Lösning till tentamen i Reglerteknik 110525

1. a) $|G(j\omega)| = \dfrac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}^{\,2}} = \dfrac{1}{\sqrt{1+\omega^2}}$ $\quad \angle G(j\omega) = -\arctan\omega - 2\arctan\omega$

$G_1(s) = \dfrac{1}{s+1} \quad |G_1(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2}} = |G(j\omega)| \quad \angle G_1(j\omega) = -\arctan\omega > \angle G(j\omega)$

b) $G(s) = \dfrac{1-Ts}{(1+s)^2}$ $\quad$ Nollställe i $s = \dfrac{1}{T}$

stegssvar: $U(s) = \dfrac{1}{s}$ $\quad Y(s) = \dfrac{1}{(1+s)^2 s} - T\dfrac{1}{(1+s^2)} =$

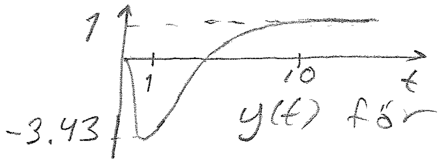
$= \dfrac{1}{s} - \dfrac{1}{s+1} - \dfrac{1+T}{(s+1)^2} \Rightarrow y(t) = 1 - (1 + (1+T)t)e^{-t}$

c) Minvärde då $\dot{y}(t) = (1+(1+T)t)e^{-t} - (1+T)e^{-t} = 0$

$1 + (1+T)t = 1+T \Rightarrow t = \dfrac{T}{1+T}$

$y_{min} = y\left(\dfrac{T}{1+T}\right) = 1 - \left(1 + (1+T)\dfrac{T}{1+T}\right)e^{-T/(1+T)} = 1-(1+T)e^{-\frac{T}{1+T}}$

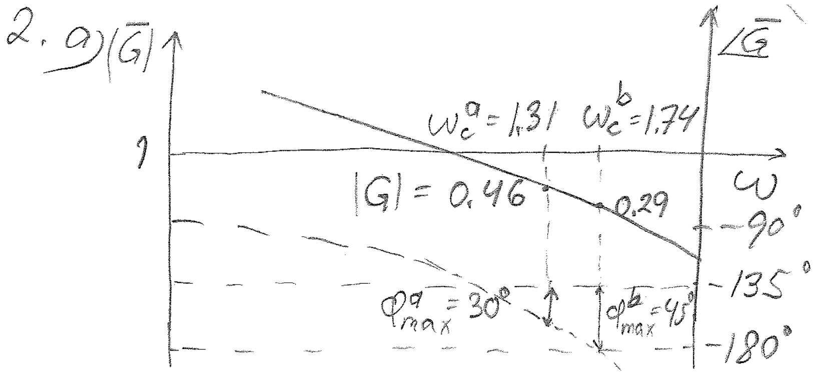
≈ -3.43 för $T=10$



d) $T >> 1 \Rightarrow T+1 \approx T \Rightarrow y_{min} \approx 1 - Te^{-1} \approx -T/e$

$\rightarrow -\infty$ då nollstället $1/T \rightarrow 0$

2. a)



$L(s) = \dfrac{e^{-0.3s}}{(1+s)^2} K_i \dfrac{(1+s)(1+s\tau_d)}{s(1+s\tau_d/b)}$

$= \underbrace{\dfrac{e^{-0.3s}}{s(1+s)}}_{\bar{G}(s)} \underbrace{K_i\dfrac{1+s\tau_d}{1+s\tau_d/b}}_{F_{PD}(s)}$

$\varphi_{max}^a = 30° \Rightarrow b^a = \dfrac{1+\sin 30°}{1-\sin 30°} = 3 \quad \tau_d^a = \sqrt{b^a}/\omega_c^a \approx 1.32$

$\sqrt{b^a} K_i^a |\bar{G}^a(j\omega_c^a)| = 1 \Rightarrow K_i^a = \dfrac{1}{\sqrt{b^a}|\bar{G}^a(j\omega_c^a)|} = 1.25$

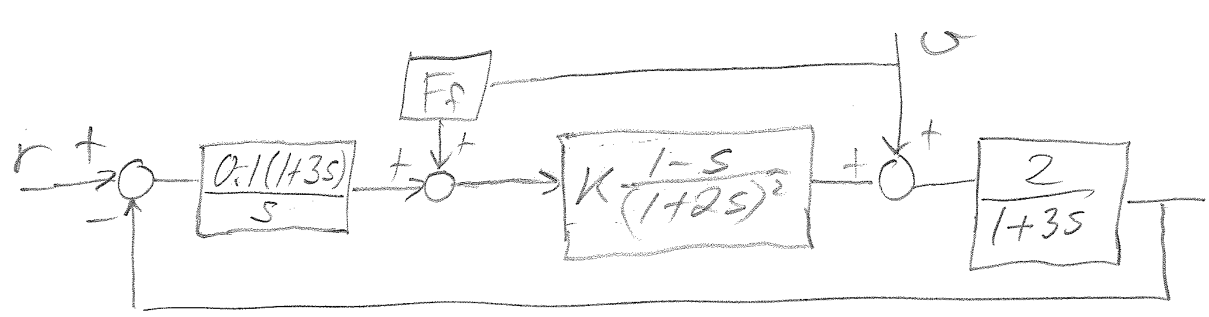
$\varphi_{max}^b = 45° \Rightarrow b^b = \dfrac{1+\sin 45°}{1-\sin 45°} = 5.83 \quad \tau_d^b = \sqrt{b^b}/\omega_c^b = 1.39$

$K_i^b = \dfrac{1}{\sqrt{b^b}|\bar{G}^b(j\omega_c^b)|} = 1.45 \quad K_\infty = F(\infty) = bK_i \Rightarrow K_\infty^a = 3.74$

$K_\infty^b = 8.44$

b) $K_i^b > K_i^a \quad K_\infty^b > K_\infty^a \Rightarrow$ alt. b ger bättre kompensering av laststörningar men högre känslighet för HF mätstörningar

3.



a) $$G_{vy}(s) = \frac{\frac{2}{1+3s}\left(1+F_f(s)K\frac{(1-s)}{(1+2s)^2}\right)}{1+\frac{2}{1+3s}K\frac{1-s}{(1+2s)^2}\cdot\frac{0.1(1+3s)}{s}} = 0$$

då $F_f(s) = -\frac{(1+2s)^2}{K(1-s)} = (K=1) = -\frac{(1+2s)^2}{1-s}$

b) Systemet blir instabilt eftersom ickeminfasnollstället blir en pol i $F_f(s)$ i $s=1$. Välj därför statisk framkoppling

$F_f = F_f(0) = -1$

c) $|G_{vy}|_{FB} \approx \frac{\omega}{K \cdot 0.1} = \frac{10\omega}{1.5} = 6.7\omega$ för små ω

$|G_{vy}|_{FB+FF} \approx 6.7s(1-1\cdot K) = |-0.5|\, 6.7\omega$
$\approx 3.3\omega$ för små ω

dvs trots den felaktiga förstärkningen i framkopplingen så blir det ändå en halvering av amplituden från v till y då framkopplingen ingår.

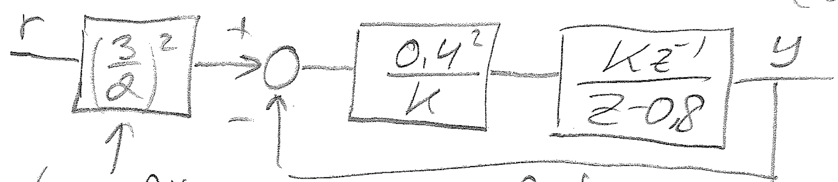
4. a) KE $1+\frac{K_r K z^{-1}}{z-0{,}8} = \frac{z^2-0{,}8z+K_rK}{z^2-0{,}8z} = 0$

$z = 0{,}4 \pm \sqrt{0{,}4^2 - K_rK}$ Reella poler då $K_r \le \frac{0{,}16}{K}$

∴ välj $K_r = 0{,}16/K$

b) $u = K_r(r-y) \Rightarrow G_{ry}(z) = \frac{0{,}16 z^{-1}}{1+\frac{0{,}16z^{-1}}{z-0{,}8}} = \frac{0{,}16}{z^2-0{,}8z+0{,}16}$

$= \frac{0{,}16}{(z-0{,}4)^2}$ $G_{ry}(1) = \frac{0{,}16}{0{,}36} = \left(\frac{0{,}4}{0{,}6}\right)^2 = \left(\frac{2}{3}\right)^2$

c) 

$\Rightarrow G_{ry}(1) = \left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{2}\right)^2 = 1$

extra förstärkning från referenssignalen

5.

a) $\frac{\Omega(s)}{U(s)} = \frac{1}{s+1}$ $\dot{\omega} = -\omega + u$ $\frac{\Theta(s)}{\Omega(s)} = \frac{1}{s}$ $\dot{\theta} = \omega$

$$\begin{bmatrix}\dot\theta\\ \dot\omega\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \qquad u = -[l_\theta \ l_\omega]\begin{bmatrix}\theta\\ \omega\end{bmatrix} + K_r r$$

$$\begin{bmatrix}\dot\theta\\ \dot\omega\end{bmatrix} = \begin{bmatrix}0 & 1\\ -l_\theta & -1-l_\omega\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0\\ K_r\end{bmatrix}r$$

$$y = [1 \ 0]\begin{bmatrix}\theta\\ \omega\end{bmatrix}$$

$$G_{ry}(s) = [1 \ 0]\begin{bmatrix}s & -1\\ l_\theta & s+1+l_\omega\end{bmatrix}^{-1}\begin{bmatrix}0\\ K_r\end{bmatrix} = \frac{[1 \ 0]\overbrace{\begin{bmatrix}s+1+l_\omega & 1\\ -l_\theta & s\end{bmatrix}\begin{bmatrix}0\\ K_r\end{bmatrix}}^{\begin{bmatrix}K_r\\ sK_r\end{bmatrix}}}{s^2+(1+l_\omega)s+l_\theta} =$$

$$= \frac{K_r}{s^2+(1+l_\omega)s+l_\theta} = \frac{\omega_n^2}{s^2+2\cdot 0{,}7\omega_n s+\omega_n^2} \qquad (\Rightarrow G_{ry}(0)=1)$$

$l_\omega = 1{,}4\omega_n - 1$ $l_\theta = \omega_n^2$ $K_r = \omega_n^2 = l_\theta$

b) $G_{ru}(s) = -\frac{[l_\theta \ l_\omega]\begin{bmatrix}K_r\\ sK_r\end{bmatrix}}{s^2+(1+l_\omega)s+l_\theta} + K_r$

$u(0) = \lim_{s\to\infty} s\, G_{ru}(s) \underbrace{\frac{1}{s}}_{R(s)} = G_{ru}(\infty) = K_r = \omega_n^2$

$u(0)_{2\omega_n} = 4\omega_n^2 = 4u(0)_{\omega_n}$ ∴ Dubblering av ω_n ger fyra gånger högre $u(0)$