Lösning till tentamen i Reglerteknik 090528

1. a) $L(s) = K_p G(s) = \dfrac{K_p K (s+a)^2}{s^3} = \dfrac{\bar{K}s^2 + 2a\bar{K}s + \bar{K}a^2}{s^3}$ $\quad \bar{K} = K_p K$

KE $\quad s^3 + \bar{K}s^2 + 2a\bar{K}s + \bar{K}a^2$

Routh-Hurwitz tabell $\qquad$ Stabilitetskrav

$$\begin{array}{lll} s^3 & 1 & 2a\bar{K} \\ s^2 & \bar{K} & \bar{K}a^2 \\ s^1 & \dfrac{2a\bar{K}^2 - \bar{K}a^2}{\bar{K}} & 0 \\ s^0 & \bar{K}a & \end{array}$$

$\bar{K} > 0$

$2a\bar{K} - a^2 > 0$

$\bar{K}a > 0$

$a > 0 \Rightarrow \bar{K} = K_p K > \dfrac{a}{2}$

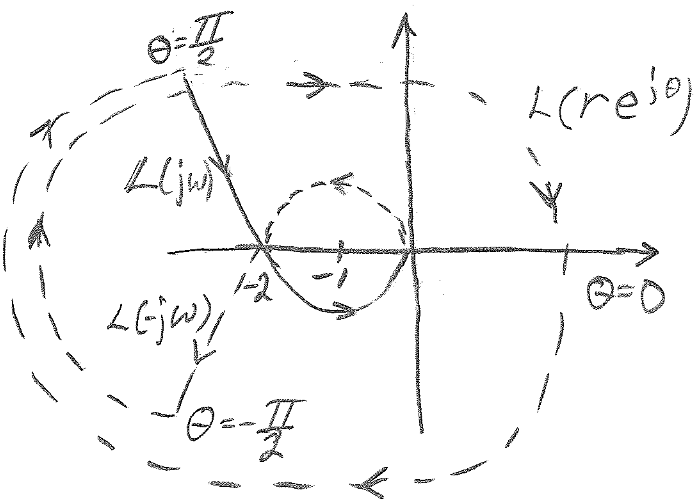
$\therefore \quad \underline{\underline{K_p > \dfrac{a}{2K}}}$

b) $A_m K_p = \dfrac{a}{2K} \qquad K_p = \dfrac{a}{A_m 2K}$ Kritiskt då förstärkningen minskar med en faktor 2 $\Rightarrow A_m = 0.5$ och $K_p = a/K$

c) $a = K = 1 \Rightarrow K_p = 1 \qquad L(s) = \dfrac{(s+1)^2}{s^3}$

$L(j\omega) = \dfrac{(j\omega+1)^2}{(j\omega)^3} = -\dfrac{1-\omega^2+2j\omega}{j\omega^3} = -\dfrac{2}{\omega^2} + j\dfrac{1-\omega^2}{\omega^3} = -2 \quad \omega = 1$

$L(re^{j\theta}) \underset{r \ll 1}{\approx} \dfrac{1}{(re^{j\theta})^3} = \dfrac{1}{r^3} e^{-j3\theta} \qquad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$



$$= \begin{cases} \dfrac{1}{r^3} e^{j\frac{3}{2}\pi} & \theta = -\dfrac{\pi}{2} \\ \dfrac{1}{r^3} & \theta = 0 \\ \dfrac{1}{r^3} e^{-j\frac{3}{2}\pi} & \theta = \dfrac{\pi}{2} \end{cases}$$

N = en moturs + en medurs omslingring $= 0$

Z = antalet nollställen till KE i HHP = $= P + N = 0 + 0 = 0 \quad \therefore$ stabilt

2. a) $G(j\omega) = \frac{1}{(1+j\omega)^2}$ $\quad |G(j\omega)| = \frac{1}{1+\omega^2}$ $\quad \underline{/G(j\omega)} = -2\arctan\omega$

$F_{PI}(j\omega) = K_i \frac{1+T_i j\omega}{j\omega}$ $\quad |F_{PI}(j\omega)| = K_i \frac{\sqrt{1+(T_i\omega)^2}}{\omega}$

$\underline{/F_{PI}(j\omega)} = -90° + \arctan \omega T_i$

$\underline{/L(j\omega_c)} = \underline{/G(j\omega_c)} + \underline{/F_{PI}(j\omega_c)} = -180° + \varphi_m \qquad \varphi_m = 50°$

$-2\arctan\omega_c - 90° + \arctan\omega_c T_i = -180° + 50° = -130°$

$\omega_c T_i = \tan(2\arctan\omega_c - 40°)$

$|L(j\omega_c)| = |G(j\omega_c)|\,|F_{PI}(j\omega_c)| = \frac{K_i\sqrt{1+(\omega_c T_i)^2}}{\omega_c(1+\omega_c^2)} = 1$

$K_i = \frac{\omega_c(1+\omega_c^2)}{\sqrt{1+\tan^2(2\arctan\omega_c - 40°)}}$

b) Minimalt $J_0 = \frac{1}{K_i}$ $\Rightarrow$ maximalt K_i

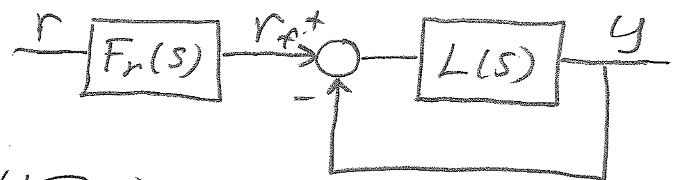
ω_c	1.3	1.36	1.4	1.5
K_i	1.486	1.493	1.490	1.456

Optimalt $\omega_c = 1.36$

$\Rightarrow K_i = 1.493$

$T_i = 1.762$

3. a) $L(s) = \frac{K_i(1+T_i s)}{s(1+s)^2}$



$G_{r_f y}(s) = \frac{L(s)}{1+L(s)} = \frac{K_i(1+T_i s)}{s^3+2s^2+(1+K_iT_i)s+K_i}$

$F_r(s) = G_{r_f y}^{-1}(s)\frac{1}{(1+T_f s)^m} = \frac{s^3+2s^2+(1+K_iT_i)s+K_i}{K_i(1+T_i s)(1+T_f s)^2}$

$\Rightarrow G_{ry}(s) = G_{r_f y}(s)F_r(s) = \frac{1}{(1+T_f s)^2}$

FS sid 13 $\Rightarrow$ $t_{5\%} = (3+1.6)T_f = 1 \Rightarrow T_f = \frac{1}{4.6} = 0.217$

b) Ökningen i $u(0) = \frac{r_f(0)}{r(0)} = \lim_{s\to\infty} s\,F_r(s)\frac{r(0)}{s}\frac{1}{r(0)} =$

$= \lim_{s\to\infty} F_r(s) = \lim_{s\to\infty} \frac{s^3+\ldots}{K_iT_iT_f^2 s^3+\ldots} = \frac{1}{K_iT_iT_f^2} = 8.07$

4. a) $\xrightarrow{u(kh)} \boxed{\text{Håll}} \longrightarrow \boxed{e^{-hs}} \xrightarrow[\bar{u}(kh)]{u(kh-h)} \boxed{\frac{1}{s}} \longrightarrow \underset{h}{\downarrow} \xrightarrow{y(kh)}$

$$\frac{Y(z)}{\bar{U}(z)} = \frac{h}{z-1} \qquad \bar{U}(z) = \mathcal{Z}\{u(kh-h)\} = z^{-1}U(z)$$

$$G_d(z) = \frac{Y(z)}{U(z)} = \frac{Y(z)}{\bar{U}(z)}\frac{\bar{U}(z)}{U(z)} = \frac{h}{z-1}z^{-1} = \frac{hz^{-2}}{1-z^{-1}}$$

b) $F_d(z) = K_p \qquad G_{ry_d}(z) = \dfrac{G_d(z)F_d(z)}{1+G_d(z)F_d(z)} =$

$$= \frac{K_p h z^{-2}}{1-z^{-1}+K_p h z^{-2}} = \frac{hK_p}{z^2 - z + hK_p} = \frac{hK_p}{(z-\alpha)^2} =$$

$$= \frac{hK_p}{z^2 - 2\alpha z + \alpha^2} \qquad \begin{cases} 2\alpha = 1 \\ \alpha^2 = hK_p \end{cases} \qquad \begin{aligned} &\alpha = 0.5 \\ &K_p = \alpha^2/h = 0.25/h \end{aligned}$$

c) $$G_{vy_d}(z) = \frac{G_d(z)}{1+G_d(z)F_d(z)} = \frac{G_{ry_d}(z)}{F_d(z)} = \frac{h}{(z-\alpha)^2}$$

$G_{vy_d}(e^{j\omega h})$ för $\omega = 0 \Leftrightarrow G_{vy_d}(1) = \dfrac{h}{(1-\alpha)^2} = \dfrac{h}{0.25} = \dfrac{1}{K_p}$

5. a) $$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2\sqrt{x_{10}}} & 1 \\ u_0 & -1 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ x_{10} \end{bmatrix}\Delta u$$

$$\Delta y = \begin{bmatrix} 2x_{10} & u_0 \cos x_{20} \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \sin x_{20}\,\Delta u$$

b) $$\begin{bmatrix} -\sqrt{x_{10}} + x_{20} \\ x_{10}u_0 - x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{cases} x_{10} = x_{20}^2 \\ x_{20} = x_{10}u_0 = x_{20}^2 u_0 \end{cases} \qquad \begin{cases} x_{10} = \frac{1}{u_0^2} \\ x_{20} = \frac{1}{u_0} \end{cases}$$

c) $$A = \begin{bmatrix} -0.5u_0 & 1 \\ u_0 & -1 \end{bmatrix} \qquad \det(sI_n - A) = \det\begin{bmatrix} s+0.5u_0 & -1 \\ -u_0 & s+1 \end{bmatrix} =$$

$$= (s+0.5u_0)(s+1) - u_0 = s^2 + (1+0.5u_0)s - 0.5u_0$$

stabilt då $\begin{cases} 1+0.5u_0 > 0 \\ -0.5u_0 > 0 \end{cases} \qquad \underline{\underline{-2 < u_0 < 0}}$