Lösning Reglerteknik Z21F2 2010-01-13

1. a) $G(s) = \frac{K}{(1+Ts)^2}$ $\quad |G(j\omega)| = \frac{K}{(1+T^2\omega^2)}$

$$\frac{|G(j\omega_b)|}{|G(0)|} = \frac{K}{(1+T^2\omega_b^2)K} = \frac{1}{\sqrt{2}} \qquad 1+T^2\omega_b^2 = \sqrt{2}$$

$$\omega_b = \frac{1}{T}\sqrt{\sqrt{2}-1} = \frac{0.644}{T}$$

b) $$\frac{G(s)}{s} = K\left(\frac{A}{s} + \frac{B}{(1+Ts)} + \frac{C}{(1+Ts)^2}\right) = K\frac{A(1+Ts)^2 + Bs(1+Ts) + Cs}{s(1+Ts)^2}$$

$$= K\frac{A(1+2Ts+T^2s^2) + B(s+Ts^2) + Cs}{s(1+Ts)^2} =$$

$$= K\frac{(AT^2+BT)s^2 + (2AT+B+C)s + A}{s(1+Ts)^2}$$

$A = 1$

$B = -AT = -T$

$C = -B-2AT =$

$= T-2T = -T$

$$= K\left(\frac{1}{s} - \frac{1}{s+1/T} - \frac{1/T}{(s+1/T)^2}\right)$$

$$y(t_{5\%}) = K\left(1 - e^{-t_{5\%}/T} - \frac{t_{5\%}}{T}e^{-t_{5\%}/T}\right) = 0.95K$$

$$0.05 = \left(1+\frac{t_{5\%}}{T}\right)e^{-t_{5\%}/T} = (1+4.74)e^{-4.74} = 0.05$$

2. a) $G(s) = \frac{1}{(s+1)^2}$ $\nearrow 1$ små s, $\searrow \frac{1}{s^2}$ stora s

$F_{PI}(s) = 1.7\frac{s+0.8}{s}$ $\nearrow \frac{1.36}{s}$ små s, $\searrow 1.7$ stora s

$F_{PI_f}(s) = 5.6\frac{s+0.8}{s(s+5)}$ $\nearrow \frac{0.896}{s}$ små s, $\searrow 5.6/s$ stora s

För små s gäller då

$$G(s)S(s) = S(s) = \frac{1}{1+G(s)F(s)} = \begin{cases} \frac{1}{1+1.36/s} = \frac{s}{1.36} = 0.735s & F_{PI} \\ \frac{1}{1+0.896/s} = \frac{s}{0.896} = 1.12s & F_{PI_f} \end{cases}$$

För stora s gäller då

$$F(s)S(s) = F(s) = \begin{cases} 1.7 & F_{PI} \\ 5.6/s & F_{PI_f} \end{cases}$$

b) F_{PI}: Lägre $S = GS$ för små s ger effektivare följning av referenssignaler och kompensering av lastströrningar. F_{PI_f}: Lägre FS för stora s ger mindre känslighet för HF mätstörningar i styrsignalen

3. a) $\ddot{\theta} = \omega_0^2 \sin\theta + u \frac{\omega_0^2}{g} \cos\theta$

$$\begin{cases} \dot{\theta} = \omega \\ \dot{\omega} = \omega_0^2 \sin\theta + u \frac{\omega_0^2}{g} \cos\theta \end{cases}$$

Arbetspunkt $\theta = \omega = 0$, $u_0 = 0$

Linjärisering kring $\theta = \omega = 0$ ger $\Delta\theta = \theta$, $\Delta\omega = \omega$, $\Delta u = u$

$\dot{\theta} = \omega$

$$\dot{\omega} = \omega_0^2 \cos 0 \cdot \theta + \underset{\uparrow \atop u_0}{0} \cdot \frac{\omega_0^2}{g}(-\sin 0)\,\theta + \frac{\omega_0^2}{g} \cos 0 \cdot u = \omega_0^2 \theta + \frac{\omega_0^2}{g} u$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_0^2/g \end{bmatrix} u$$

b) Polerna ges av $\det(sI - A) = 0$

$$\det\begin{pmatrix} s & -1 \\ -\omega_0^2 & s \end{pmatrix} = s^2 - \omega_0^2 = 0 \Rightarrow s = \pm\omega_0$$

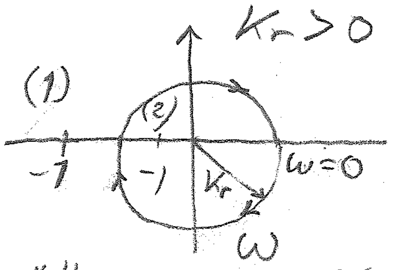
En pol i HHP ($s = \omega_0$) ger instabilt system

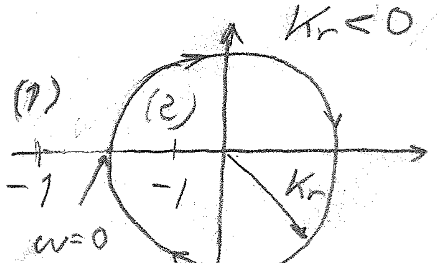
4.

$Y(s) = e^{-sT_d} U(s) \Rightarrow G(s) = e^{-sT_d}$ $\quad F(s) = K_r$

$L(s) = G(s)F(s) = K_r e^{-sT_d}$

$L(j\omega) = K_r e^{-j\omega T_d}$ $\quad |L(j\omega)| = K_r$ $\quad \angle L(j\omega) = -\omega T_d \frac{180^\circ}{\pi} + \angle K_r$





$\angle K_r = -180^\circ$

Z = antal rötter i HHP för KE $1 + L(s) = 0$

$$Z = P + N = N = \begin{cases} 0 & K_r < 1 \quad (1) \\ 1 & K_r \geq 1 \quad (2) \end{cases}$$

$$Z = P + N = N = \begin{cases} 0 & K_r > -1 \\ 1 & K_r \leq -1 \end{cases}$$

$\therefore$ stabilt då $-1 < K_r < 1$

5. a) $\dot\theta = \omega$

$\dot\omega = \ddot\theta = u$

$$\begin{bmatrix}\dot\theta\\ \dot\omega\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}}_{A}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{B}u$$

$$u = -\underbrace{[l_\theta \quad l_\omega]}_{L_u}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + K_r r$$

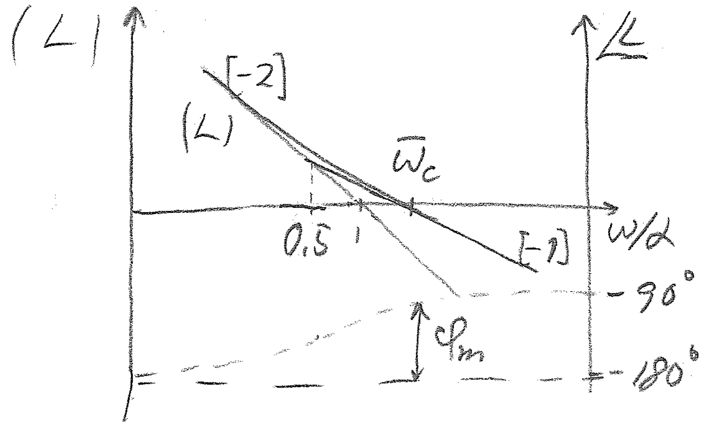
$$\det(sI_n - A + BL_u) = \det\begin{bmatrix}s & -1\\ l_\theta & s+l_\omega\end{bmatrix} = s^2 + l_\omega s + l_\theta =$$

$$= (s+\alpha)^2 = s^2 + 2\alpha s + \alpha^2 \Rightarrow l_\omega = 2\alpha \quad l_\theta = \alpha^2$$

b) $$L(s) = L_u(sI_n - A)^{-1}B = [\alpha^2 \ 2\alpha]\begin{bmatrix}s & -1\\ 0 & s\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix} =$$

$$= [\alpha^2 \ 2\alpha]\begin{bmatrix}s & 1\\ 0 & s\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\frac{1}{s^2} = \frac{\alpha^2 + 2\alpha s}{s^2} = \frac{1 + 2\,s/\alpha}{(s/\alpha)^2}$$

$$L(j\bar\omega) = \frac{1 + 2j\bar\omega}{\bar\omega^2} \qquad \bar\omega = \omega/\alpha$$



$$\bar\omega_c = \frac{\omega_c}{\alpha} = 2.1 \text{ rad/s}$$

$$\varphi_m = 76^\circ$$

$$|L(j\bar\omega_c)| = \frac{\sqrt{1 + 4\bar\omega_c^2}}{\bar\omega^2} = 1$$

$$\bar\omega_c^4 - 4\bar\omega_c^2 - 1 = 0$$

$$\bar\omega_c^2 = 2 \overset{+}{(-)} \sqrt{2^2 + 1} = 2 + \sqrt5$$

$$\bar\omega_c = \sqrt{2+\sqrt5} = 2.058$$

$$\varphi_m = 180^\circ + \underline{\angle L(j\bar\omega_c)} = 180^\circ - 180^\circ + \arctan 2\bar\omega_c =$$

$$= \arctan 4.116 = 76.3^\circ$$

6.

a) $G(s) = \frac{b}{a}\frac{a}{s+a} \Rightarrow \alpha = e^{-ah} \quad \beta = \frac{b}{a}(1-\alpha)$

b) Visas med induktion eftersom

$y(h) = \alpha y(0)$ och $y(kh+h) = \alpha y(kh) = \alpha\alpha^k y(0) = \dots = \alpha^{k+1} y(0)$

$y(kh) = (e^{-h})^k y(0) = e^{-kh} y(0)$

$kh = 1 \Rightarrow y(1) = e^{-1} y(0)$

eller

$y(2h) = \alpha y(h) = \alpha^2 y(0)$

$y(3h) = \alpha y(2h) = \alpha^3 y(0)$

$y(4h) = \dots$

c) $h \to 0 \Rightarrow \alpha \to 1 \quad y(\infty) = y(0)$

Ingen avklingning motsvarar en integral-process, dvs $a \to 0$, vilket också ger $\alpha = 1$

d) $y_\varepsilon(kh) = ((1+\varepsilon)\alpha)^k y(0) = (1+\varepsilon)^k \alpha^k y(0) =$

$= (1+\varepsilon)^k e^{-kh} y(0) = (1+\varepsilon)^k e^{-1} y(0)$

$y_\varepsilon(1) - y(1) = (1+\varepsilon)^k e^{-1} y(0) - e^{-1} y(0) = ((1+\varepsilon)^k - 1) e^{-1} y(0)$

h	$y(1)/y(0)$	$y_\varepsilon(1)/y(0)$	$(y_\varepsilon(1) - y(1))/y(0)$
10^{-2}	0.3679	0.3679	3.7E-6
10^{-4}	– " –	0.3683	3.7E-4
10^{-6}	– " –	0.4066	3.9E-2

$\varepsilon = 10^{-7}$

Felet ökar ungefär proportionellt mot $1/h$ dvs ett kort samplingsintervall ger större fel. Ett större ε ger större fel, dvs korta samplingsintervall kräver högre noggrannhet.