SSY042 för Z2

2020-03-19

1a) $a_0 = \frac{5}{2}$, signalens medelvärde $\quad a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{6}\cdot 5\cdot 3 = \frac{5}{2}$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$ rad/s

$\sin(n\omega_0 t)$ udda, $x(t)$ är jämn $\Rightarrow b_1 = 0$ ($b_n = 0$, $n=1,2,\ldots$)

b) sinusformad signal ger högst värde i $|X[k]|$ vid $k=6$

$\frac{k}{N} = \frac{\omega}{\omega_s} \Rightarrow \omega = \frac{k}{N}\cdot\omega_s = \frac{k}{N}\frac{2\pi}{T} = \frac{6}{64}\,\frac{2\pi}{10\cdot 10^{-3}} = 9.4\cdot 2\pi$ rad/s

c) $y(t) = \pi\sin(9t) + \sqrt{2}\cos\left(\frac{15}{2}t\right)$, $\forall t$

$\omega_1 = 9$, $T_1 = \frac{2\pi}{\omega_1}$

$\omega_2 = \frac{15}{2}$, $T_2 = \frac{2\pi}{\omega_2}$

$T = k_1 T_1 = k_2 T_2$

$$\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{9\cdot 2}{15} = \frac{3\cdot 2}{5} = \frac{6}{5}$$

$$T = k_1 T_1 = 6\cdot\frac{2\pi}{9} = \frac{4\pi}{3}\ \text{s}$$

eller

$$T = k_2 T_2 = 5\cdot\frac{2\pi\cdot 2}{15} = \frac{4\pi}{3}\ \text{s}$$

Svar: Ja! $T = \frac{4\pi}{3}$ s

2. $y(t) = A\left[1 - e^{-t}\left(\cos(4t) + 0{,}25\sin(4t)\right)\right]u(t)$; $A = \frac{8}{17}$

Insignal $x(t) = u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}$

$$\mathcal{L}\{y(t)\} = Y(s) = A\left[\frac{1}{s} - \frac{s+1}{(s+1)^2+16} - 0{,}25\cdot\frac{4}{(s+1)^2+16}\right] =$$

$$= A\left[\frac{1}{s} - \left(\frac{s+1+0{,}25\cdot 4}{(s+1)^2+16}\right)\right] = A\left[\frac{1}{s} - \frac{s+2}{(s+1)^2+16}\right] =$$

$$= A\left[\frac{(s+1)^2+16 - s^2 - 2s}{s\left[(s+1)^2+16\right]}\right] =$$

$$= A\left[\frac{s^2+2s+17-s^2-2s}{s\left[(s+1)^2+16\right]}\right] = A\left[\frac{17}{s\left[(s+1)^2+16\right]}\right] =$$

$$= \left\{A = \frac{8}{17}\right\} = \frac{1}{s}\cdot\left(\frac{8}{(s+1)^2+16}\right)$$

$$Y(s) = X(s)\cdot H(s) \Rightarrow H(s) = \frac{8}{(s+1)^2+16}$$

Impulssvar

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{2\cdot\frac{4}{(s+1)^2+4^2}\right\} =$$

$$= 2e^{-t}\sin(4t)\cdot u(t)$$

3. $h[n] = [2(0{,}2)^n - 0{,}5(-0{,}4)^n]\, u[n]$

z-transf.

$$H(z) = 2\frac{z}{z-0{,}2} - 0{,}5\frac{z}{z+0{,}4} = z\left[\frac{2(z+0{,}4)-0{,}5(z-0{,}2)}{(z-0{,}2)(z+0{,}4)}\right] =$$

$$= \frac{z(1{,}5z+0{,}9)}{(z-0{,}2)(z+0{,}4)}$$

$$x[n] = 3(0{,}8)^n \xleftrightarrow{z} X(z) = \frac{3z}{z-0{,}8}$$

$$Y(z) = H(z)\cdot X(z) = \frac{3z^2(1{,}5z+0{,}9)}{(z-0{,}8)(z-0{,}2)(z+0{,}4)} =$$

$$= z\cdot \underbrace{\frac{3z(1{,}5z+0{,}9)}{(z-0{,}8)(z-0{,}2)(z+0{,}4)}}_{\text{PBU } = \frac{A}{z-0{,}8}+\frac{B}{z-0{,}2}+\frac{C}{z+0{,}4}}$$

$$3z(1{,}5z+0{,}9) = A(z-0{,}2)(z+0{,}4) + B(z-0{,}8)(z+0{,}4) + C(z-0{,}8)(z-0{,}2)$$

$z=0{,}8 \Rightarrow 3\cdot 0{,}8(1{,}5\cdot 0{,}8+0{,}9) = A(0{,}6)\cdot 1{,}2 \Rightarrow A = 7$

$z=0{,}2 \Rightarrow 3\cdot 0{,}2(1{,}5\cdot 0{,}2+0{,}9) = B(-0{,}6)(0{,}6) \Rightarrow B=-2$

$z=-0{,}4 \Rightarrow -3\cdot 0{,}4(-1{,}5\cdot 0{,}4+0{,}9) = C(-1{,}2)(-0{,}6) \Rightarrow C = -0{,}5$

(Multiplizera in z igen)

$$Y(z) = 7\frac{z}{z-0{,}8} - 2\frac{z}{z-0{,}2} - 0{,}5\frac{z}{z+0{,}4}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = [7\cdot(0{,}8)^n - 2(0{,}2)^n - 0{,}5(-0{,}4)^n]\, u[n]$$

4.

$$y[n] - a\,y[n-1] = b\,x[n-1] \qquad a = \frac{1}{\sqrt{2}}\,,\ b = 2$$

z-transformera

$$Y(z) - a z^{-1} Y(z) = b\, z^{-1} X(z)$$

$$Y(z)(1 - a z^{-1}) = X(z)\, b z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b z^{-1}}{1 - a z^{-1}} = \frac{b}{z - a}$$

Frekvenssvar

$$H(e^{j\Omega}) = H(z)\Big|_{z = e^{j\Omega}} = \frac{b}{e^{j\Omega} - a} =$$

$$= \frac{b}{\cos(\Omega) + j\sin(\Omega) - a} = \frac{b}{\cos(\Omega) - a + j\sin(\Omega)}$$

Insignal $x[n] = \cos(\Omega_0 n)$ med $\Omega_0 = \frac{\pi}{4}$

$$H(e^{j\Omega})\Big|_{\Omega = \frac{\pi}{4}} = \frac{2}{\cos(\frac{\pi}{4}) - \frac{1}{\sqrt{2}} + j\sin(\frac{\pi}{4})} =$$

$$= \frac{2}{j\frac{1}{\sqrt{2}}} = -j\,2\sqrt{2} = 2\sqrt{2}\, e^{-j\frac{\pi}{2}}$$

$$y[n] = \left|H(e^{j\frac{\pi}{4}})\right| \cos\left(\frac{\pi}{4}n + \arg\{H(e^{j\frac{\pi}{4}})\}\right) =$$

$$= 2\sqrt{2}\cos\left(\frac{\pi}{4}n - \frac{\pi}{2}\right), \quad \forall n$$

5.

$$H(s) = \frac{K}{(s-p_1)(s-p_2)} \quad ; K>0 \qquad p_1 = -1360$$

Frekvenssvar $H(s)\Big|_{s=j\omega} = \dfrac{K}{(j\omega-p_1)(j\omega-p_2)}$

För insignal $x(t) = \sin(\omega t)$ blir utsignalen

$y(t) = |H(j\omega)| \sin(\omega t + \phi)$

där $\phi = \arg\{H(j\omega)\}$

Utsignalen är fördröjd, teckna som

$$y(t) = |H(j\omega)| \sin(\omega(t-t_0)) = |H(j\omega)| \sin(\omega t - \omega t_0)$$

$\phi = \arg\{H(j\omega)\} = -\omega t_0$ där $\omega = 125{,}2\pi$ rad/s

och $t_0 = 1{,}0$ ms

$$\phi = -\omega t_0 = -0{,}25\pi = -\frac{\pi}{4}$$

$H(j\omega) = \dfrac{K}{(j\omega+1360)(j\omega+x)}$ med $\boxed{\omega = 250\pi \text{ rad/s}}$

$$\arg\{H(j\omega)\} = \underbrace{\arg\{K\}}_{=0} - \underbrace{\arctan\frac{\omega}{1360}}_{\frac{\pi}{6}\,(30^\circ)} - \underbrace{\arctan\frac{\omega}{x}}_{\frac{\pi}{12}\,(15^\circ)} = \underbrace{-\frac{\pi}{4}}_{(-45^\circ)}$$

Alltså: $\arctan\dfrac{\omega}{x} = \dfrac{\pi}{12}\ (15^\circ)$

$$\tan\frac{\pi}{12} = \frac{\omega}{x} \Rightarrow x = \frac{\omega}{\tan(\pi/12)} = 2931$$

$$p_2 = -x = -2931$$