Signaler & System, Z2
SSY042 2020-06-09

1a) c_0 svarar mot signalens medelvärde

$$c_0 = \frac{1}{T}\int_0^T x(t)\,dt = \ldots = 4$$

Periodtid $T = 5$ $\quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{5} = 0{,}4\pi$

b) $$X(z) = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n} = \underbrace{z^{-6}}_{X_1(z)} + \underbrace{\frac{z^{-3}}{z+0{,}5}}_{X_2(z)}$$

$$X_1(z) = 1 \cdot z^{-6} \Rightarrow x_1[n] = \delta[n-6]$$

$$X_2(z) = \frac{z^{-3}}{z+0{,}5} = z^{-4}\,\frac{z}{z+0{,}5} = (-0{,}5)^{n-4} \cdot u[n-4]$$

$$x[n] = \delta[n-6] + (-0{,}5)^{n-4} \cdot u[n-4]$$

c) Använd tex Parsevals formel (Fourierserie)

$$x(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) =$$

$$= 2\cos(\omega_0 t) + 5\sin(\omega_0 t)$$

Alltså $n = 1$ och $a_1 = 2$, $b_1 = 5$
övriga $a_n, b_n = 0$

$$\bar{P} = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{1}{2}(a_1^2 + b_1^2) = \frac{1}{2}(4+25) = \frac{29}{2}$$

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2. a) Pol-nollställe plottar

A: Nollställe i origo
$\Rightarrow H_A(j\omega) = 0$ för $\omega = 0$
Stämmer med H_1

C: Nollställe vid $j\omega = \pm j2$
$\Rightarrow H_C(j\omega) = 0$ för $\omega = 2$
Stämmer med H_3

B: Inga nollställen
Komplexa poler "nära" im-axeln vid $j\omega = \pm j3$
$\Rightarrow |H_B|$ får högt värde vid $\omega = 3$
Stämmer med H_4

D: Inga nollställen
Två reella poler.
$\Rightarrow |H_D(j\omega)|$ faller monotont då ω ökar
Stämmer med H_2

Svar:

Plot	$\|H(j\omega)\|$
A	H_1
B	H_4
C	H_3
D	H_2

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2b)

$N = 64$

$T_s = 5{,}0\,ms \Rightarrow f_s = \frac{1}{T_s}$; $\omega_s = \frac{2\pi}{T_s} = 200 \cdot 2\pi$ r/s $=$

$= 400\pi$ r/s

I) $x_1(t) = \underbrace{\cos(104\pi t)}_{\omega_{11} = 104\pi} + \underbrace{e^{j2\pi \cdot 120t}}_{\omega_{12} = 240\pi \text{ r/s}}$

- $\frac{k}{N} = \frac{\omega_{11}}{\omega_s} \Rightarrow k = \frac{\omega_{11}}{\omega_s} \cdot N = \frac{104}{400} \cdot 64 = 16{,}64 \approx 17$

Reell signal ger "topp" även vid $N-k \approx 47$

$x_1(t)$ har medelvärde noll $\Rightarrow X[0] = 0$

- $\frac{k}{N} = \frac{\omega_{12}}{\omega_s} \Rightarrow k = \frac{\omega_{12}}{\omega_s} \cdot N = \frac{240}{400} \cdot 64 = 38{,}4 \approx 38$

II) $x_2(t) = \underbrace{\cos(2\pi \cdot 148t)}_{\omega_{21} = 296\pi} + \underbrace{e^{j160\pi t}}_{\omega_{22} = 160\pi \text{ r/s}}$

- $\frac{k}{N} = \frac{\omega_{21}}{\omega_s} \Rightarrow k = \frac{\omega_{21}}{\omega_s} \cdot N = \frac{296}{400} \cdot 64 = 47{,}36 \approx 47$

Reell signal ger "topp" även vid $N-k \approx 17$

- $\frac{k}{N} = \frac{\omega_{22}}{\omega_s} \Rightarrow k = \frac{160}{400} \cdot 64 = 25{,}6 \approx 26$

$x_2(t)$ har medelvärde noll $\Rightarrow X[0] = 0$

Svar: $x_1(t)$: $X[k]$ stort vid $k = 17, 47, 38 \Rightarrow$ C
$x_2(t)$: $X[k]$ stort vid $k = 17, 47, 26 \Rightarrow$ A

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3. $y[n] = [34(-0{,}4)^n - 15(-0{,}2)^n + 6(0{,}6)^n]\, u[n]$

z-transf.

$$Y(z) = 34\frac{z}{z+0{,}4} - 15\frac{z}{z+0{,}2} + 6\frac{z}{z-0{,}6} =$$

$$= z\cdot\left(\frac{34(z+0{,}2)(z-0{,}6) - 15(z+0{,}4)(z-0{,}6) + 6(z+0{,}4)(z+0{,}2)}{(z+0{,}4)(z+0{,}2)(z-0.6)}\right)$$

$$= z\left(\frac{34(z^2-0{,}4z-0{,}12) - 15(z^2-0{,}2z-0{,}24) + 6(z^2+0{,}6z+0{,}08)}{(z+0{,}4)(z+0{,}2)(z-0{,}6)}\right)$$

$$= z\left(\frac{25z^2-7z}{(z+0{,}4)(z+0{,}2)(z-0{,}6)}\right)$$

$x[n] = 5[(-0{,}4)^n]\, u[n]$ z-transf.

$$X(z) = 5\frac{z}{z+0{,}4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\cancel{z}(25z^2-7z)}{\cancel{(z+0{,}4)}(z+0{,}2)(z-0{,}6)}\cdot\frac{\cancel{(z+0{,}4)}}{5z} \Rightarrow$$

$$H(z) = \frac{z}{5}\cdot\underbrace{\frac{25z-7}{(z+0{,}2)(z-0{,}6)}}_{\text{P.B.U}} = \frac{z}{5}\cdot\left[\frac{A}{z+0{,}2} + \frac{B}{z-0{,}6}\right]$$

$25z-7 = A(z-0{,}6) + B(z+0{,}2)$; $z=-0{,}2 \Rightarrow A = \frac{-25\cdot0{,}2-7}{-0{,}8} = 15$

$z=0{,}6 \Rightarrow B = \frac{25\cdot0{,}6-7}{0{,}8} = 10$

$$H(z) = 3\frac{z}{z+0{,}2} + 2\frac{z}{z-0.6}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = [3(-0{,}2)^n + 2(0{,}6)^n]\, u[n]$$

$$H(z) = z\left(\frac{3(z-0{,}6)+2(z+0{,}2)}{(z+0{,}2)(z-0{,}6)}\right) = \frac{5z^2-1{,}4z}{z^2-0{,}4z-0{,}12}$$

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4. $$y(t) + a_1 \frac{dy(t)}{dt} + a_2 \frac{d^2y(t)}{dt^2} = b_1 \frac{dx(t)}{dt}$$

Laplacetransf. (system i vila)

$$Y(s)\left(1 + a_1 s + a_2 s^2\right) = X(s) \cdot s b_1$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s b_1}{1 + a_1 s + a_2 s^2}$$

Frekvenssvar, sätt $s = j\omega$

$$H(j\omega) = \frac{j\omega b_1}{1 + j\omega a_1 - a_2\omega^2} = \frac{j\omega b_1}{1 - a_2\omega^2 + j\omega a_1}$$

Amplitudpåverkan:

$$|H(j\omega)| = \frac{\omega b_1}{\sqrt{(1 - a_2\omega^2)^2 + (\omega a_1)^2}}$$

Fasändring: $$\arg\{H(j\omega)\} = \arg\{j\omega b_1\} - \arg\{1 - a_2\omega^2 + j\omega a_1\} = \frac{\pi}{2} - \arctan\left\{\frac{\omega a_1}{1 - a_2\omega^2}\right\}$$

Insignal $x(t) = \cos(\omega_0 t)$

Utsignal $y(t) = |H(j\omega_0)| \cos\left(\omega_0 t + \arg\{H(j\omega_0)\}\right)$

$$y(t) = \frac{\omega_0 b_1}{\sqrt{(1 - a_2\omega_0^2)^2 + (\omega_0 a_1)^2}} \cos\left(\omega_0 t + \frac{\pi}{2} - \arctan\left\{\frac{\omega_0 a_1}{1 - a_2\omega_0^2}\right\}\right)$$

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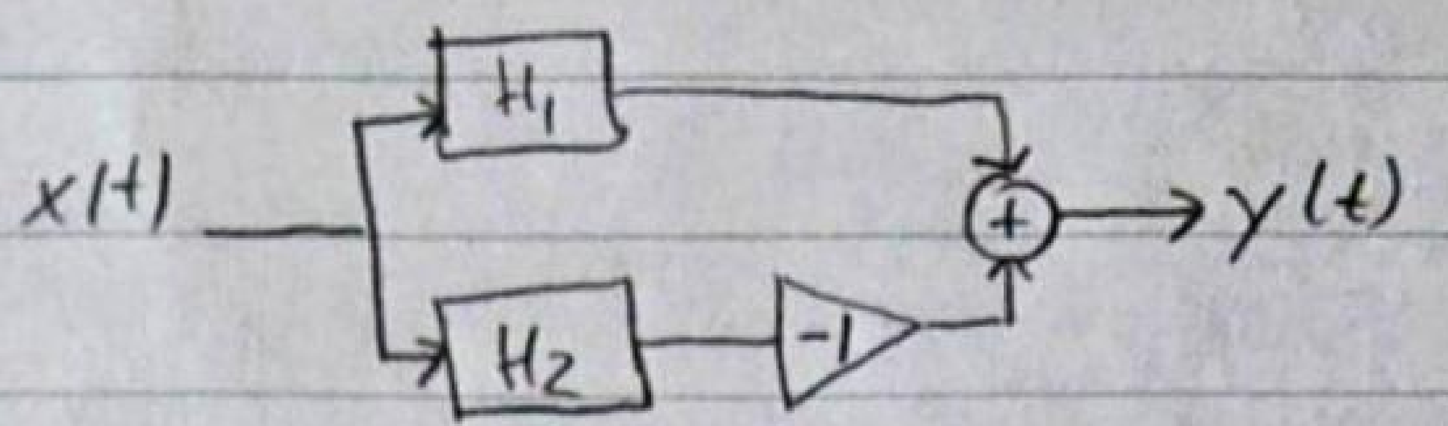

$$h_1(t) = Ae^{-2t}u(t) \; ; \; H_1(s) = \mathcal{L}\{h_1(t)\} = \frac{A}{s+2}$$

$$H_2(s) = \frac{B}{s+5} \Rightarrow h_2(t) = \mathcal{L}^{-1}\{H_2(s)\} = Be^{-5t}u(t)$$

Totala systemets impulssvar $h(t) = h_1(t) - h_2(t)$

$$h(t) = \left(Ae^{-2t} - Be^{-5t}\right)u(t)$$

$$h(0) = 2 \Rightarrow A - B = 2 \quad (-B = 2 - A)$$

$$H(s) = H_1(s) - H_2(s) = \frac{A}{s+2} - \frac{B}{s+5} = \frac{A(s+5) - B(s+2)}{(s+2)(s+5)} =$$

$$= \frac{s(A-B) + 5A - 2B}{(s+2)(s+5)}$$; Stegsvar
Insignal $x(t) = u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}$

$$Y_s(s) = \frac{1}{s}\,\frac{2s + 5A + 4 - 2A}{(s+2)(s+5)} = \frac{1}{s}\cdot\frac{2s + 3A + 4}{(s+2)(s+5)}$$

P.BU. $Y_s(s) = \frac{1}{s}\cdot\frac{2s+3A+4}{(s+2)(s+5)} = \frac{P_1}{s} + \frac{P_2}{s+2} + \frac{P_3}{s+5}$

P_1 ger $y_s(t)$ då $t \to \infty$ ty $y_s(t) = \left(P_1 + P_2e^{-2t} + P_3e^{-5t}\right)u(t)$

$$2s + 3A + 4 = P_1(s+2)(s+5) + P_2\, s(s+5) + P_3\, s(s+2)$$

$$s = 0 \; ; \; 3A + 4 = P_1(2)(5) = \frac{16}{10}\cdot 10$$

$$3A = 16 - 4 \Rightarrow A = 4$$

$$B = 2$$