Sensorer Signaler & System
del A SSY041 A
081218

1/ a)

Superposition ger $y[n] = h[n+3] + 2h[n+2] - h[n] - 2h[n-1] + h[n-2]$

	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$h[n+3]$		1	0,5	0,25								
$2h[n+2]$			2	1	0,5							
$-h[n]$					-1	-0,5	-0,25					
$-2h[n-1]$						-2	-1	-0,5				
$h[n-2]$							1	0,5	0,25			
$\Sigma \Rightarrow y[n]$		1	2,5	1,25	-0,5	-2,5	-0,25	0	0,25			

Svar: $y[n] = \delta[n+3] + 2{,}5\,\delta[n+2] + 1{,}25\,\delta[n+1] - 0{,}5\,\delta[n] - 2{,}5\,\delta[n-1] - 0{,}25\,\delta[n-2] + 0{,}25\,\delta[n-4]$

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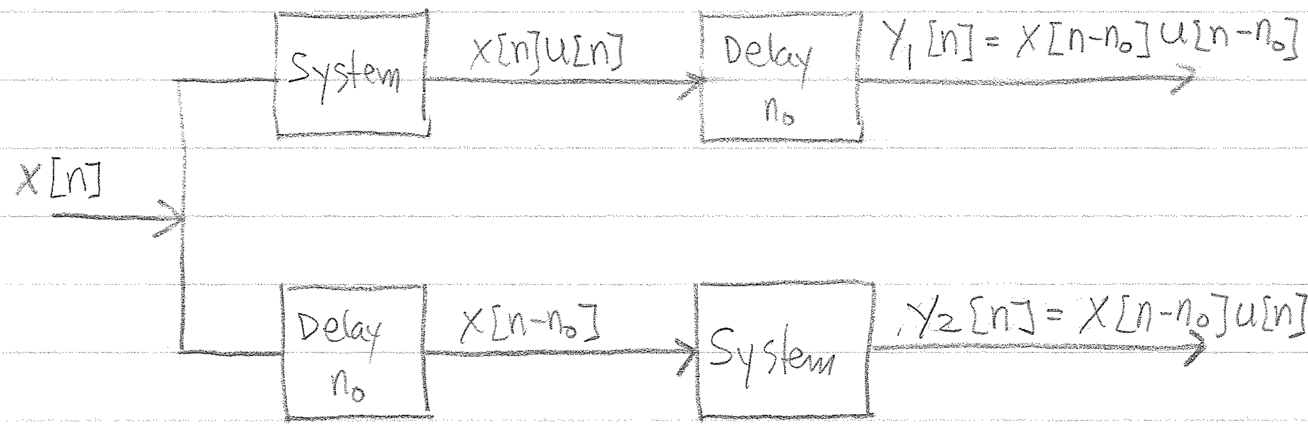
1b/

$$y[n] = x[n]\,u[n]$$

Test



$$y_1[n] \neq y_2[n]$$

Ej tidsinvariant

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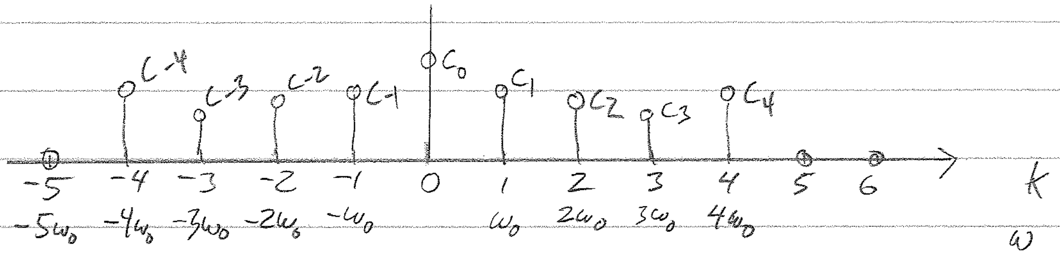
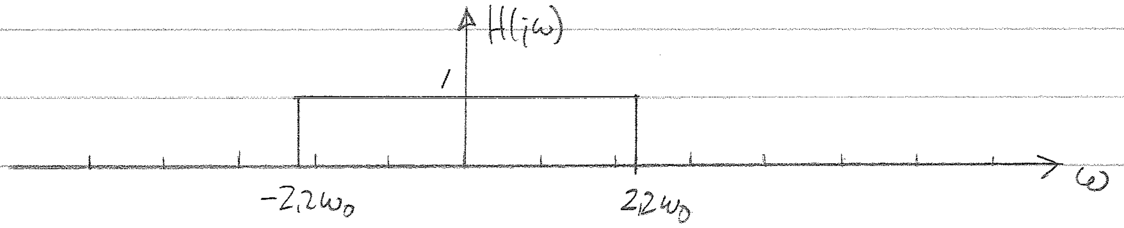
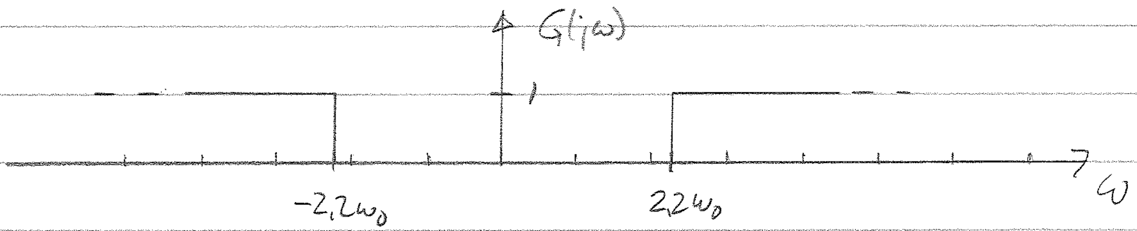
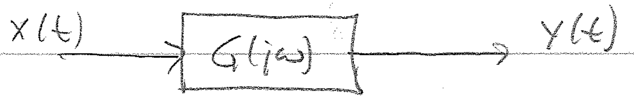
FS

2)









$G(j\omega)$ släpper endast igenom frekvenser $|\omega| > 2{,}2\,\omega_0$

FS-koeff till $y(t)$ blir då $\begin{cases} c_3,\ c_{-3},\ c_4 \text{ och } c_{-4} \\ \text{övriga } c_k = 0 \end{cases}$

Medeleffekt $P = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$P_x = |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 + 2|c_3|^2 + 2|c_4|^2 =$$
$$= 4 + 2\cdot 1 + 2\cdot 0{,}5^2 + 2\cdot 0{,}2^2 + 2\cdot 0{,}4^2 = 6{,}9$$

$$P_y = 2|c_3|^2 + 2|c_4|^2 = 2\cdot 0{,}2^2 + 2\cdot 0{,}4^2 = 0{,}4$$

$$\frac{P_y}{P_x} = \frac{0{,}4}{6{,}9} \approx 0{,}058$$

3.

$$U_R(j\omega) = U_i(j\omega)\, H(j\omega)$$

$$H(j\omega) = \frac{R}{R + j\omega L - j\frac{1}{\omega C}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$|H(j\omega)|$ påverkar utsignalens amplitud

$$|H(j\omega)| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$|H(j\omega)|$ störst då $\omega L - \frac{1}{\omega C} = 0$

$\therefore \quad \omega^2 = \omega_p^2 = \frac{1}{LC} \qquad \omega_p = \frac{1}{\sqrt{LC}} = \ldots = 10^4$ r/s

Vid $\omega = \omega_p$ är $H(j\omega)\Big|_{\omega=\omega_q} = 1 = 1\angle 0^\circ$

$\angle H(j\omega)\Big|_{\omega=\omega_q} = 0$ Ingen fasskillnad!

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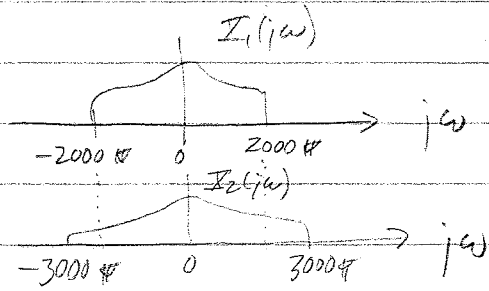
4) $y_1(t) = x_1(t) * x_2(t)$ $\qquad X_1(j\omega) = 0, \ |\omega| > 2000\pi$

$X_2(j\omega) = 0, \ |\omega| > 3000\pi$

$$Y_1(j\omega) = X_1(j\omega)\,X_2(j\omega)$$

Alltså är $Y_1(j\omega) = 0$ för $|\omega| > 2000\pi$

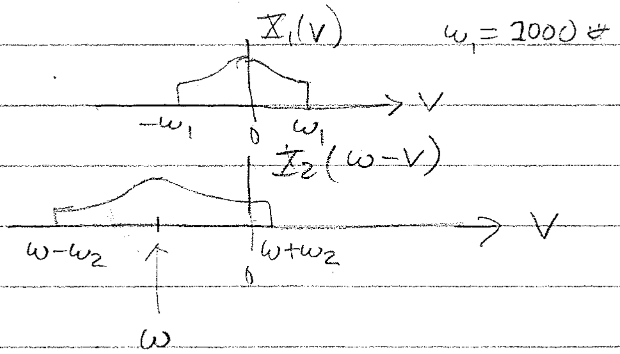


Samplingsteoremet ger $\omega_s > 2\cdot\omega_{max} = 2\cdot 2000\pi$
(ω_s = samplingsintervall)

$$T = \frac{2\pi}{\omega_s} \Rightarrow T < \frac{2\pi}{2\cdot 2000\pi} = \frac{1}{2000} = 0{,}5 \text{ ms}$$

$$y_2(t) = x_1(t)\,x_2(t) \Rightarrow Y_2(j\omega) = \frac{1}{2\pi} X_1(j\omega) * X_2(j\omega) =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(jv)\,X_2(j(\omega - v))\,dv$$



Låt ω variera från $-\infty$ till $+\infty$

$$\Rightarrow Y_2(j\omega) = 0 \ ; \ |\omega| > \omega_1 + \omega_2 = 5000\pi$$

$$T < \frac{2\pi}{2\cdot 5000\pi} = \frac{1}{5000} = 0{,}2 \text{ ms}$$

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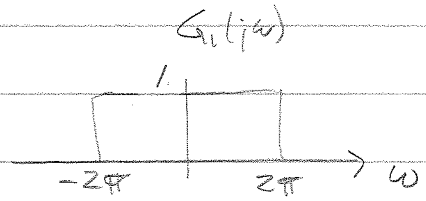
5. $h(t) = \underbrace{\frac{\sin(2\pi t)}{\pi t}}_{g_1(t)} \cdot \underbrace{\cos(7\pi t) \cdot 2}_{g_2(t)}$

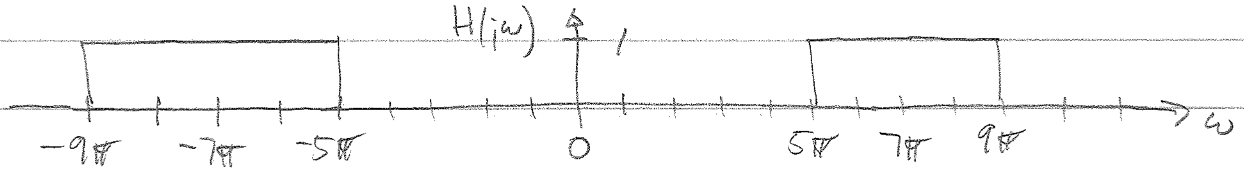
$g_1(t) = 2\,\frac{\sin(2\pi t)}{2\pi t} = 2\,\mathrm{sinc}(2\pi t) \overset{FT}{\longleftrightarrow} \underbrace{\frac{2 \cdot \pi}{2\pi}\,\mathrm{rect}\left(\frac{\omega}{4\pi}\right)}_{G_1(j\omega)}$



$g_2(t) = 2 \cdot \frac{1}{2}\left(e^{j7\pi t} + e^{-j7\pi t}\right)$

Allmänt $e^{j\omega_0 t}\, x(t) \overset{FT}{\longleftrightarrow} X(j(\omega - \omega_0))$

Alltså blir $H(j\omega) = G(j(\omega - 7\pi)) + G(j(\omega + 7\pi))$



$$H(j\omega) = \begin{cases} 1, & \omega \in [-9\pi, -5\pi] \\ 1, & \omega \in [5\pi, 9\pi] \\ 0, & \text{annars} \end{cases}$$

Insignal $x(t) = \cos(2\pi t) + \sin(6\pi t) \overset{FT}{\longleftrightarrow} X(j\omega) = \pi\left[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)\right] + \frac{\pi}{j}\left[\delta(\omega - 6\pi) - \delta(\omega + 6\pi)\right]$

$Y(j\omega) = H(j\omega)\, X(j\omega) = \frac{\pi}{j}\left[\delta(\omega - 6\pi) - \delta(\omega + 6\pi)\right]$

Svar: $y(t) = \sin(6\pi t)$

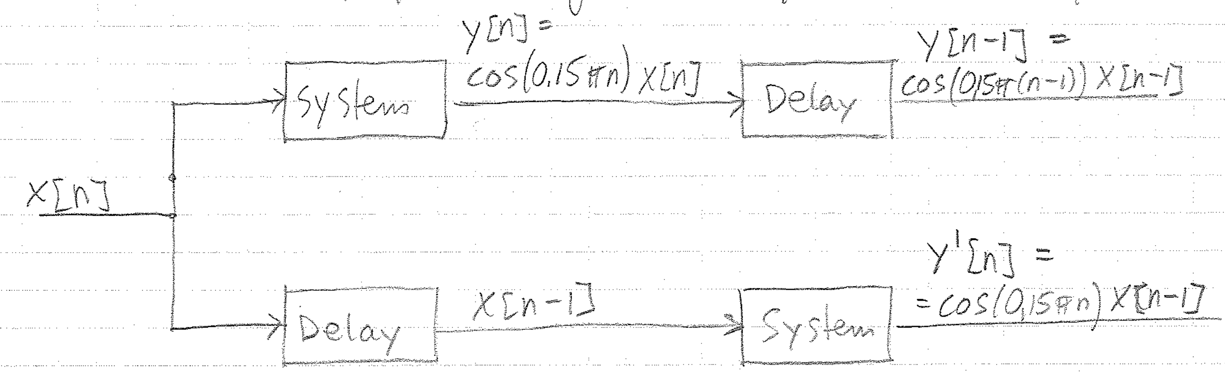
1 a) $y[n] = \cos(0.15\pi n)\, x[n]$

i)

Insignal	Utsignal
$x[n]$	$y[n] = \cos(0{,}15\pi n)\, x[n]$
$a\,x[n]$	$\cos(0{,}15\pi n)\, a\,x[n] = a\,y[n]$
$x_1[n] + x_2[n]$	$\cos(0{,}15\pi n)\,(x_1[n] + x_2[n]) = y_1[n] + y_2[n]$
$a x_1[n] + b x_2[n]$	$a y_1[n] + b y_2[n]$

Superposition gäller – systemet är linjärt

ii)



$y'[n] \neq y[n-1] \Rightarrow$ Ej tidsinvariant

iii) Impulssvar $x[n] = \delta[n]$

$y[n] = h[n] = \cos(0{,}15\pi n)\,\delta[n] = \cos(0)\,\delta[n] = \delta[n]$

b) $x(t) = 1 + \cos(8\pi \cdot 10^3 t) + 7{,}4 \sin(\pi \cdot 10^4 t)$, $\forall t$

$\omega_1 = 8\pi \cdot 10^3 \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{8\pi \cdot 10^3} = 0{,}25 \cdot 10^{-3}$ s

$\omega_2 = \pi \cdot 10^4 \Rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi \cdot 10 \cdot 10^3} = 0{,}2 \cdot 10^{-3}$ s

Gemensam periodtid $m, n \in \mathbb{Z}$

$T_0 = m T_1 = n T_2$

$\frac{m}{n} = \frac{T_2}{T_1} = \frac{0{,}2}{0{,}25} = \frac{20}{25} = \frac{4}{5}$

$T_0 = 4 \cdot T_1 = 5 \cdot T_2 = 1 \cdot 10^{-3}\,\text{s} = 1{,}0$ ms

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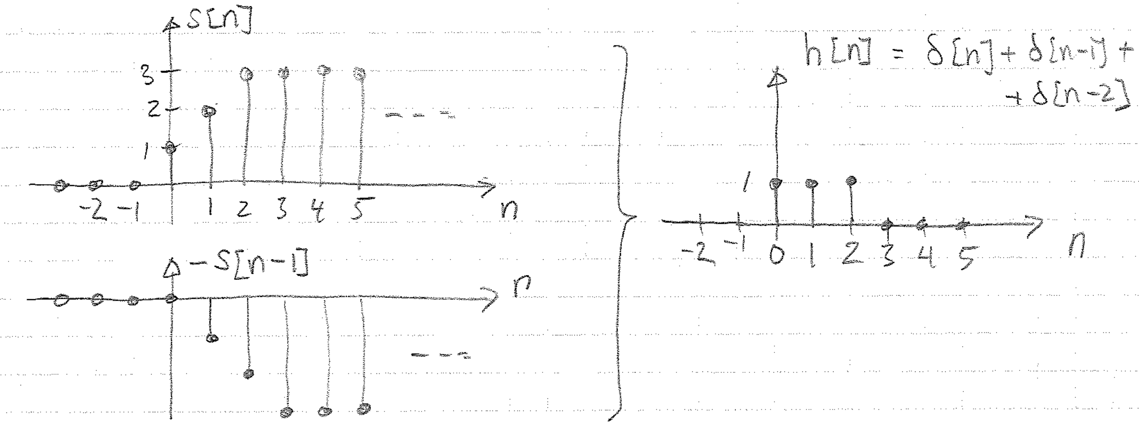
2.1

Insignal → System → Utsignal

$x[n]$	$y[n]$
$\delta[n]$	$h[n]$ Impulssvar
$u[n]$	$s[n]$ Stegsvar
$\delta[n] = u[n] - u[n-1]$	$h[n] = s[n] - s[n-1]$ Superpos.

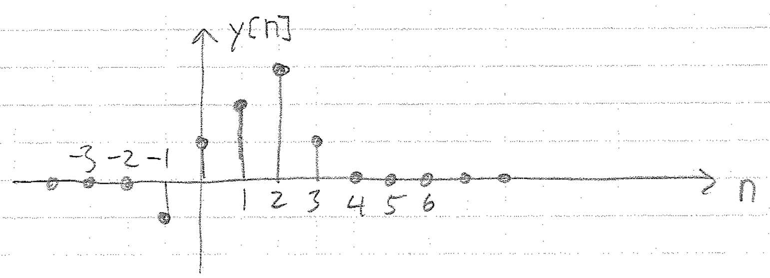


Insignal $x[n] = -\delta[n+1] + 2\delta[n] + \delta[n-1]$

Utsignal $y[n] = -h[n+1] + 2h[n] + h[n-1]$

n	-3 -2 -1 0 1 2 3 4 5
$-h[n+1]$	0 0 -1 -1 -1 0 0 0 0
$2h[n]$	0 0 0 2 2 2 0 0 0
$h[n-1]$	0 0 0 0 1 1 1 0 0
$\Sigma \Rightarrow y[n]$	0 0 -1 1 1 2 3 1 0 0

Svar: $y[n] = -\delta[n+1] + \delta[n] + \delta[n-1] + 3\delta[n-2] + \delta[n-3]$



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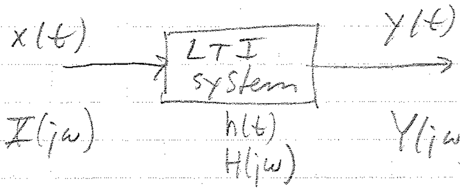
3)

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{k}{1+k^2} e^{jk4t} \qquad (\omega_0 = 4)$$

$$x_1(t) = e^{jk\omega_0 t} \xleftrightarrow{FT} 2\pi\delta(\omega - k\omega_0) = X_1(j\omega)$$



$$Y(j\omega) = H(j\omega)X(j\omega)$$

Insignal transf.

$$X_1(j\omega) = 2\pi\delta(\omega - k\omega_0)$$

Utsignal transf.

$$H(j\omega)\,2\pi\delta(\omega - k\omega_0) = 2\pi H(jk\omega_0)\delta(\omega - k\omega_0)$$

$$\Rightarrow y_1(t) = H(jk\omega_0)e^{jk\omega_0 t}$$

Superpos. ger

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0) \frac{k}{1+k^2} e^{jk4t}$$

$$h(t) = \delta(t) - e^{-3t}u(t) \xleftrightarrow{FT} H(j\omega) = 1 - \frac{1}{3+j\omega} = \frac{3+j\omega-1}{3+j\omega} =$$

$$= \frac{2+j\omega}{3+j\omega}$$

Utsignalens kompleta Fourierserie

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0) \frac{k}{1+k^2} e^{jk\omega_0 t} = \left\{ \omega_0 = 4,\ H(j\omega) = \frac{2+j\omega}{3+j\omega} \right\} =$$

$$= \sum_{k=-\infty}^{\infty} \frac{2+jk4}{3+jk4} \cdot \frac{k}{1+k^2} e^{jk4t}$$

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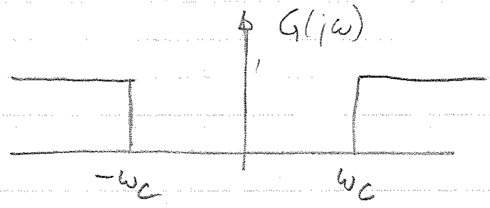
4./ Parsevals relation, total energi hos signalen $x(t)$ är

$$E_t = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$x(t) = e^{-bt}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{b+j\omega} \qquad \left| \begin{array}{l} b \in \mathbb{R} \\ b > 0 \end{array} \right.$$



$$E_t = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} e^{-2bt} dt = \left[\frac{e^{-2bt}}{-2b}\right]_0^{\infty} =$$

$$= 0 - \left(-\frac{1}{2b}\right) = \frac{1}{2b}$$

Bortfiltrerad energi

$$E_f = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} |X(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{1}{b^2+\omega^2}\cdot d\omega = \{\text{jämn fkn}\} =$$

$$= \frac{2}{2\pi}\int_0^{\omega_c} \frac{d\omega}{b^2+\omega^2} = \frac{1}{\pi}\cdot\frac{1}{b}\left[\arctan\frac{\omega}{b}\right]_0^{\omega_c} =$$

$$= \frac{1}{\pi b}\left(\arctan\frac{\omega_c}{b} - 0\right)$$

$$\frac{\text{Utsignalens energi}}{\text{Total energi}} = \frac{E_t - E_f}{E_t} = 1 - \frac{E_f}{E_t} =$$

$$= 1 - \frac{\frac{1}{\pi b}\arctan\left(\frac{\omega_c}{b}\right)}{\frac{1}{2b}} =$$

$$= 1 - \frac{2}{\pi}\arctan\left(\frac{\omega_c}{b}\right)$$

5./ 24 varv/s $\Rightarrow$ $\omega = 2\pi \cdot 24$ r/s

Vinkelförändring per bild $\phi = T_s \cdot \omega$

a/

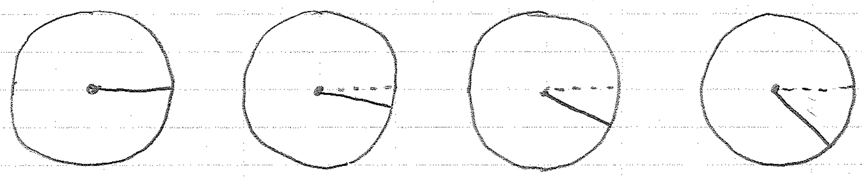
$$\phi = \frac{23}{576} \cdot 2\pi \cdot 24 = 2\pi \cdot \frac{23 \cdot 24}{24 \cdot 24} = 2\pi \cdot \frac{23}{24} \text{ rad}$$

(Alltså $\frac{23}{24}$ av ett helt varv)

$$\frac{2\pi}{24} \cdot \text{rad} \;\hat{=}\; \frac{2\pi}{24} \cdot \frac{360}{2\pi} = 15^\circ$$

Radiens läge (vinkel mot "x-axeln")

n	ϕ [rad]	ϕ [grader]
0	0	0
1	$2\pi \cdot 23/24 = -\pi/12$	$345 = -15^\circ$
2	$2\pi \cdot 2 \cdot 23/24 = -\pi/6$	$690 = -30^\circ$
3	$2\pi \cdot 3 \cdot 23/24 = -3\pi/12$	$1035 = -45^\circ$



n=0 n=1 n=2 n=3

b/ En illustration av sampling och aliasing.
Vi tar bilder för sällan för att spegla det verkliga händelseförloppet (samplingsfrekv. för låg).
Det ser ut som om skivan roterar medurs med en vinkelhastighet som är

$$\omega' = -\frac{2\pi}{24} \cdot \frac{1}{T_s} = -\frac{2\pi}{24} \cdot \frac{24 \cdot 24}{23} = -2\pi \frac{24}{23} \text{ rad/s}$$

Alltså $\frac{24}{23}$ varv/s Medurs (Aliasing !)

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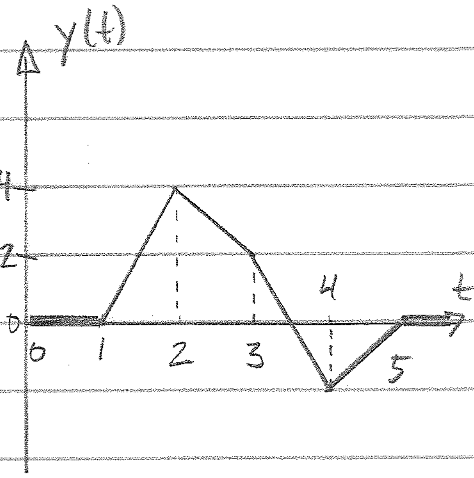
1a/ $y(t) = \delta(t)$ c/ ej tidsinvariant
b/ $y(t) = \cos(0{,}5\pi)\,\delta(t-1)$ d/ Systemet är linjärt

2.
$$y(t) = \begin{cases} 0, & t < 1 \\ 4(t-1), & 1 \le t < 2 \\ 4-2(t-2), & 2 \le t < 3 \\ 4(4-t)-2, & 3 \le t < 4 \\ -2(5-t), & 4 \le t < 5 \\ 0, & t \ge 5 \end{cases}$$



3. a/ $x_5 - A$, $x_2 - D$, $x_4 - B$, $x_1 - C$
b/ $N = 18$
c/ $\omega_s = 4\omega_0$, $\omega_s = \frac{4}{5}\omega_0$ (Aliasing)

4. $c_1 = \frac{1}{2}(A_1 - jB_1)$; $c_{-1} = c_1^* = \frac{1}{2}(A_1 + jB_1)$

$c_2 = \frac{3}{2}(A_2 - jB_2)$; $c_{-2} = c_2^* = \frac{3}{2}(A_2 + jB_2)$

övriga $c_k = 0$

5. $x(t)$ har högsta signalfrekvens $\omega_M = 100\pi$ r/s
LP-filtrets brytfrekvens $\omega_c > \omega_M$
Störsignalens vinkelfrekv. $\omega_n > \omega_c$

Svar: $\omega_n > \omega_c > \omega_M = 100\pi$ r/s