow \quad t = \frac{1}{2}$

$x(2)$ mappas mot $y\left(\frac{1}{2}\right)$



SSY041 A
110822

1b/ $x(t) = 3 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

(i) $\omega_1 = \frac{2\pi}{3} \Rightarrow T_1 = \frac{2\pi}{\omega_1} = 3 \quad s$

$\omega_2 = \frac{5\pi}{3} \Rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi \cdot 3}{5\pi} = \frac{6}{5} \quad s$

$T_0 = k_1 T_1 = k_2 T_2 \qquad \frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{6}{5} \cdot \frac{1}{3} = \frac{2}{5}$

$\therefore \; k_1 = 2, \quad k_2 = 5 \Rightarrow T_0 = 2 \cdot 3 = 5 \cdot \frac{6}{5} = 6 \; s$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{6} \quad r/s$

(ii) $x(t) = 3 + \cos(2\omega_0 t) + 4\sin(5\omega_0 t) =$

$= 3 + \frac{1}{2}\left(e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right) + \frac{4}{2j}\left(e^{j5\omega_0 t} - e^{-j5\omega_0 t}\right)$

$= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

Identifiera!
$$\begin{cases} c_0 = 3 \\ c_2 = c_{-2} = \frac{1}{2} \\ c_5 = \frac{4}{2j} = -j2 \\ c_{-5} = -\frac{4}{2j} = j2 = c_5^* \\ \text{övriga } c_k = 0 \end{cases}$$

SSY041 A
110822

2 $\quad x(t) = e^{-t} u(t)$

$$E_t = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} e^{-2t} dt =$$

$$= \left[ \frac{e^{-2t}}{-2} \right]_0^{\infty} = 0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$$

Fouriertransformera!

$$FT\{x(t)\} = \frac{1}{1+j\omega} = X(j\omega) \Rightarrow |X(j\omega)|^2 = \frac{1}{1+\omega^2}$$

$$E_\omega = \frac{1}{2\pi} \int_{-1}^{1} |X(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-1}^{1} \frac{1}{1+\omega^2} d\omega = \{\text{"jämn"}\}$$

$$= \frac{1}{\pi} \int_{0}^{1} \frac{1}{1+\omega^2} d\omega = \frac{1}{\pi} \left[\arctan \omega\right]_0^1 =$$

$$= \frac{1}{\pi}\left[\frac{\pi}{4} - 0\right] = \frac{1}{4}$$

Kvot: $\dfrac{E_\omega}{E_t} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \dfrac{1}{2} \;\hat{=}\; 50\%$

SSY041 A
110822

3, $\frac{R}{L} V_0(t) + \frac{d}{dt} V_0(t) = \frac{d}{dt} V_i(t)$

Fouriertransformera !

$$\frac{R}{L} V_0(j\omega) + j\omega V_0(j\omega) = j\omega V_i(j\omega)$$

$$V_0(j\omega)\left(\frac{R}{L} + j\omega\right) = j\omega V_i(j\omega)$$

$$H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{j\omega}{\frac{R}{L} + j\omega}$$ Systemets Frekv. svar

Amplitud först:

a) $$|H(j\omega)| = \left|\frac{1}{1 + \frac{R}{L} \cdot \frac{1}{j\omega}}\right| = \frac{1}{\sqrt{1 + \left(\frac{R}{L\omega}\right)^2}} = \frac{1}{2}$$

$$1 + \left(\frac{R}{L\omega}\right)^2 = 2^2 = 4$$

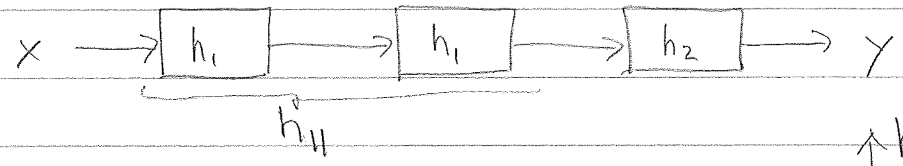
$$\left(\frac{R}{L\omega}\right)^2 = 3 \quad ; \quad \frac{R}{L} = \sqrt{3} \cdot \omega = \sqrt{3} \cdot 1000$$

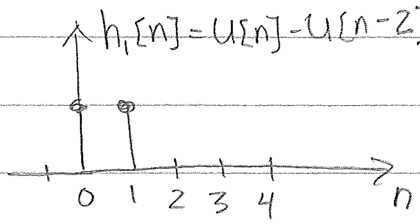
b) $$|H(j\omega)| = \left|\frac{1}{\sqrt{1 + \left(\frac{R}{L\omega}\right)^2}}\right| = 1 \text{ då } \omega \to \infty$$

Utsignal och insignal får samma amplitud (sinusformad signal) då $\omega \to \infty$

SSY041 A
110822

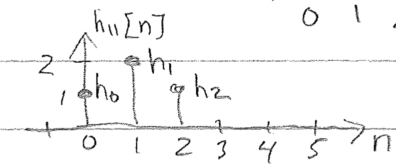
4.



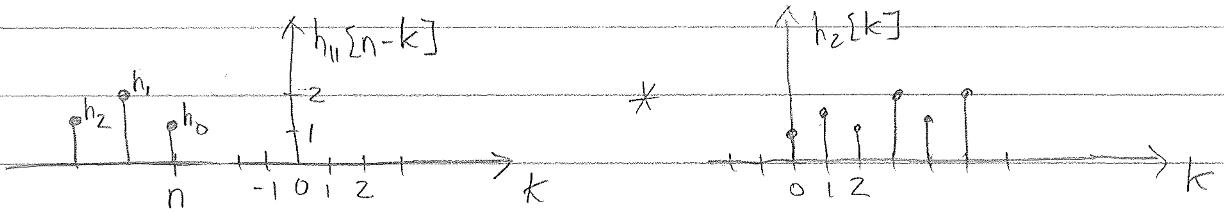




$$h_{11}[n] = h_1[n] * h_1[n] = \sum_{k=-\infty}^{\infty} h_1[k]h_1[n-k]$$

$$h_{11}[n] = \delta[n] + 2\delta[n-1] + \delta[n]$$




Hela systemets impulssvar: $h[n] = h_2[n] * h_{11}[n] = \sum_{k=-\infty}^{\infty} h_2[k] \cdot h_{11}[n-k]$




$n < 0 \Rightarrow h_2[n] = 0$ ty $h[n] = 0$ för $n < 0$

$n=0$ $h[0] = h_2[0] \cdot h_0 = h_2[0] \cdot 1 = 1$ $\Rightarrow h_2[0] = 1$

$n=1$ $h[1] = h_2[0] \cdot h_1 + h_2[1] \cdot h_0 = 1 \cdot 2 + h_2[1] \cdot 1 = 5$ $\Rightarrow h_2[1] = 3$

$n=2$ $h[2] = h_2[0] \cdot h_2 + h_2[1] \cdot h_1 + h_2[2] \cdot h_0 = 1 \cdot 1 + 3 \cdot 2 + h_2[2] \cdot 1 = 10$ $\Rightarrow h_2[2] = 3$

$n=3$ $h[3] = h_2[1] \cdot h_2 + h_2[2] \cdot h_1 + h_2[3] \cdot h_0 = 3 + 3 \cdot 2 + h_2[3] \cdot 1 = 11$ $\Rightarrow h_2[3] = 2$

$n=4$ $h[4] = h_2[2] \cdot h_2 + h_2[3] \cdot h_1 + h_2[4] \cdot h_0 = 3 \cdot 1 + 2 \cdot 2 + h_2[4] \cdot 1 = 8$ $\Rightarrow h_2[4] = 1$

$n=5$ $h[5] = h_2[3] \cdot h_2 + h_2[4] \cdot h_1 + h_2[5] \cdot h_0 = 2 \cdot 1 + 1 \cdot 2 + h_2[5] \cdot 1 = 4$ $\Rightarrow h_2[5] = 0$

$n=6$ $h[6] = h_2[4] \cdot h_2 + h_2[5] \cdot h_1 + h_2[6] \cdot h_0 = 1 \cdot 1 + 0 \cdot 2 + h_2[6] \cdot 1 = 1$ $\Rightarrow h_2[6] = 0$

$n=7$ $h[7] = h_2[5] \cdot h_2 + h_2[6] \cdot h_1 + h_2[7] \cdot h_0 = 0 + 0 + h_2[7] \cdot 1 = 0$ $\Rightarrow h_2[7] = 0$

$$h_2[n] = \{\ldots 0, 0, \underset{\substack{\uparrow \\ n=0}}{\underline{1}}, 3, 3, 2, 1, 0, 0, 0, \ldots\}$$

$$h_2[n] = 0 \begin{cases} n < 0 \\ n \geq 5 \end{cases}$$

554041 A
110822

5. $y_1(t) = x_1(t) * x_2(t)$ $\qquad X_1(j\omega) = 0 \ , \ |\omega| > 500\pi$ r/s

$X_2(j\omega) = 0 \ , \ |\omega| > 1500\pi$ r/s

(i) $Y_1(j\omega) = X_1(j\omega) \cdot X_2(j\omega)$

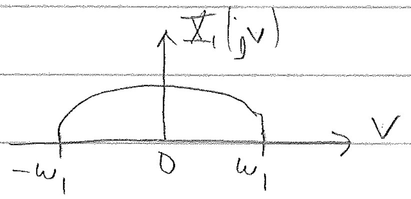
Då är $Y_1(j\omega) = 0$ för $|\omega| > 500\pi$ ty
$X_1(j\omega) = 0$ för $|\omega| > 500\pi$

Samplingsteoremet ger $\omega_s > 2\omega_{max} = 2 \cdot 500\pi = 1000\pi$
($\omega_s$ = samplingsvinkelfrekv.)
Samplingsintervall $T = \frac{2\pi}{\omega_s} \Rightarrow T < \frac{2\pi}{2 \cdot \omega_{max}} = \frac{2\pi}{2 \cdot 500\pi} =$

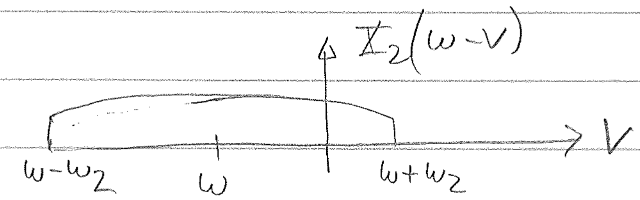
$= \frac{1}{500} = \underline{\underline{2.0 \text{ ms}}}$

(ii) $y_2(t) = x_1(t)\, x_2(t) \Rightarrow Y_2(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega) =$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(jv) X_2(j(\omega - v))\, dv$




Låt $\omega$ variera från $-\infty$ till $\infty$.




$Y_2(j\omega) = 0$ då $|\omega| > \omega_1 + \omega_2 =$
$= (500 + 1500)\pi =$
$= 2000\pi$

$T < \frac{2\pi}{2 \cdot 2000\pi} = \underline{\underline{0{,}5 \text{ ms}}}$