Sensorer, Signaler & System
SSY040 del A, 061222

1a) Grundvinkelfrekvens : $\omega_0 = 2\pi$
Högsta signalvinkelfrekvens : $\omega_M = 3 \cdot 2\pi \quad (k=3)$
Samplingsvinkelfrekvens $\omega_s > 2\,\omega_M$ rad/s
Samplingsfrekvens $f_s = \frac{\omega_s}{2\pi} > 2 \cdot \frac{\omega_M}{2\pi} = \frac{2 \cdot 3 \cdot 2\pi}{2\pi} = 6\,Hz$

Svar: $f_s > 6\,Hz$

b) Parsevals relation ger signalens medeleffekt

$$\overline{P} = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2 =$$

$$= |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 + 2|c_3|^2 =$$

$$= 2 + \frac{2}{4} + \frac{2}{64} + \frac{2}{256} = \frac{256+64+4+1}{128} = \frac{325}{128}$$

Svar: $\overline{P} = \frac{325}{128} \approx 2{,}54$

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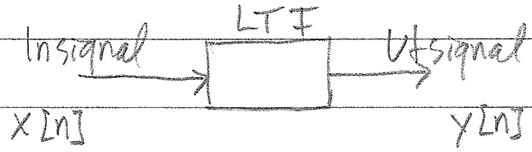
2.



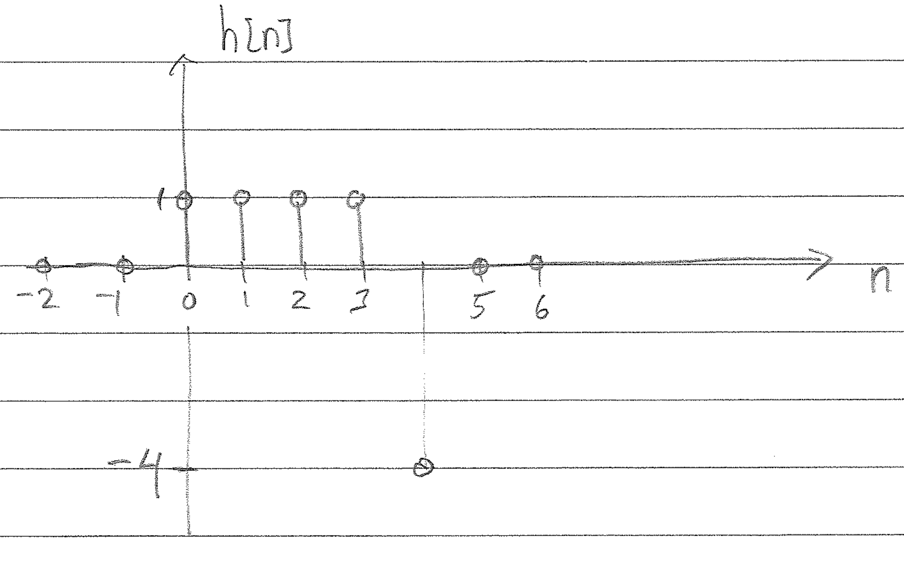
Insignal	Utsignal	
$x[n]$	$y[n]$	
$u[n]$	$y_s[n]$	Givet i uppgiften

$\delta[n] = u[n] - u[n-1]$ ← Impuls

$h[n] = y_s[n] - y_s[n-1]$ ← Impulssvar

$$h[n] = y_s[n] - y_s[n-1] =$$
$$= \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$$
$$- \quad \delta[n-1] - 2\delta[n-2] - 3\delta[n-3] - 4\delta[n-4] =$$

$$= \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - 4\delta[n-4]$$



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3. $$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v(t)$$

Fouriertransformera!

$$j\omega V_c(j\omega) + \frac{1}{RC} V_c(j\omega) = \frac{1}{RC} V(j\omega)$$

$$V_c(j\omega)\left[j\omega + \frac{1}{RC}\right] = \frac{1}{RC} V(j\omega)$$

$$H(j\omega) = \frac{V_c(j\omega)}{V(j\omega)} = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} = \frac{1}{1 + j\omega RC}$$ Frekv. svar!

Faskarakteristik (Fasskillnad mellan in- och utsignal)

$$\angle H(j\omega) = \arg\{H(j\omega)\} = -\arctan(\omega RC)$$

Fasvridning $-\frac{\pi}{3}$ vid $\omega = \omega_c$

$$\arctan(\omega_c RC) = \frac{\pi}{3} \Rightarrow \omega_c RC = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\omega_c = \frac{\sqrt{3}}{RC}$$

Amplitudkarakteristik (Ampl. förändring hos utsignal)

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \{\omega = \omega_c\} =$$

$$= \frac{1}{\sqrt{1 + (\sqrt{3})^2}} = \frac{1}{2}$$

Svar: $\frac{1}{2}$

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4. $y(t) = A\,x(t-t_0)$

a) Låt $x(t) = \delta(t)$

Impulssvar $h(t) = A\,\delta(t-t_0)$

b) Frekvenssvar? Fouriertransformera!

$$Y(j\omega) = A\,e^{-j\omega t_0}\,X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = A\,e^{-j\omega t_0}$$

c) Amplitudförändring: $|H(j\omega)| = A, \ \forall\,\omega$

Fasförändring: $\angle H(j\omega) = -\omega t_0$

$$\angle H(j\omega)\Big|_{\omega=\omega_0} = -\omega_0 t_0$$

Alt: $x(t) = \cos(\omega_0 t)$

$$y(t) = A\cos(\omega_0(t-t_0)) =$$
$$= \underbrace{A}_{\text{Amplitudförändring}}\cos(\omega_0 t \underbrace{- \omega_0 t_0}_{\text{Fasförändring}})$$

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5. Faltningssumman: $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$h[n] = a^n u[n]$, $x[n] = z^n$

$|a| < 1$

$$y[n] = \sum_{k=-\infty}^{\infty} a^k u[k]\, z^{n-k} =$$

$$= \sum_{k=0}^{\infty} a^k z^n z^{-k} = z^n \sum_{k=0}^{\infty} \left(a z^{-1}\right)^k =$$

$$= \left\{ \text{Geom. serie, om } |a z^{-1}| < 1 \right\} =$$

$$= z^n \cdot \underbrace{\frac{1}{1 - a z^{-1}}}$$

(Detta är z-transformen, $H(z)$, av impulssvaret $h[n]$.)