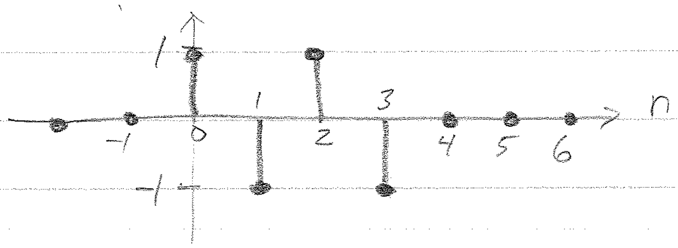
SSY040
Sensorer, Signaler och System, del A

060828

1. $x[n] \rightarrow \boxed{h[n]} \xrightarrow{w[n]} \boxed{h[n]} \rightarrow y[n]$

Impulssvar: $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$



Om $x[n] = \delta[n]$ blir $w[n] = h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

För LTI-system gäller (Superposition)

$w[n] \rightarrow \boxed{h[n]} \rightarrow y_h[n] = h[n] - h[n-1] + h[n-2] - h[n-3]$

n	-1	0	1	2	3	4	5	6	7	8
$h[n]$		1	-1	1	-1					
$-h[n-1]$			-1	1	-1	1				
$h[n-2]$				1	-1	1	-1			
$-h[n-3]$					-1	1	-1	1		
$y[n]$		1	-2	3	-4	3	-2	1		

$$y_h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - 4\delta[n-3] +$$
$$+ 3\delta[n-4] - 2\delta[n-5] + \delta[n-6]$$

Hela systemets impulssvar är $y_h[n]$

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/forts 1

För insignal $x[n] = \delta[n+1] + \delta[n-1] + \delta[n-2]$

Blir totala utsignalen $y[n] = y_h[n+1] + y_h[n-1] + y_h[n-2]$

(Superposition)

n	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
$y_h[n+1]$	1	-2	3	-4	3	-2	1							
$y_h[n-1]$			1	-2	3	-4	3	-2	1					
$y_h[n-2]$				1	-2	3	-4	3	-2	1				
$\Sigma \Rightarrow y[n]$	1	-2	4	-5	4	-3	0	1	-1	1				

Svar: $y[n] = \delta[n+1] - 2\delta[n] +$
$+ 4\delta[n-1] - 5\delta[n-2] +$
$+ 4\delta[n-3] - 3\delta[n-4] +$
$+ \delta[n-6] - \delta[n-7] + \delta[n-8]$

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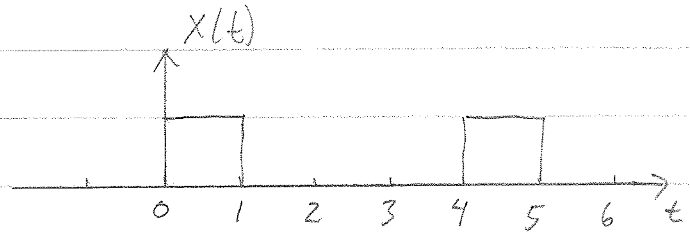
2. $x(t) = \sum_{n=-\infty}^{\infty} u(t-4n) - u(t-4n-1)$



a) Periodtid $T = 4s$

Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$ r/s

b) $c_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt = \frac{1}{4} \int_0^1 e^{-jk\omega_0 t} dt =$

$$= \frac{1}{4} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_0^1 = \frac{1}{4} \frac{1}{(-jk\omega_0)} \left[e^{-jk\omega_0} - 1 \right] =$$

$$= \frac{j\,2}{k \cdot 4 \cdot \pi} \left[e^{-jk\frac{\pi}{2}} - 1 \right] = \frac{j}{2k\pi} \left[e^{-j\frac{k\pi}{2}} - 1 \right]$$

Studera $k=0$ separat

$$c_0 = \frac{1}{4} \int_0^1 1 \cdot dt = \frac{1}{4} \left[t \right]_0^1 = \frac{1}{4}$$

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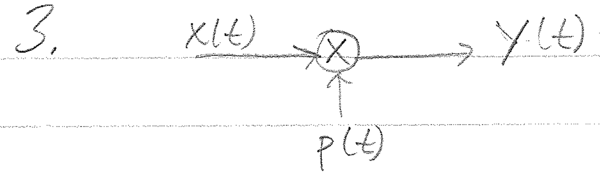
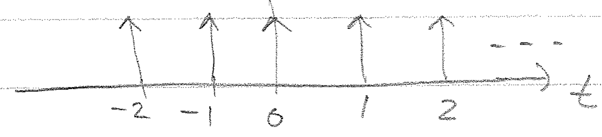
3.



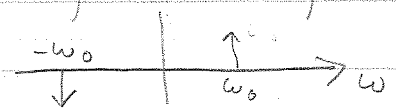
$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-k)$$



$x(t) = \sin\left(\frac{6\pi}{5} t\right) = \sin(\omega_0 t)$

Fouriertransf.

$$X(j\omega) = \frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$$



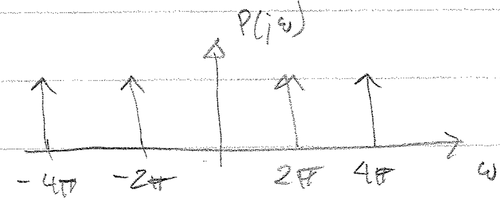
Periodtid $T = 1\,s$

Fouriertransform

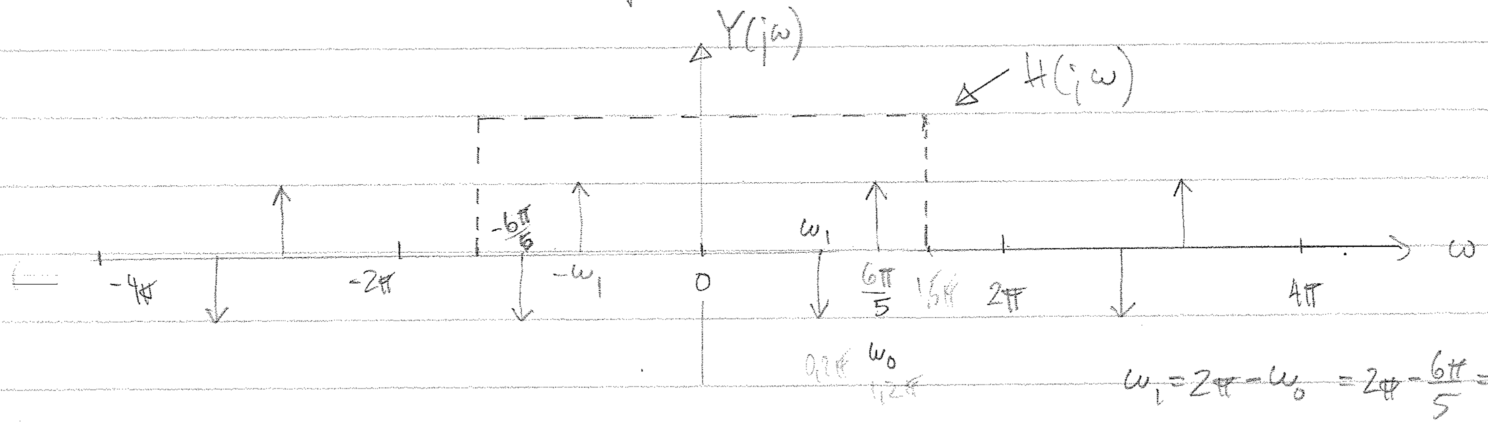
$$P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right) =$$

$$= 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - k2\pi)$$

$$x(t)\,p(t) \xleftrightarrow{FT} \frac{1}{2\pi} X(j\omega) * P(j\omega) = Y(j\omega)$$



$X(j\omega)$ Faltas med impulser



$$\omega_1 = 2\pi - \omega_0 = 2\pi - \frac{6\pi}{5} = \frac{4\pi}{5}$$

Filtrera med $H(j\omega) = \begin{cases} 1, & |\omega| \le \frac{3\pi}{2} \\ 0, & \text{för övrigt} \end{cases}$

Filtrets utsignal: $\hat{Y}(j\omega) = \frac{1}{T}\left[\frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0) - \frac{\pi}{j}\delta(\omega-\omega_1) + \frac{\pi}{j}\delta(\omega+\omega_1)\right]$

$(T=1)$ Inv. Fouriertransf.

$$\hat{y}(t) = \sin\omega_0 t - \sin\omega_1 t = \sin\left(\frac{6\pi}{5}t\right) - \sin\left(\frac{4\pi}{5}t\right)$$

Aliasing!

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4. $\frac{R}{L} v_o(t) + \frac{d}{dt} v_o(t) = \frac{d}{dt} v_i(t)$

Antag sinusformat stationärtillstånd

Laplacetransf. $\frac{R}{L} V_o(s) + s V_o(s) = s V_i(s)$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s + \frac{R}{L}}$$

b) Impulssvar $H(s) = \dfrac{s + \frac{R}{L} - \frac{R}{L}}{s + \frac{R}{L}} = 1 - \dfrac{R/L}{s + \frac{R}{L}}$

Impulssvar

$$\mathcal{L}^{-1}\{H(s)\} = h(t) = \delta(t) - \frac{R}{L} e^{-\frac{R}{L}t} u(t)$$

a) Frekvenssvar

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{R}{L}}$$

$\arg\{H(j\omega)\} = \frac{\pi}{2} - \arctan\left(\frac{\omega L}{R}\right) = \frac{\pi}{4}$ vid $\omega = 100$ r/s

$\therefore \arctan \frac{\omega L}{R} = \frac{\pi}{4} \Rightarrow \frac{\omega L}{R} = 1 \Rightarrow \frac{R}{L} = \omega = 100$

Svar: a) $\frac{R}{L} = 100$

b) $h(t) = \delta(t) - 100 e^{-100t} u(t)$

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5. $x[n] \longrightarrow \boxed{H} \longrightarrow y[n]$

$$y[n] = H[x[n]] = n\,x[n]$$

a)

Insignal	Utsignal
$x[n]$	$y[n] = H[x[n]] = n\,x[n]$
$a\,x[n]$	$y[n] = H[x[n]] = n\,a\,x[n] = a\,y[n]$ "Homogent"
$x_1[n]$	$y_1[n] = n\,x_1[n]$
$x_2[n]$	$y_2[n] = n\,x_2[n]$
$x[n] = x_1[n] + x_2[n]$	$y[n] = n\,x[n] = n[x_1[n] + x_2[n]] =$ $= n\,x_1[n] + n\,x_2[n] =$ $= y_1[n] + y_2[n]$

"Superposition"

Systemet är linjärt!

b) Utsignalen $y[n]$ beror endast på det samtida värden av insignalen, $x[n]$.

Systemet är kausalt!