BL 100819

Lösning till tentamen i Reglerteknik 2010-08-19

1. a) $G_{ry}(s) = \dfrac{G(s)F(s)}{1+G(s)F(s)} = \dfrac{\frac{0{,}5K_i}{s(s+1)}}{1+\frac{0{,}5K_i}{s(s+1)}} =$

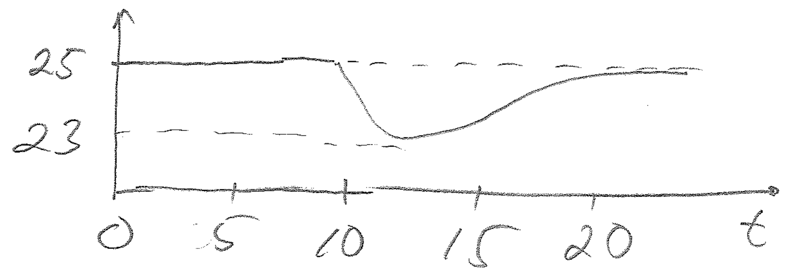
$= \dfrac{0{,}5K_i}{s^2+s+0{,}5K_i}$ $\quad y_0 = G_{ry}(0)r_0 = 1\cdot 25\,m/s$

b) $G_{vy}(s) = \dfrac{G(s)}{1+G(s)F(s)} = \dfrac{\frac{0{,}5}{s+1}}{1+\frac{0{,}5K_i}{s^2+s}} = \dfrac{0{,}5s}{s^2+s+0{,}5K_i}$

$\Delta Y(s) = G_{vy}(s)\Delta V(s) = \dfrac{0{,}5s}{s^2+s+0{,}5K_i}\cdot\dfrac{-5e^{-10s}}{s} =$

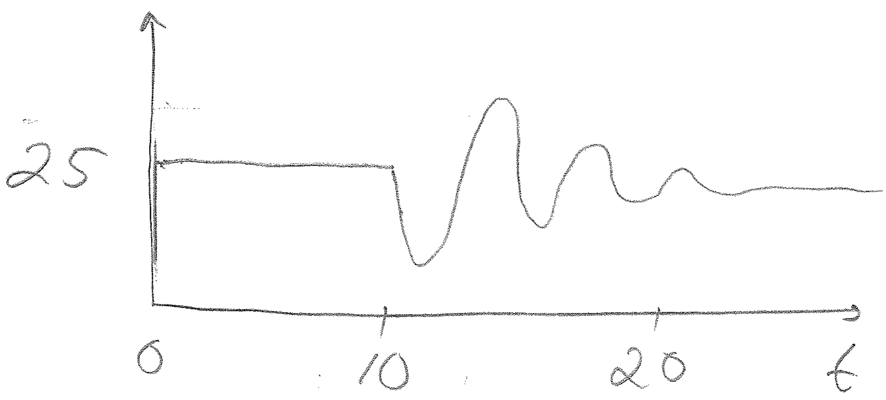
$= -\dfrac{2{,}5e^{-10s}}{s^2+s+0{,}25} = -\dfrac{2{,}5e^{-10s}}{(s+0{,}5)^2}$ $\quad$ för $K_i = 0{,}5$

$y(t) = y_0 + \mathcal{L}^{-1}\{\Delta Y(s)\} = 25 - 2{,}5(t-10)e^{-0{,}5(t-10)}\sigma(t-10)$



c) $G_{vy}(s) = \dfrac{0{,}5s}{s^2+s+8} = \dfrac{0{,}5s}{s^2+2\frac{1}{2\sqrt{8}}\sqrt{8}s+\sqrt{8}^2}$

$\omega_n = \sqrt{8} = 2{,}83 \qquad \zeta = \dfrac{1}{2\sqrt{8}} = 0{,}177$



2. a) $L(s) = G(s)F_{PI}(s) = \frac{1-Ts}{(1+s)^3}\,\frac{K_i(1+T_is)}{s} = (T_i=1) = \frac{K_i(1-Ts)}{\underbrace{s(1+s)^2}_{s^3+2s^2+s}}$

KE $\quad s^3+2s^2+(1-K_iT)s+K_i$

R-H		
s^3	1	$1-K_iT$
s^2	2	K_i
s^1	$\frac{2-2K_iT-K_i}{2}$	0
s^0	K_i	0

Stabilt då $2 > 2K_iT+K_i = (2T+1)K_i$

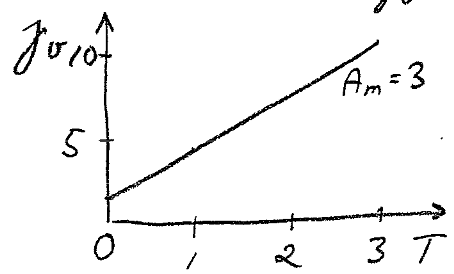
$K_i > 0$

$0 < K_i < \frac{2}{1+2T} = \frac{1}{T+0.5}$

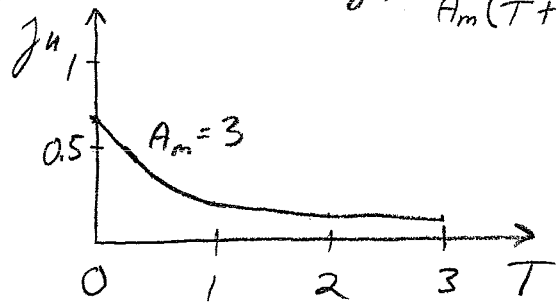
Amplitudmarginal $A_m \Rightarrow K_i = \frac{1}{A_m(T+0.5)}$

b) $F_{PI}(s) = \frac{K_i(1+s)}{s}$ $\qquad J_v = \frac{1}{K_i}$ $\qquad J_u = F_{PI}(\infty) = K_i$

$J_v = A_m(T+0.5)$



$J_u = \frac{1}{A_m(T+0.5)}$

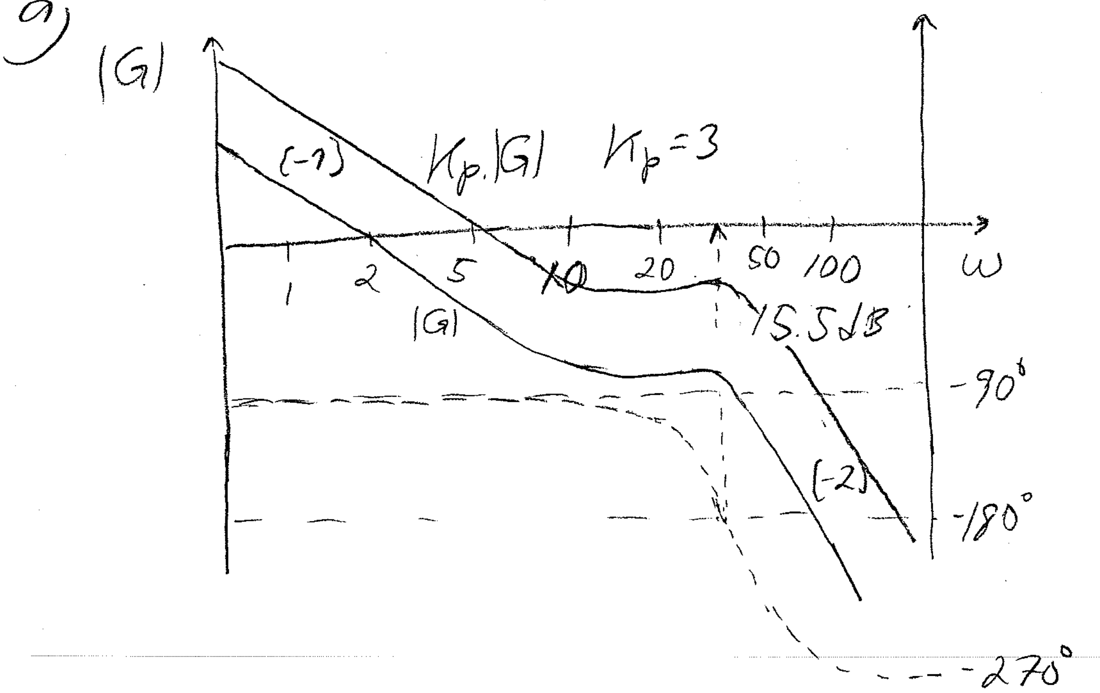


Försämrad kompensering av laststörningar (ökat J_v) men i gengäld lägre styrsignal aktivitet då nollstället i $s=1/T$ närmar sig origo. Samma tendens då A_m ökar.

3.

$$G(s) = \frac{2}{s(1+2\cdot 0.2\, s/30 + (s/30)^2)}$$

a)



$\angle G(j30) = -180° \Rightarrow \omega_{180} = 30$ rad/s

$|G(j30)| \approx -15.5$ dB $\approx 1/6$

$K_p = 3 \Rightarrow A_m = 2 \quad \varphi_m \approx 90°$

b) Faskurvan skär $-180°$ ungefär vid $\omega = \omega_n$ vilket innebär att ω_{180} skjunker till 20 rad/s. Vid denna frekvens kommer lågfrekvens delen $2/w$ att bidra med en högre förstärkning $2/20$ i stället för $2/30$, dvs en faktor $\frac{2/20}{2/30} = 1.5$ högre, vilket innebär att amplitudmarginalen skjunker till $2/1.5 = 1.33$.

4. a) $G(s)=e^{-sT_d}$ $\quad h=T_d \Rightarrow G_d(z)=z^{-1}$

$$F(s)=K_p+\frac{K_p/T_i}{s} \Rightarrow F_d(z)=K_p+\frac{K_pT_d/T_i\, z^{-1}}{1-z^{-1}}$$

$$L_d(z)=G_d(z)F_d(z)=K_p\frac{1+\left(\frac{T_d}{T_i}-1\right)z^{-1}}{1-z^{-1}}z^{-1}$$

b) $T_i=T_d \Rightarrow L_d(z)=K_p\frac{z^{-1}}{1-z^{-1}}=\frac{K_p}{z-1}$

$$\frac{Y(z)}{R(z)}=G_{ry}(z)=\frac{L_d(z)}{1+L_d(z)}=\frac{K_p}{z-(1-K_p)}$$

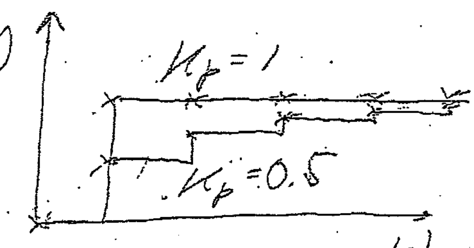
polplacering $\quad z=1-K_p=\begin{cases}0.5 & K_p=0.5\\ 0 & K_p=1\end{cases}$

Differensekv.

$$y(kh+h)=(1-K_p)y(kh)+K_p r(kh)$$

$y(kh)$ $K_p=0.5$	$y(kh)$ $K_p=1$
0	0
0.5	1
0.75	1
0.875	1

$r(kh)=1 \quad k>0$



c) $L_d(e^{j\omega h})=\frac{K_p}{e^{j\omega h}-1}=\frac{K_p}{\cos\omega h+j\sin\omega h-1}=$

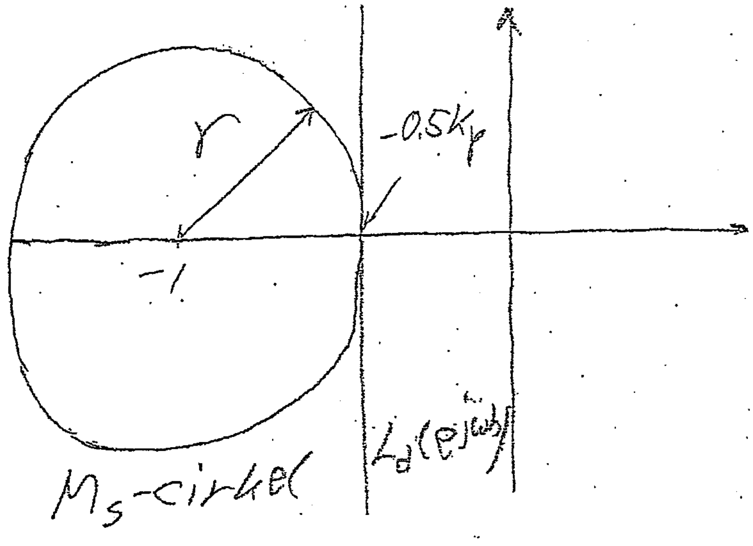
$$=K_p\frac{\cos\omega h-1-j\sin\omega h}{(\cos\omega h-1)^2-j^2\sin^2\omega h}=K_p\frac{\cos\omega h-1-j\sin\omega h}{\underbrace{\cos^2\omega h+\sin^2\omega h}_{1}+1-2\cos\omega h}$$

$$=-K_p\frac{1-\cos\omega h+j\sin\omega h}{2(1-\cos\omega h)}=-\frac{K_p}{2}-j\frac{K_p\sin\omega h}{1-\cos\omega h}$$

För små ω gäller $\frac{\sin\omega h}{1-\cos\omega h}\approx\frac{\omega h}{1-1+\frac{(\omega h)^2}{2}}=\frac{2}{\omega h}$

$$\Rightarrow \mathrm{Im}\,L_d \to \begin{matrix}-\infty & \omega\to 0^+\\ +\infty & \omega\to 0^-\end{matrix}$$

Nyquist diagram



$$r = 1 - 0.5K_p$$

$$M_s = \frac{1}{r} = \frac{1}{1-0.5K_p}$$

5. a) $x = \begin{bmatrix} y \\ v \end{bmatrix}$ $\quad \dot{y} = v$
$\dot{v} = -2v + 3u$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 3 \end{bmatrix} u \qquad y = [1 \ 0] x$$

b) $\det(B \ AB) = \det \begin{bmatrix} 0 & 3 \\ 3 & -6 \end{bmatrix} = 9 \neq 0$

$\therefore$ styrbart

c) $\det(sI - A + BL_u) = (s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2$

$L_u = [l_1, \ l_2]$

$$\det(sI - A + BL_u) = \det \begin{bmatrix} s & -1 \\ 3l_1 & s+2+3l_2 \end{bmatrix} =$$

$$= s^2 + (2+3l_2)s + 3l_1 = s^2 + 2\alpha s + \alpha^2$$

$$\Rightarrow 2 + 3l_2 = 2\alpha \quad \Rightarrow l_2 = 2(\alpha - 1)/3$$
$$3l_1 = \alpha^2 \quad \Rightarrow l_1 = \alpha^2/3$$

$$G_y(s) = C(sI - A + BL_u)^{-1} K_r \qquad G_y(0) = 1$$

$$K_r = \frac{1}{C(-A + BL_u)^{-1} B} = \frac{1}{[1 \ 0] \begin{bmatrix} 0 & -1 \\ \alpha^2 & 2\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 3 \end{bmatrix}}$$

$$= \frac{\alpha^2}{[1 \ 0] \begin{bmatrix} 2\alpha & 1 \\ -\alpha^2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}} = \alpha^2/3$$

d) $G_{rn}(s) = -L_u(sI - A + BL_u)^{-1}BK_r + K_r =$

$$= -\left[\alpha^2/3 \quad 2(\alpha-1)/3\right] \underbrace{\begin{bmatrix} s & -1 \\ \alpha^2 & s+2\alpha \end{bmatrix}^{-1}}_{\frac{\begin{bmatrix} s+2\alpha & 1 \\ -\alpha^2 & s \end{bmatrix}}{(s+\alpha)^2}} \begin{bmatrix} 0 \\ \alpha^2 \end{bmatrix} + \frac{\alpha^2}{3}$$

$$= -\left[\alpha^2/3 \quad 2(\alpha-1)/3\right] \frac{\begin{bmatrix} \alpha^2 \\ \alpha^2 s \end{bmatrix}}{(s+\alpha)^2} + \frac{\alpha^2}{3} =$$

$$= \frac{\alpha^2}{3}\left(1 - \frac{2(\alpha-1)s + \alpha^2}{(s+\alpha)^2}\right)$$

$$u(0) = \lim_{s\to\infty} G_{rn}(\infty) = \frac{\alpha^2}{3}$$

$u(0)$ växer kvadratiskt med polens avstånd α från origo.