Signaler och System, Z2
SSY042 2019-08-28

1. $T = 625\ \mu s \Rightarrow f_s = \frac{1}{T} = 1600\ Hz$

$N = 64 \qquad \frac{k}{N} = \frac{f}{f_s} \Rightarrow k = \frac{f}{f_s} \cdot N$

Signal	f [Hz]	$\frac{f}{f_s} \cdot N$	→ k	N-k
1	420	16,8	17	47
2	455	18,2	18	46
3	1110	44,4	44	20
4	544	21,76	22	42

Notera: Aliasing för signal x_3.

Jämför $|X[k]|$ värden i figur med framräknade index k och N-k.
Vilka värden på $|X[k]|$ är störst?

Vi ser att
x_1 – B
x_2 – D
x_3 – C
x_4 – A

SSY042

2019-08-28

2

$$h(t) = 10e^{-10t}u(t) \xleftrightarrow{\mathcal{L}} H(s) = \frac{10}{s+10}$$

$$x(t) = \cos(\omega_0 t)u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{s}{s^2+\omega_0^2} \quad ; \omega_0 = 4\pi \ 1/s$$

$$Y(s) = H(s)X(s) = \frac{10}{s+10} \cdot \frac{s}{s^2+\omega_0^2} = \frac{A}{s+10} + \frac{Bs+C}{s^2+\omega_0^2}$$

$$10s = A(s^2+\omega_0^2) + \overbrace{(Bs+C)(s+10)}^{Bs^2+(10B+C)s+10C}$$

s^2; $0 = A+B$	$A = -B$	$0 = A(\omega_0^2+10^2)+10^2$
s^1: $10 = 10B+C$	$C = 10(1-B)$	$A = -\frac{100}{\omega_0^2+100} = -B$
s^0: $0 = A\omega_0^2+10C$	$0 = A\omega_0^2+10^2(1+A)$	

$$C = 10\left(1-\frac{100}{\omega_0^2+100}\right) \approx 6{,}12 \qquad B \approx 0{,}388$$

$$y(t) = Ae^{-10t} + B\frac{s}{s^2+\omega_0^2} + \frac{C}{\omega_0} \cdot \frac{\omega_0}{s^2+\omega_0^2} =$$

$$= \left[-0{,}388\left(e^{-10t} - \cos\omega_0 t\right) + 0{,}487 \sin\omega_0 t\right] u(t)$$

Kan skrivas om som

$$B\cos(\omega_0 t) + \frac{C}{\omega_0}(\sin\omega_0 t) = \sqrt{B^2+\left(\frac{C}{\omega_0}\right)^2} \cdot \sin(\omega_0 t+\varphi)$$

med $\varphi = \arcsin\frac{B}{\sqrt{B^2+\left(\frac{C}{\omega_0}\right)^2}} = 38{,}5^\circ$

och $y(t) = \left[0{,}623 \cdot \sin\left(\omega_0 t + 38{,}5^\circ\right) - 0{,}388 \cdot e^{-10t}\right] u(t)$

3. $y[n] + 0{,}6\,y[n-1] - 0{,}16\,y[n-2] = x[n-1] + 0{,}5\,x[n-2]$

z-transf.

$$Y(z)\left(1 + 0{,}6z^{-1} - 0{,}16z^{-2}\right) = X(z)\left(z^{-1} + 0{,}5z^{-2}\right)$$

$$H(z) = \frac{z^{-1} + 0{,}5z^{-2}}{1 + 0{,}6z^{-1} - 0{,}16z^{-2}} = \frac{z + 0{,}5}{z^2 + 0{,}6z - 0{,}16} = \frac{Y(z)}{X(z)}$$

Poler: $z_{1,2} = -0{,}3 \pm \sqrt{0{,}3^2 + 0{,}16} = -0{,}3 \pm 0{,}5 = \begin{cases} 0{,}2 \\ -0{,}8 \end{cases}$

$$H(z) = \frac{z + 0{,}5}{(z - 0{,}2)(z + 0{,}8)}$$

$y[n] = \left[0{,}2^n - (-0{,}8)^n\right]u[n]$ z-transf.

$$Y(z) = \frac{z}{z - 0{,}2} - \frac{z}{z + 0{,}8} = z \cdot \frac{z + 0{,}8 - z + 0{,}2}{(z - 0{,}2)(z + 0{,}8)}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{z}{(z - 0{,}2)(z + 0{,}8)} \cdot \frac{(z - 2)(z + 0{,}8)}{(z + 0{,}5)} =$$

$$= \frac{z}{z + 0{,}5}$$

$$x[n] = \mathcal{Z}^{-1}\left\{X(z)\right\} = (-0{,}5)^n\,u[n]$$

SSY042

2019-08-28

4.

$$x(t) = \sum_{n=1}^{50} \sqrt{\frac{\pi}{n}} \cdot \sin\left(\frac{2\pi n}{T} \cdot t\right)$$

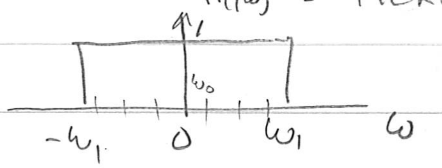
Grundvinkelfrekv. $\omega_0 = \frac{2\pi}{T}$

a)

$$h(t) = \frac{\sin(\omega_1 t)}{\pi t} \xleftrightarrow{FT} H(j\omega) = u(\omega+\omega_1) - u(\omega-\omega_1)$$

$H(j\omega)$ = Frekvenssvar

$$\omega_1 = \frac{20\pi}{3T}$$



b)

$$\frac{\omega_1}{\omega_0} = \frac{20\pi}{3T} \cdot \frac{T}{2\pi} = \frac{20}{6} \approx 3{,}3$$

Frekvenser som passerar filtret $\omega = n \cdot \omega_0$; $n = 1, 2, 3$

$$y(t) = \sum_{n=1}^{3} \sqrt{\frac{\pi}{n}} \cdot \sin\left(\frac{2\pi n}{T} \cdot t\right)$$

c) Medeleffekt $\overline{P}$ (Parsevals formel)

Om $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right\}$

$$\overline{P} = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

Vi har endast $b_1 = \sqrt{\pi}$; $b_2 = \sqrt{\frac{\pi}{2}}$; $b_3 = \sqrt{\frac{\pi}{3}}$
som är $\neq 0$.

$$\overline{P_y} = \frac{1}{2}(b_1^2 + b_2^2 + b_3^2) = \frac{\pi}{2}\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{\pi}{2} \cdot \frac{11}{6} = \frac{11\pi}{12}$$

SSY042
2019-08-28

5a/ $y[n] + 0{,}5\,y[n-1] = 1{,}5\,x[n]$

z- transf..

$$Y(z)\left(1+0{,}5z^{-1}\right) = 1{,}5\,X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1{,}5}{1+0{,}5z^{-1}}$$

Frekvenssvar: $z = e^{i\Omega}$

$$H(e^{i\Omega}) = \frac{1{,}5}{1+0{,}5e^{-i\Omega}}$$

$e^{i\cdot 0} = 1$

$e^{-i\pi} = -1$

Test vid $\Omega = 0$ och $\Omega = \pi$

Stämmer med

$$H(e^{i\Omega})\Big|_{\Omega=0} = \frac{1{,}5}{1+0{,}5} = 1$$ 2, 3

$$H(e^{i\Omega})\Big|_{\Omega=\pi} = \frac{1{,}5}{1-0{,}5} = 3$$ 2

Svar: Kurva nr. 2

SSY042
2018-08-28

5. a/ $H(s) = \dfrac{K s^2}{(s+100)^n}$; $H(j\omega) = \dfrac{-K\omega^2}{(j\omega+100)^n}$

Låga frekvenser $\omega \ll 100$ $|H(j\omega)|$ stiger med ω^2
vilket svarar mot $2 \cdot 20 = 40$ dB/dekad
(stämmer med figur)
Höga frekvenser $\omega \gg 100$ figur visar att $|H(j\omega)|$
sjunker med 60 dB/dekad $= 3 \cdot 20$ dB dekad
och $H(j\omega) \propto \dfrac{\omega^2}{\omega^n} = \dfrac{1}{\omega^{n-2}} = \dfrac{1}{\omega^3}$

$n-2 = 3 \Rightarrow n = 5$

Fas ändras från $180°$ till $-270°$ vilket är
$\Delta\phi = 180 - (-270) = n \cdot 90 = 450 \Rightarrow n = 5$

b/ $x(t) = \cos(t) \cdot \delta(t - \frac{\pi}{4}) = \cos(\frac{\pi}{4}) \cdot \delta(t - \frac{\pi}{4}) =$
$= 1/\sqrt{2}\ \delta(t - \frac{\pi}{4})$

$\delta(t) \xleftrightarrow{FT} 1$
$x(t) \xleftrightarrow{FT} X(j\omega)$
$x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$

Alltså: vårt $x(t) \xleftrightarrow{FT} X(j\omega) = \dfrac{1}{\sqrt{2}} \cdot e^{-j\omega \frac{\pi}{4}}$