Tentamen Reglerteknik Z/Kf 130530

BL 130604

1. a) $(s^2+1)Y(s) = e^{-s}U(s)$ $\quad G(s) = \frac{e^{-s}}{s^2+1}$

Poler i $s=\pm j$ $\Rightarrow$ marginellt stabilt system

b) $u(t)=\sigma(t) \Rightarrow U(s)=\frac{1}{s} \Rightarrow Y(s)=\frac{e^{-s}}{(s^2+1)s} =$

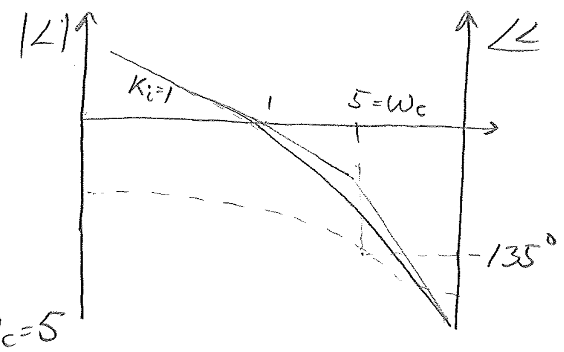
$$= \left(\frac{A}{s}+\frac{Bs}{s^2+1}+\frac{C}{s^2+1}\right)e^{-s} = \frac{As^2+A+Bs^2+Cs}{s(s^2+1)}e^{-s}$$

$\Rightarrow A=1,\ B=-1,\ C=0$

$Y(s)=\left(\frac{1}{s}-\frac{s}{s^2+1}\right)e^{-s}$; $\quad y(t)=\mathcal{L}^{-1}\{Y(s)\}=(1-\cos(t-1))\sigma(t-1)$

2. a) $L(s)=G(s)F_{PI}(s)=\frac{1}{(s+1)(1+0.2s)}\frac{K_i(1+T_is)}{s}$ $\quad T_i=1 \Rightarrow$

$$L(s)=\frac{K_i}{s(1+0.2s)} = \frac{K_i}{s(1+s/5)}$$



$\varphi_m=45° \Rightarrow$

$\angle L(j\omega_c) = -135° \Rightarrow \omega_c=5$

$$|L(j\omega_c)| = \frac{K_i}{\omega_c\sqrt{1+(\omega_c/5)^2}} = \frac{K_i}{5\sqrt{2}} = 1 \Rightarrow K_i = 7.07$$

b) $Y(s)=G(s)(V(s)+F_{PI}(s)(-Y(s))) \Rightarrow G_{vy}(s)=\frac{G(s)}{1+G(s)F_{PI}(s)}$

$=\frac{1}{F_{PI}(s)+1/G(s)}$ $\quad F_{PI}(s)\approx\frac{K_i}{s}$ $\quad G(s)\approx 1$ små s.

$\Rightarrow G_{vy}(s)\approx\frac{1}{K_i/s+1} = \frac{s}{K_i}$ för små s $\quad |G^{LF}_{vy}(j\omega)| \approx \frac{\omega}{K_i}$, $\omega \ll 1$

c) $\frac{|G^{LF}_{vy}| \text{ optimal}}{|G^{LF}_{vy}| \text{ uppgift a)}} = \frac{\omega/7.49}{\omega/7.07} = 0.94$

∴ Den optimala regulatorn är ≈ 6% bättre när det gäller kompensering av LF lastfstörningar.

3. a) $x = \begin{bmatrix} \theta \\ \omega \end{bmatrix}$ $\quad \dot{\theta} = \omega$

$\dot{\omega} = \frac{g}{l}\sin\theta - \frac{u}{l}\cos\theta$

Arbetspunkt: $x_0 = \begin{bmatrix} \theta_0 \\ 0 \end{bmatrix}$ $\quad \frac{u_0}{l}\cos\theta_0 = \frac{g}{l}\sin\theta_0$

$\Rightarrow \quad u_0 = g\tan\theta_0$

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} \Delta\omega \\ \frac{g}{l}\cos\theta_0\,\Delta\theta + \frac{u_0}{l}\sin\theta_0\,\Delta\omega - \frac{1}{l}\cos\theta_0\,\Delta u \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \\ \frac{g}{l}\cos\theta_0 & \frac{u_0}{l}\sin\theta_0 \end{bmatrix}}_{A} \begin{bmatrix} \Delta\theta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{l}\cos\theta_0 \end{bmatrix} \Delta u$$

b)

$\theta_0 = 0 \Rightarrow A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad \det(sI - A) = \det\begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix} = s^2 - 1 = 0$

$s = \pm 1$ $\therefore$ $\theta_0 = 0$ har en instabil mod, dvs pendeln kan inte balanseras utan återkoppling.

$\theta_0 = 180^\circ \Rightarrow A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\quad \det(sI - A) = \det\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix} = s^2 + 1 = 0$

$s = \pm j$ $\therefore$ $\theta_0 = 180^\circ$ motsvarar en ickedämpad resonans.

4. a) $L(s) = K_p G(s) = \frac{K_p}{s(1 + 0{,}25 s)} = \frac{4K_p}{s^2 + 4s} \triangleq \frac{B(s)}{A(s)}$

KE $\quad A(s) + B(s) = s^2 + 4s + 4K_p = (K_p = 1) = (s + 2)^2$

b) $L_0(s) = K_p G_0(s) = \frac{1 \cdot 4(1 - T_z s)}{s^2 + 4s} = \frac{-4T_z s + 4}{s^2 + 4s} \triangleq \frac{B(s)}{A(s)}$

KE $\quad A(s) + B(s) = s^2 + 4(1 - T_z)s + 4 \quad$ stabilt $T_z < 1$

(2:a ordn. system stabilt då samtliga koeff. är > 0)

c) $\Delta_G(s) = \frac{G_0(s) - G(s)}{G(s)} = \frac{G_0(s)}{G(s)} - 1 = 1 - T_z s - 1 = -T_z s$

$T(s) = \frac{L(s)}{1 + L(s)} = \frac{4}{s^2 + 4s + 4} = \frac{4}{(s+2)^2}$ $\quad |T(j\omega)| = \frac{4}{4 + \omega^2} < \frac{1}{|\Delta_G(j\omega)|} = \frac{1}{T_z \omega}$

$\omega^2 - 4T_z\omega + 4 = 0$ för endast ett $\omega = 2T_z \pm \sqrt{4T_z^2 - 4}$

$\Rightarrow 4T_z^2 - 4 = 0$ dvs $T_z = 1$, motsvarar tangering av $|T| \overset{\circ}{=} \frac{1}{|\Delta_G|}$. Resultatet blir därför detsamma (som i uppg b), dvs det konservativa kriteriet är i detta fallet ett absolut krav.

5. $G_d(z) = \dfrac{1-e^{-h}}{z-e^{-h}}$ $\quad G_{ry}(z) = \dfrac{K_p G_d(z)}{1+K_p G_d(z)} =$

$= \dfrac{K_p(1-e^{-h})}{z-e^{-h}+K_p(1-e^{-h})}$ $\quad \therefore$ slutna systemets pol i $z = e^{-h} - K_p(1-e^{-h})$

Instabilitetsgräns för $z=-1 \Rightarrow$

$z = e^{-h} - K_p A_m (1-e^{-h}) = -1 \Rightarrow K_p = \dfrac{1+e^{-h}}{A_m(1-e^{-h})}$

Polen hamnar i $z = e^{-h} - (1+e^{-h})/A_m$

6. a) $x = \begin{bmatrix} y \\ v \end{bmatrix}$ $\quad \begin{aligned} \dot{y} &= v \\ \dot{v} &= -2v + 3u \end{aligned}$ $\quad \begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}}_{A} \begin{bmatrix} y \\ v \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{B} u$

b) $S = [B \ AB] = \begin{bmatrix} 0 & 3 \\ 3 & -6 \end{bmatrix}$ $\quad \det S = \det\begin{pmatrix} 0 & 3 \\ 3 & -6 \end{pmatrix} = -9 \neq 0$

$\therefore$ styrbart

c) $\det(sI_n - A + BL_u) = \det\begin{pmatrix} s & -1 \\ 3l_1 & s+2+3l_2 \end{pmatrix} = s^2 + (2+3l_2)s + 3l_1 =$

$= (s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2$

$\Rightarrow L_u = [l_1 \ l_2] = [\alpha^2/3 \ \ (2\alpha-2)/3]$

$G_{ry}(s) = C(sI_n - A + BL_u)^{-1}BK_r$ $\quad y = \underbrace{[1 \ 0]}_{C} \begin{bmatrix} y \\ v \end{bmatrix}$

$G_{ry}(0) = C(-A+BL_u)^{-1}BK_r = [1 \ 0]\begin{bmatrix} 0 & -1 \\ \alpha^2 & 2\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 3K_r \end{bmatrix} =$

$= \left[\frac{1}{\alpha^2} \ 0\right] \begin{bmatrix} 2\alpha & -\alpha^2 \\ 1 & 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 3K_r \end{bmatrix} = \dfrac{3K_r}{\alpha^2} = 1 \Rightarrow K_r = \alpha^2/3$

$u = -\underbrace{[\alpha^2/3 \ \ (2\alpha-2)/3]}_{L_u} \begin{bmatrix} y \\ v \end{bmatrix} + \underbrace{\alpha^2/3}_{K_r} r$

d) $G_{ru}(s) = -L_u(sI_n - A + BL_u)^{-1}BK_r + K_r$

$\rightarrow K_r$ då $s \rightarrow \infty$

$u(0) = \lim_{s\to\infty} s\, G_{ru}(s) \underbrace{\frac{1}{s}}_{R(s)} = G_{ru}(\infty) = K_r = \dfrac{\alpha^2}{3}$

$u(0)$ växer kvadratiskt med α dvs dubbelpolens avstånd från origo.