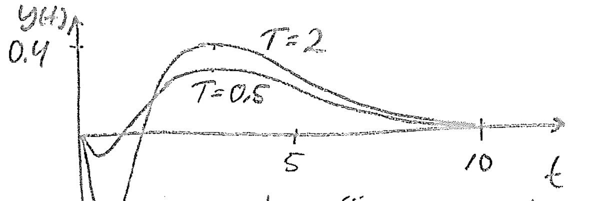
Lösning till tentamen i Reglerteknik 151027

1. a) $G(s) = \dfrac{1}{(1+s)^3} - T\dfrac{s}{(1+s)^3}$ $\quad u(t) = \delta(t) \quad U(s) = 1$

$$y(t) = \mathcal{L}^{-1} G(s) = \frac{t^2}{2}e^{-t} - T\frac{d}{dt}\left(\frac{t^2}{2}e^{-t}\right) = \left[(1+T)\frac{t^2}{2} - Tt\right]e^{-t}$$

t	$y(t)_{T=0.5}$	$y(t)_{T=2}$
0	0	0
0.5	−0.04	−0.38
3	0.26	0.37
∞	0	0

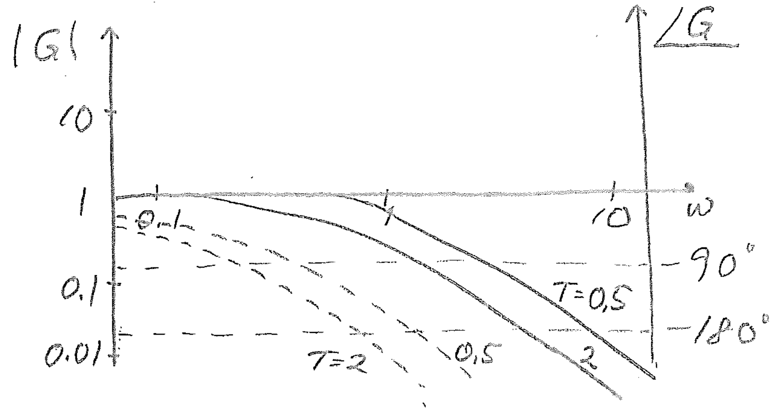


Underslängen ökar med T

b) $G(j\omega) = \dfrac{1 - Tj\omega}{(1+j\omega)^3}$

$|G(j\omega)| = \dfrac{\sqrt{1+T^2\omega^2}}{\sqrt{1+\omega^2}^{\,3}}$

$\angle G(j\omega) = -\arctan \omega T - 3\arctan\omega$



c) $K = \dfrac{|G(j\omega_\pi)|}{|G(0)|} = \begin{cases} 0.31 & T = 0.5 \quad (\omega_\pi = 1.2) \\ 0.88 & T = 2 \quad (\omega_\pi = 0.85) \end{cases}$

Då nollstället i $s = 1/T$ närmar sig origo blir processen svårare att reglera, vilket motsvaras av ökad underslång och ökad negativ fasvridning

2a) $\angle G(j\omega_{G150}) = -4\arctan \omega_{G150} = -150°$

$\Rightarrow \omega_{G150} = \tan 37.5° = 0.767$ rad/s $\qquad \omega_c = 0.4\,\omega_{G150} = 0.307$ ra

$\angle G(j\omega_c) + \angle F_{PI}(j\omega_c) = -4\arctan\omega_c - 90° + \arctan \omega_c T_i$
$= -180° + \varphi_m = -130°$

$T_i = \dfrac{1}{\omega_c}\tan(4\arctan\omega_c - 40°) = 1.751$

$|G(j\omega_c)|\,|F_{PI}(j\omega_c)| = \dfrac{1}{(1+\omega_c^2)^2}\,\dfrac{K_i\sqrt{1+(\omega_c T_i)^2}}{\omega_c} = 1$

$K_i = \dfrac{\omega_c(1+\omega_c^2)^2}{\sqrt{1+(\omega_c T_i)^2}} = 0.324 \qquad \gamma_o = 1/K_i = 3.09$

$\gamma_u = K_p = K_i T_i = 0.567$

2 b)

$$\omega_c = 0.6 \cdot \omega_{G150} = 0.6 \cdot 0.767 = 0.460 \text{ rad/s}$$

$$|G(j\omega_c)| = \frac{1}{(1+\omega_c^2)^2} = 0.835 \qquad \angle G(j\omega_c) = -4 \arctan \omega_c = -98.8^\circ$$

$$\angle F_{PID}(j\omega_c) = -180^\circ + \varphi_m - \angle G(j\omega_c) = -180^\circ + 50^\circ + 98.8^\circ = -31.2^\circ$$

$$FS \Rightarrow \quad \omega_c \tau = 0.61 \quad \Rightarrow \tau = 1.326$$

$$K_\infty |G(j\omega_c)| = 4.45 \quad \Rightarrow K_\infty = K_i \tau \beta = \frac{4.45}{0.835} = 5.33$$

$$K_i = \frac{4.45}{10 \cdot 1.326 \cdot 0.835} = 0.402$$

PI: $J_v = 3.09 \quad J_u = 0.567$ PID: $J_v = 2.49$, $J_u = 5.33$

c)

$$\frac{J_{v\,PID}}{J_{v\,PI}} = 0.81 \qquad \frac{J_{u\,PID}}{J_{u\,PI}} = 9.4$$

∴ Nästan 10 gånger högre J_u [illegible]

19% förbättring av J_v

3. a) $L(s) = \frac{KK_p(s+2)}{s(s-2)}$

$G_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{KK_p s + 2KK_p}{s^2+(KK_p-2)s+2KK_p}$

Instabilt då $KK_p - 2 \leq 0 \Rightarrow KK_p \leq 2$

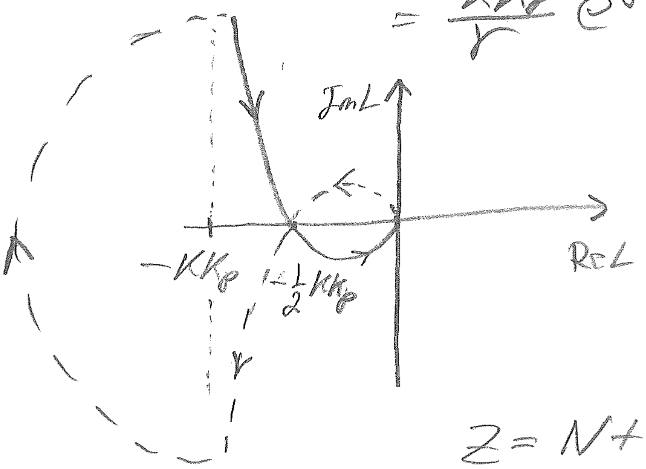
b) $L(j\omega) = \frac{KK_p(j\omega+2)(-j\omega-2)}{j\omega\underbrace{(j\omega-2)(-j\omega-2)}_{\omega^2+4}} = -j\,\frac{KK_p(\omega^2-4-4j\omega)}{\omega(\omega^2+4)}$

$$= -\frac{4KK_p}{(\omega^2+4)} + j\,\frac{KK_p(4-\omega^2)}{\omega(4+\omega^2)} = \begin{cases} -KK_p + j\infty & \omega=0 \\ -\frac{1}{2}KK_p & \omega=2 \\ 0 & \omega=\infty \end{cases}$$

$L(-j\omega) = \mathrm{Re}\,L(j\omega) - j\,\mathrm{Im}\,L(j\omega)$

$$L(re^{j\theta}) = -\frac{KK_p}{re^{j\theta}} = \underbrace{e^{j\pi}}_{=-1}\frac{KK_p}{r}e^{-j\theta} = \frac{KK_p}{r}e^{j(\pi-\theta)} = \qquad -\frac{\pi}{2}<\theta<\frac{\pi}{2}$$

$$= \frac{KK_p}{r}e^{j\varphi} \qquad \varphi = \begin{cases} \frac{3\pi}{4} & \theta=-\frac{\pi}{2} \\ \pi & \theta=0 \\ \frac{\pi}{2} & \theta=\frac{\pi}{2} \end{cases}$$



$KK_p<2 \qquad N=1$

$KK_p>2 \qquad N=-1$

$$Z=N+P=\begin{cases} 1+1=2 & KK_p\leq 2 \text{ instabilt} \\ -1+1=0 & KK_p>2 \text{ stabilt} \end{cases}$$

c) $L_0(s) = \frac{K_0K_p(s+2)}{s(s-2)}$ stabilt då $K_0K_p>2$

$K_p = \frac{4}{K}$ (K=nominellt värde på K_0) $\Rightarrow$

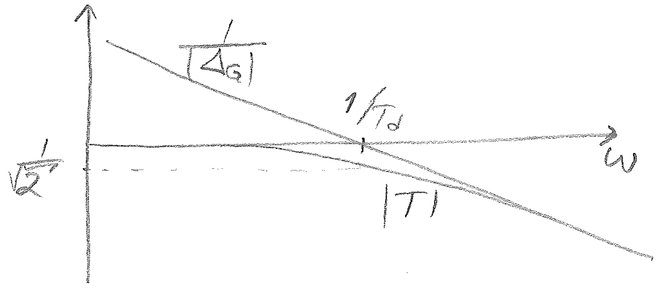
$K_0\frac{4}{K}=2$ då $K_0=\frac{K}{2}$ dvs instabilitet inträffar precis förstärkningen halveras, men stabilitet erhålls för högre processförstärkning.

4. $G_0(s) = \frac{1-T_d s}{1+5s}$ $\quad G(s) = \frac{1}{1+5s}$

$$\Delta_G(s) = \frac{G_0 - G}{G} = \frac{G_0}{G} - 1 = 1 - T_d s - 1 = -T_d s$$

$$F_{PI}(s) = \frac{K_i(1+5s)}{s} \Rightarrow L(s) = \frac{K_i}{s} \text{ och}$$

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{K_i}{s+K_i} = \frac{1}{1+s/K_i}$$



$|T| < \frac{1}{|\Delta_G|} = \frac{1}{\omega T_d} \quad \forall \omega$

kräver i detta fall att HF komponenten av $|T|$ dvs $\frac{K_i}{\omega} < \frac{1}{\omega T_d}$

$$|T(j\omega_b)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\frac{\omega_b}{K_i}\right)^2}} \Rightarrow K_i = \omega_b$$

$\therefore \quad \omega_b < \frac{1}{T_d}$

5. $G_d(z) = z^{-1}$ $\quad F_d(z) = K_p + \frac{K_i z^{-1}}{1-z^{-1}} = K_p + \frac{K_i}{z-1} = \frac{K_p z + K_i - K_p}{z-1}$

a) $L_d(z) = G_d(z) F_d(z) = \frac{K_p z + K_i - K_p}{z^2 - z}$

$$G_{ry}(z) = \frac{L_d(z)}{1+L_d(z)} = \frac{K_p z + K_i - K_p}{\underbrace{z^2 - (1-K_p)z + K_i - K_p}_{(z-0.3)^2 = z^2 - 0.6z + 0.09}}$$

$K_p = 0.4 \quad K_i = K_p + 0.09 = 0.49$

b) Statisk förstärkning: $z = e^{j0} = 1$

$$G_{ry}(1) = \frac{K_p + K_i - K_p}{1-(1-K_p)+K_i-K_p} = \frac{K_i}{K_i} = 1$$

$\Rightarrow$ kvarstående fel undviks från referenssignal till utsignal

6. a) $l\ddot{\theta} = g\sin\theta - u\cos\theta$

$$\dot{\omega} = \frac{g}{l}\sin\theta - \frac{u}{l}\cos\theta$$

$$\dot{\theta} = \omega$$

$$\begin{bmatrix}\Delta\dot{\theta}\\ \Delta\dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & 1\\ \underbrace{\frac{g}{l}\cos\theta_0 + \frac{u_0}{l}\sin\theta_0}_{g/l} & 0\end{bmatrix}_{\theta_0=0}\begin{bmatrix}\Delta\theta\\ \Delta\omega\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{l}\cos\theta_0\end{bmatrix}_{\theta_0=0}\Delta u$$

Arbetspunkt $\theta_0 = \omega_0 = u_0 = 0 \Rightarrow$

$$\begin{bmatrix}\dot{\theta}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & 1\\ g/l & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ -1/l\end{bmatrix}u \qquad y = \theta = [1\ 0]\begin{bmatrix}\theta\\ \omega\end{bmatrix}$$

b) $l = g = 10\ m/s^2 \Rightarrow \quad A = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \quad B = \begin{bmatrix}0\\ -0.1\end{bmatrix}$

$$S = [B\ AB] = \begin{bmatrix}0 & -0.1\\ -0.1 & 0\end{bmatrix} \quad \det S = -0.01 \neq 0$$

$\therefore$ styrbart system

c)

$L_u = [l_\theta\ l_\omega]$ Slutna systemets poler

$$\det(sI_n - A + BL_u) = \det\left(\begin{bmatrix}s & -1\\ -1 & s\end{bmatrix} + \begin{bmatrix}0\\ -0.1\end{bmatrix}[l_\theta\ l_\omega]\right) =$$

$$= \det\begin{bmatrix}s & -1\\ -1-0.1l_\theta & s-0.1l_\omega\end{bmatrix} = s^2 - 0.1l_\omega s - 1 - 0.1l_\theta$$

$l_\omega = 0 \Rightarrow s^2 = 1 + 0.1l_\theta$

$$\text{poler i } s = \begin{cases}\pm\sqrt{1+0.1l_\theta} & l_\theta \geq -10\\ \pm j\sqrt{0.1|l_\theta| - 1} & l_\theta < -10\end{cases}$$

dvs alltid pol i HHP eller på imaginäraxeln

d) $\alpha_c(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 3.6s + 9$

$= s^2 - 0.1l_\omega s - 1 - 0.1l_\theta \Rightarrow l_\omega = -36 \quad l_\theta = -100$

$$G_{r\theta} = C(sI_n - A + BL_u)^{-1}BK_r = [1\ 0]\begin{bmatrix}s & -1\\ 9 & s+3.6\end{bmatrix}^{-1}\begin{bmatrix}0\\ -0.1\end{bmatrix}K_r =$$

$$= \frac{[1\ 0]\begin{bmatrix}s+3.6 & 1\\ -9 & s\end{bmatrix}\begin{bmatrix}0\\ -0.1\end{bmatrix}K_r}{s^2+3.6s+9} = \frac{-0.1K_r}{s^2+3.6s+9}$$

$$G_{r\theta}(0) = \frac{-0.1K_r}{9} = 1 \Rightarrow K_r = -90$$