Tentamen i Reglerteknik 2014-01-15

1. $G(s) = \frac{1}{1+4s}$ $\quad u(t) = t - (t-2)\sigma(t-2)$

$U(s) = \frac{1}{s^2}(1 - e^{-2s})$

$$Y(s) = \frac{0.25}{s^2(s+0.25)}(1-e^{-2s}) = \left(\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+0.25}\right)(1-e^{-2s}) =$$

$$= \left(\frac{1}{s^2} - \frac{4}{s} + \frac{4}{s+0.25}\right)(1-e^{-2s})$$

$$y(t) = (t - 4 + 4e^{-t/4})\sigma(t) - \left((t-2) - 4 + 4e^{-(t-2)/4}\right)\sigma(t-2)$$

2. a) $\dot{x} = \sin x + u^3$ $\quad x_0 = \pi/3$

Arbetspunkt $\quad \sin \pi/3 + u_0^3 = 0 \Rightarrow u_0 = -0.75^{1/6}$

≈ -0.9532

Linjärisering $\quad \Delta\dot{x} = \cos x_0 \Delta x + 3u_0^2 \Delta u =$

$= \cos\frac{\pi}{3} \Delta x + 3 \cdot 0.75^{1/3} \Delta u =$

$= 0.5\Delta x + 2.73 \Delta u$

b) $\frac{\Delta X(s)}{\Delta U(s)} = \frac{2.73}{s - 0.5} \Rightarrow$ pol i $s = 0.5$ dvs instabilt system

3. a) KE $\quad 1 + \frac{K_p(1-sT)}{(1+s)^2(1+0.5s)} = \frac{1+2s+s^2+0.5s+s^2+0.5s^3-K_pTs+K_p}{\ldots\ldots} = 0$

$0.5s^3 + 2s^2 + (2.5 - K_pT)s + K_p + 1 = 0$

R-H tabell

s^3	0.5	$2.5 - K_pT$
s^2	2	$K_p + 1$
s^1	$\frac{5 - 2K_pT - 0.5K_p - 0.5}{2}$	0
s^0	$K_p + 1$	

stabilt då

$4.5 > (2T + 0.5)K_p$

$K_p > -1$

$-1 < K_p < \frac{9}{1+4T}$

b) $A_m K_p = \frac{9}{1+4T} = 1.8 \Rightarrow K_p = 1.8/A_m = \begin{cases} 0.9 & A_m = 2 \\ 0.45 & A_m = 4 \end{cases}$

c) $K_p < \frac{9}{1+4T} \Rightarrow 1 + 4T < 9/K_p \Rightarrow T < \frac{9}{4K_p} - \frac{1}{4}$

$A_m = 2 \Rightarrow T < \frac{9}{4 \cdot 0.9} - 0.25 = 2.25$

$A_m = 4 \Rightarrow T < \frac{9}{4 \cdot 0.45} - 0.25 = 4.75$

d) Större amplitudmarginal (stabilitetsmarginal) $\Rightarrow$ större parameterosäkerhet accepteras.

4. a) $J_m \dot{\omega}_m = T_d - K_{ml}(\theta_m - \theta_l) - B_{ml}(\omega_m - \omega_l)$

$J_l \dot{\omega}_l = K_{ml}(\theta_m - \theta_l) + B_{ml}(\omega_m - \omega_l) - T_v$

$\Delta\dot{\theta} = \frac{d}{dt}(\theta_m - \theta_l) = \omega_m - \omega_l$

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\omega}_l \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{B_{ml}}{J_m} & \frac{B_{ml}}{J_m} & -\frac{K_{ml}}{J_m} \\ \frac{B_{ml}}{J_l} & -\frac{B_{ml}}{J_l} & \frac{K_{ml}}{J_l} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \omega_l \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} \frac{1}{J_m} & 0 \\ 0 & -\frac{1}{J_l} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_d \\ T_v \end{bmatrix}$$

$$\omega_l = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \omega_l \\ \Delta\theta \end{bmatrix}$$

b) Välj $\Delta\omega = \omega_m - \omega_l$ och $\Delta\theta = \theta_m - \theta_l$ som tillståndsvariabler

$\Delta\dot{\omega} = T_d - \Delta\theta - \Delta\omega - \Delta\theta - \Delta\omega + T_v$

$\Delta\dot{\theta} = \Delta\omega$

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_d \\ T_v \end{bmatrix}$$

$$T_a = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\theta \end{bmatrix}$$

c) $\det(sI - A) = \det\begin{bmatrix} s+2 & 2 \\ -1 & s \end{bmatrix} = s^2 + 2s + 2 = s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n = \sqrt{2} \qquad \zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{2}}$

5.

a) $$L(s) = \frac{20e^{-20s}}{(1+100s)(1+5s)} \, \frac{K_i(1+100s)}{s} = \frac{20K_i e^{-20s}}{s(1+5s)}$$

$$\underline{\angle L(j\omega_c)} = -20\,\omega_c \frac{180^\circ}{\pi} - 90^\circ - \arctan 5\omega_c = -180^\circ + \underset{\nwarrow \varphi_m}{50^\circ} = -130^\circ$$

$$1146\,\omega_c + \arctan 5\omega_c = 40^\circ \quad \Rightarrow \quad \omega_c = 0.028 \text{ rad/s}$$

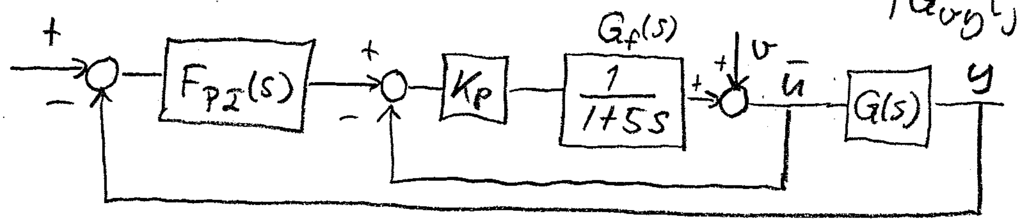
$$|L(j\omega_c)| = \frac{20K_i}{\omega_c\sqrt{1+(5\omega_c)^2}} = 707K_i = 1 \quad \Rightarrow \quad K_i = 0.0014$$

b) $$G_{vy}(s) = \frac{G(s)}{1+L(s)} \quad \text{där} \quad G(s) = \frac{20e^{-20s}}{1+100s} \approx 20 \text{ små } s$$

$$L(s) = \frac{20K_i e^{-20s}}{s(1+5s)} \approx \frac{20K_i}{s} \text{ små } s$$

$$G_{vy}(s) \approx \frac{20}{1+20K_i/s} \approx \frac{s}{K_i} \text{ små } s \quad \Rightarrow \quad |G_{vy}(j\omega)|_{LF} \approx \frac{\omega}{K_i}$$

c)



$$Y = G\bar{U} \qquad \bar{U} = V + G_f K_p(-\bar{U} - F_{PI} G\bar{U}) \qquad (1 + G_f K_p(1 + F_{PI} G))\bar{U} = V$$

$$G_{vy} = \frac{Y}{V} = G\frac{\bar{U}}{V} = \frac{G}{1+G_f K_p(1+F_{PI} G)} \approx \frac{20}{1+K_p\left(1+\frac{20K_i}{s}\right)} = \frac{20s}{s+K_p(s+20K_i)}$$

$$\approx \frac{20s}{20K_pK_i} = \frac{s}{K_pK_i} \quad \Rightarrow \quad |G_{vy}(j\omega)|_{LF} \approx \frac{\omega}{K_pK_i}$$

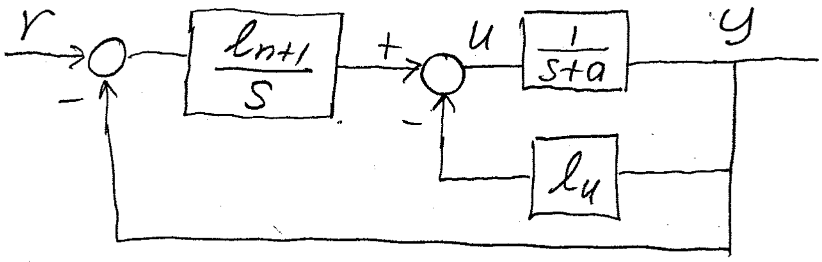
Stort K_p ⇒ effektivare kompensering av v vid kaskadreglering

6. a) $\dot{x} = -ax + u$ $\quad (s+a)Y(s) = U(s) \quad G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+a}$

$y = x$

$$u = -l_u x + l_{n+1} \int_0^t (r-y)\,d\tau$$

$$U(s) = -l_u Y(s) + l_{n+1} \frac{1}{s}(R(s) - Y(s))$$



b) $$(s+a)Y(s) = U(s) = -l_u Y(s) + l_{n+1} \frac{1}{s}(R(s) - Y(s))$$

$$(s^2 + (a+l_u)s + l_{n+1})\, Y(s) = l_{n+1} R(s)$$

$$G_{ry}(s) = \frac{Y(s)}{R(s)} = \frac{l_{n+1}}{s^2 + (a+l_u)s + l_{n+1}} = \frac{l_{n+1}}{(s+\alpha)^2}$$

$$s^2 + 2\alpha s + \alpha^2$$

$$\Rightarrow \begin{cases} l_u = 2\alpha - a \\ l_{n+1} = \alpha^2 \end{cases}$$

c) $$G_{ry}(0) = \frac{l_{n+1}}{0^2 + (a+l_u)\cdot 0 + l_{n+1}} = 1$$ oavsett värdet på a, l_u och l_{n+1}