Sensorer, Signaler & System, A

SSY041-A 100407

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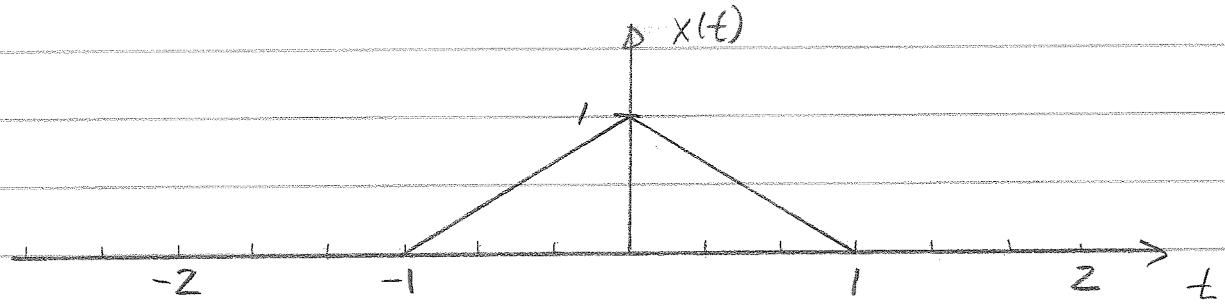
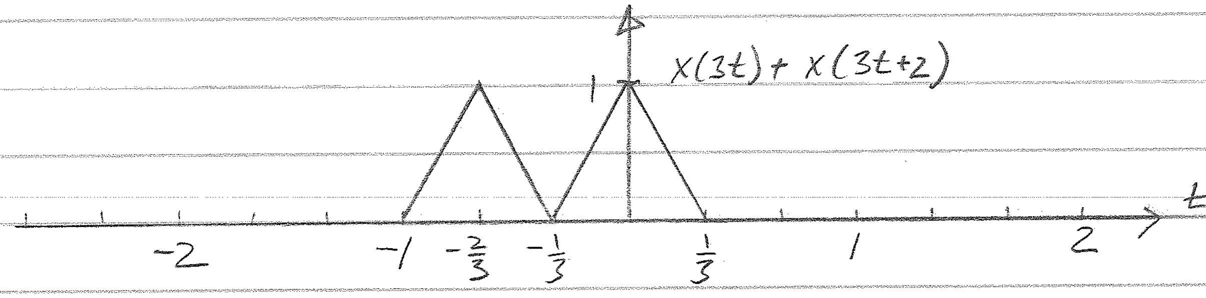
a)




$x(3t)$ : "komprimering" med faktor 3

$x(3t+2) = x(3(t+\frac{2}{3}))$ : Komprimering med faktor 3

Förskjutning (vänster) med $\frac{2}{3}$

b) $x[n] = 2\cos\left(\frac{9\pi n}{4}\right) - 3\sin\left(\frac{6\pi n}{5} + \frac{\pi}{3}\right) =$

$= 2\cos(\Omega_1 n) - 3\sin(\Omega_2 n + \frac{\pi}{3})$

$\Omega_1(n+N_1) = \Omega_1 n + \Omega_1 N_1 \Rightarrow \Omega_1 N_1 = 2\pi m_1 \qquad N_1, m_1 \in \mathbb{Z}$

$N_1 = \frac{2\pi m_1}{\Omega_1} = \frac{2\pi m_1 \cdot 4}{9\pi} = \frac{8 m_1}{9}$ ; $m_1 = 9 \Rightarrow N_1 = 8$

$\Omega_2(n+N_2) = \Omega_2 n + \Omega_2 N_2 \Rightarrow \Omega_2 N_2 = 2\pi m_2 \qquad N_2, m_2 \in \mathbb{Z}$

$N_2 = \frac{2\pi m_2}{\Omega_2} = \frac{2\pi m_2 5}{6\pi} = \frac{5 m_2}{\cancel{6}\,3} \qquad m_2 = \cancel{6}\,3 \Rightarrow N_2 = 5$

$N = k_1 N_1 = k_2 N_2 \qquad \frac{k_1}{k_2} = \frac{N_2}{N_1} = \frac{5}{8} \qquad k_1, k_2 \in \mathbb{Z}$

$N = 5 \cdot 8 = 8 \cdot 5 = 40$ Svar: $\underline{\underline{N = 40}}$

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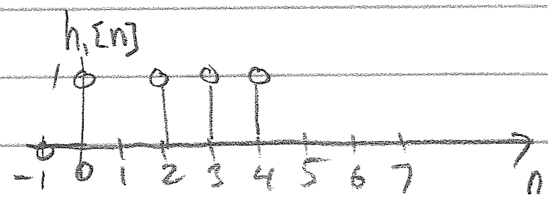
2.

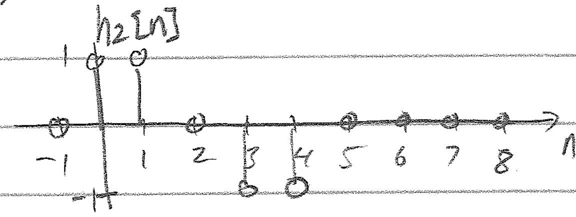
$h_1[n] = \delta[n] + u[n-2] - u[n-5]$




$h_1[n] = \ldots, 0, 0, 0, \underline{1}, 0, 1, 1, 1, 0, 0, \ldots$

$n=0$

$h_2[n] = \begin{cases} 1, & n = 0,1 \\ -1, & n = 3,4 \\ 0, & \text{annars} \end{cases}$




$h_2[n] = \ldots, 0, 0, 0, \underline{1}, 1, 0, -1, -1, 0, 0, 0, \ldots$

$n=0$

$h_1[n]$ insignal till system med impulssvar $h_2[n]$

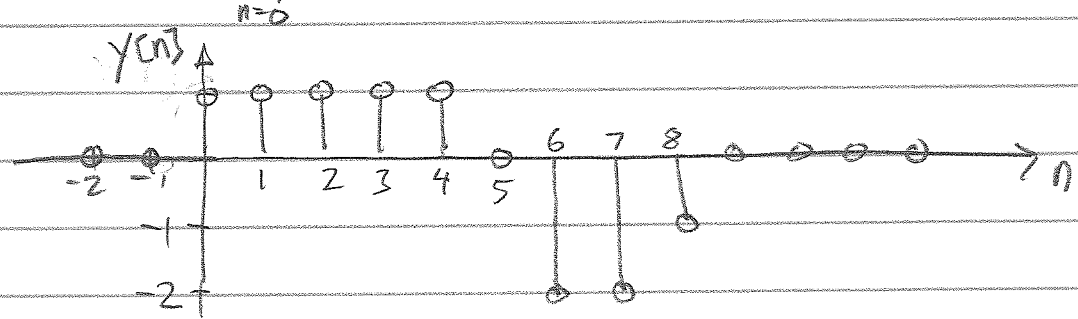
Superposition! (eller $h_1[n] * h_2[n]$)

$y[n] = h_2[n] + h_2[n-2] + h_2[n-3] + h_2[n-4]$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h_2[n]$ | 1 | 1 | 0 | -1 | -1 | | | | | | |
| $h_2[n-2]$ | | | 1 | 1 | 0 | -1 | -1 | | | | |
| $h_2[n-3]$ | | | | 1 | 1 | 0 | -1 | -1 | | | |
| $h_2[n-4]$ | | | | | 1 | 1 | 0 | -1 | -1 | | |
| $\Sigma \Rightarrow y[n]$ | 1 | 1 | 1 | 1 | 1 | 0 | -2 | -2 | -1 | | |

$y[n] = \ldots, 0, 0, \underline{1}, 1, 1, 1, 1, 0, -2, -2, -1, 0, 0, \ldots$

$n=0$



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3)

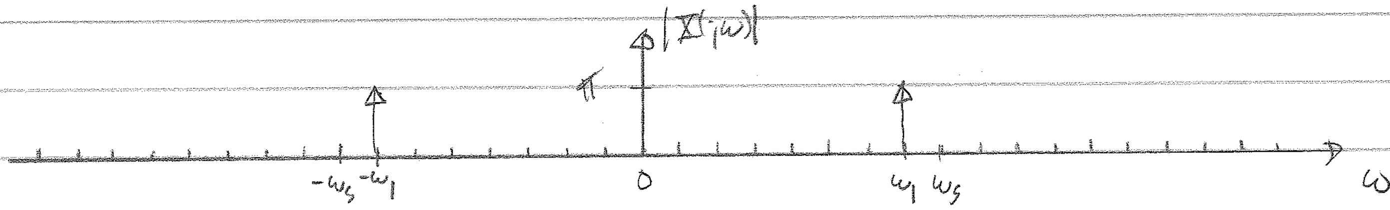
$$x(t) \longrightarrow \otimes \longrightarrow x_p(t) = x(t)p(t)$$

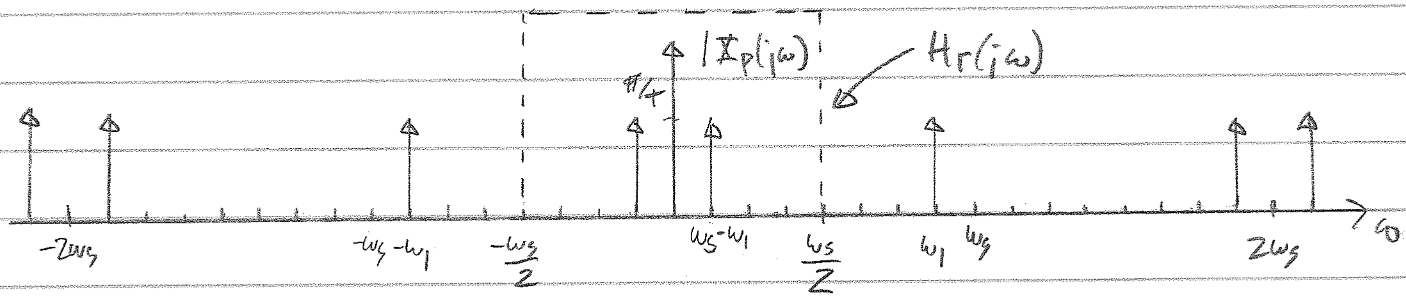
$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \qquad \omega_s = \frac{2\pi}{T}$$

$$x(t) = \sin(\omega_1 t) \xleftrightarrow{FT} X(j\omega) = \frac{\pi}{j}\left[\delta(\omega-\omega_1) - \delta(\omega+\omega_1)\right]$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \xleftrightarrow{FT} P(j\omega) = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$x_p(t) = x(t)p(t) \xleftrightarrow{FT} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty}$$




$$X_p(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(j(\omega - n\omega_s))$$

Idealt LP-filter enligt spec. $H_r(j\omega)$, se fig.

$$Y(j\omega) = X_p(j\omega)\cdot H_r(j\omega) = \frac{\pi}{j}\left[\delta(\omega-(\omega_s-\omega_1)) - \delta(\omega+(\omega_s-\omega_1))\right]$$

Signalen $y(t)$ med Fouriertransf. $Y(j\omega)$ blir

$$y(t) = \sin((\omega_s-\omega_1)t), \qquad \omega_1 = \frac{7}{8}\omega_s$$

$$\omega_s - \omega_1 = \frac{1}{8}\omega_s$$

Aliasing!

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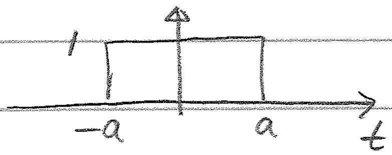
4)

Från Beta    enhetssteg: u(t)

$$\theta(t+a) - \theta(t-a) \xleftrightarrow{FT} \frac{2\sin a\omega}{\omega}$$




Med $a = T$

$$x(t) = u(t+T) - u(t-T) \xleftrightarrow{FT} X(j\omega) = \frac{2\sin T\omega}{\omega} =$$

$$= 2T\,\frac{\sin T\omega}{T\omega} =$$

$$= 2T\,\mathrm{sinc}(T\omega)$$

a) $\mathrm{sinc}(T\omega)$ har max värde $= 1$ då $\omega \to 0$

$$X(j\omega)\big|_{max} = 2T$$

$X(j\omega) = 0$ då $\sin(T\omega) = 0$ $(\omega > 0)$ $T\omega = n\cdot\pi$

$$\omega = n\,\frac{\pi}{T} \quad n = 1, 2, 3, 4, \ldots$$

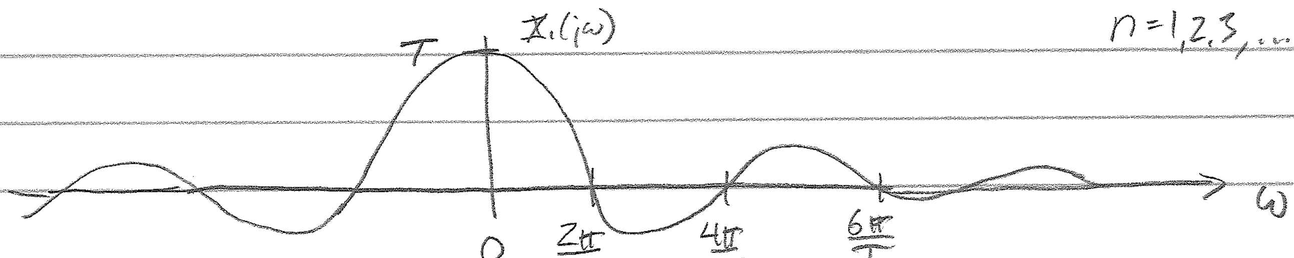
b) $a = \frac{T}{2}$, $x_1(t) = u(t+\frac{T}{2}) - u(t-\frac{T}{2}) \xleftrightarrow{FT} X_1(j\omega) = \frac{2\sin\frac{T}{2}\omega}{\omega}$

$$X_1(j\omega) = 2\cdot\frac{T}{2}\,\frac{\sin\frac{T}{2}\omega}{\frac{T}{2}\omega} = T\,\mathrm{sinc}\left(\frac{T}{2}\omega\right)$$

$X_1(j\omega)\big|_{max} = T$ då $\omega \to 0$

$X_1(j\omega) = 0$ $(\omega > 0)$ då $\frac{T}{2}\omega = n\cdot\pi \Rightarrow \omega = n\cdot\frac{2\pi}{T}$

$n = 1, 2, 3, \ldots$



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$$\begin{cases} -u_s(t) + u_L(t) + u_R(t) = 0 \\ u_R(t) = i(t)\cdot R \\ u_L(t) = L\,\dfrac{di(t)}{dt} \end{cases}$$

a/
$$L\frac{di(t)}{dt} + R\,i(t) = u_s(t)$$

$$\frac{L}{R}\frac{di(t)}{dt} + i(t) = \frac{1}{R}u_s(t)$$

Fouriertransf.

$$\frac{L}{R} j\omega I(j\omega) + I(j\omega) = \frac{1}{R}U_s(j\omega)$$

$$I(j\omega) = \frac{\frac{1}{R}U_s(j\omega)}{1 + j\omega\frac{L}{R}}$$

Mult. med $R$ i HL och V.L.

$$R\cdot I(j\omega) = U_R(j\omega)$$

$$\frac{U_R(j\omega)}{U_s(j\omega)} = H(j\omega) = \frac{1}{1 + j\omega\frac{L}{R}}$$

$H(j\omega)$: systemets frekv. svar

b/

Insignalspänning $u_s(t)$

Fourierserie från Beta

$(L = \frac{T}{2},\ h = 10)$

$$u_s(t) = \frac{2\cdot 10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n2\pi t}{T}\right)$$

Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T}$

LTI-system (för sinusformad signal)

$$\sin(\omega t) \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega)\sin(\omega t) =$$

$$|H(j\omega)|\sin(\omega t + \arg\{H(j\omega)\})$$

Superposition gör

$$u_R(t) = \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cdot H(jn\omega_0)\sin(n\omega_0 t)$$

$$u_R(t) = \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}H(jn\omega_0)\sin(n\omega_0 t) = \left\{\omega_0 = \frac{2\pi}{T}\right\} =$$

$$= \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\underbrace{\left|\frac{1}{1+jn\omega_0\frac{L}{R}}\right|}_{\text{Amplitud-påverkan}}\sin\Bigg(n\omega_0 t + \underbrace{\arg\left\{\frac{1}{1+jn\omega_0\frac{L}{R}}\right\}}_{\text{Fasförskjutn.}}\Bigg)$$