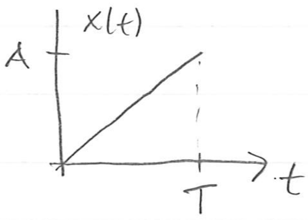
Signaler och System, Z2
SSY042 2019-06-11

1a)



Alt. (i) $$x(t) = \frac{A}{T}\cdot t\,[u(t) - u(t-T)] =$$

$$= \frac{A}{T} t\,u(t) - \frac{A}{T} t\cdot u(t-T) =$$

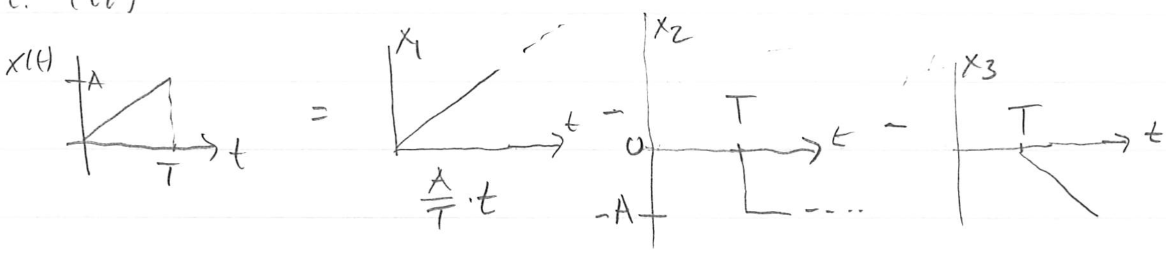
$$= \frac{A}{T} t\,u(t) - \frac{A}{T}(t - T + T)\,u(t-T) =$$

$$= \frac{A}{T}\left\{ t\,u(t) - (t-T)\,u(t-T) - T\,u(t-T)\right\}$$

Laplace transformera

$$X(s) = \frac{A}{T}\left(\frac{1}{s^2} - \frac{1}{s^2}e^{-sT} - T\cdot\frac{1}{s}\cdot e^{-sT}\right)$$

Alt. (ii)



$$x(t) = x_1(t) - x_2(t) - x_3(t)$$

$$x_1(t) = \frac{A}{T}\cdot t\cdot u(t) \Rightarrow X_1(s) = \frac{A}{T}\frac{1}{s^2}$$
$$x_2(t) = -A\,u(t-T) \Rightarrow X_2(s) = -A\frac{1}{s}e^{-sT}$$
$$x_3(t) = -\frac{A}{T}(t-T)\,u(t-T) \Rightarrow X_3(s) = -\frac{A}{T}\frac{1}{s^2}e^{-sT}$$

$$X(s) = X_1(s) + X_2(s) + X_3(s) =$$
$$= \frac{A}{T}\left(\frac{1}{s^2} - T\frac{1}{s}e^{-sT} - \frac{1}{s^2}e^{-sT}\right)$$

1b)

Signal: Periodtid $T = 320$ ms
frekvens $f = \frac{1}{T}$

Samplingsfrekvens: $f_s = 200$ Hz

$T_s = \frac{1}{f_s} = 5{,}0$ ms

Antal punkter $N = 2^{10} = 1024$

$$\frac{k}{N} = \frac{f}{f_s}$$

$$k = \frac{f}{f_s} \cdot N = \frac{T_s}{T} \cdot N = \frac{5 \cdot 10^{-3}}{0{,}320} \cdot 2^{10} =$$

$$= 16$$

Reell signal: $|X[k]|$ har "toppvärde"
även vid $N-k = 1008$

Svar: $k = 16$ och 1008

SS4042

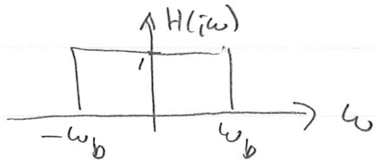
2019-06-11

2.
$$x(t)=\sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi n}} \sin\left(\frac{2\pi}{T}\cdot n \cdot t\right)$$

Grundvinkelfrekvens $\omega_0 = \frac{2\pi}{T}$

a/

Filter: $h(t) = \frac{\sin(\omega_b t)}{\pi t} \xleftrightarrow{\mathcal{F}T} H(j\omega) = u(\omega+\omega_b) - u(\omega-\omega_b)$



$$\omega_b = \frac{16\pi}{3T}$$

b/
$$\frac{\omega_b}{\omega_0} = \frac{16\pi}{3T}\cdot\frac{T}{2\pi} = \frac{8}{3} \approx 2{,}7 \qquad \omega_b \approx 2{,}7\omega_0$$

Endast frekvenserna $(n=1,2)$ ω_0 och $2\omega_0$ passerar filtret

$$y(t) = \sum_{n=1}^{2} \frac{1}{\sqrt{\pi n}} \sin\left(\frac{2\pi}{T} n \cdot t\right)$$

c/ Medeleffekt, Använd Parsevals formel

Allmänt: Om $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(\omega_0 n t) + b_n \sin(\omega_0 n t)\right)$ är

medeleffekten $\overline{P} = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$

Vi har endast $b_1 = \frac{1}{\sqrt{\pi}}$ och $b_2 = \frac{1}{\sqrt{2\pi}}$
som är $\neq 0$.

$$\overline{P}_y = \frac{1}{2}\left(\left(\frac{1}{\sqrt{\pi}}\right)^2 + \left(\frac{1}{\sqrt{2\pi}}\right)^2\right) = \frac{1}{2}\left[\frac{1}{\pi} + \frac{1}{2\pi}\right] =$$

$$= \frac{1}{\pi}\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4\pi}$$

SSY042
2019-06-11

3. $$\frac{d^2}{dt^2} u_c(t) + \frac{R}{L}\frac{d}{dt} u_c(t) + \frac{1}{LC} u_c(t) = \frac{1}{LC} u(t)$$

Laplacetransformera

a) $$s^2 U_c(s) + \frac{R}{L} s U_c(s) + \frac{1}{LC} U_c(s) = \frac{1}{LC} U(s)$$

$$H(s) = \frac{U_c(s)}{U(s)} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

b) Poler $$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\underbrace{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}_{=0}}$$

Två lika reella poler

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{L}\frac{4L^2}{R^2} = \frac{4L}{R^2}$$

$$L = 10\,mH,\ R = 100\,\Omega;\ C = \frac{4\cdot 0{,}01}{100^2} = 4\cdot 10^{-6}\ F$$

c) $$H(s) = \frac{\frac{1}{LC}}{\left(s+\frac{R}{2L}\right)^2} = \frac{25\cdot 10^6}{(s+5000)^2};$$

Stegsvar: $x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$$Y(s) = X(s)\cdot H(s) = \frac{25\cdot 10^6}{s(s+5000)^2} \quad \begin{cases} K = 25\cdot 10^6 \\ b = 5000 \end{cases}$$

$$Y(s) = \frac{K}{s(s+b)^2} = \frac{A}{s} + \frac{B}{s+b} + \frac{C}{(s+b)^2}$$

$$K = A(s+b)^2 + Bs(s+b) + sC = A(s^2 + s2b + b^2) + B(s^2 + sb) + sC$$

s^0: $K = Ab^2 \Rightarrow A = K/b^2 = 1$

s^1: $0 = 2bA + Bb + C \qquad C = -2b + b = -b = -5000$

s^2: $0 = A + B \Rightarrow B = -1$

$$Y(s) = \frac{1}{s} - \frac{1}{s+5000} - \frac{5000}{(s+5000)^2} \Rightarrow y(t) = 1 - e^{-5000t} - 5000te^{-5000t}$$

för $t \geq 0$

554042
2019-06-11

4. $y[n] - 0{,}25\,y[n-2] = x[n]$

z-transformera

$$Y(z) - 0{,}25\,z^{-2}\,Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0{,}25\,z^{-2}} = \frac{z^2}{z^2-0{,}25} =$$

$$= \frac{z^2}{(z-0{,}5)(z+0{,}5)}$$

a)

Insignal $x[n] = u[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z) = \dfrac{z}{z-1}$

$$Y(z) = X(z)\cdot H(z) = \frac{z^3}{(z-1)(z-0{,}5)(z+0{,}5)}$$

$$\frac{Y(z)}{z} = \frac{z^2}{(z-1)(z-0{,}5)(z+0{,}5)} = \frac{A}{z-1} + \frac{B}{z-0{,}5} + \frac{C}{z+0{,}5}$$

$$z^2 = A(z-0{,}5)(z+0{,}5) + B(z-1)(z+0{,}5) + C(z-1)(z-0{,}5)$$

$$z=1 \Rightarrow 1 = A(0{,}5\cdot 1{,}5) = A\,\frac{1}{2}\cdot\frac{3}{2} \Rightarrow A = \frac{4}{3}$$

$$z=0{,}5 \Rightarrow 0{,}25 = B\left(-\frac{1}{2}\cdot 1\right) \Rightarrow B = -\frac{1\cdot 2}{4} = -\frac{1}{2}$$

$$z=-0{,}5 \Rightarrow 0{,}25 = C(-1{,}5)(-1) = \frac{3}{2} \Rightarrow C = \frac{1}{4}\cdot\frac{2}{3} = \frac{1}{6}$$

Alltså: $Y(z) = \dfrac{4}{3}\cdot\dfrac{z}{z-1} - \dfrac{1}{2}\cdot\dfrac{z}{z-0{,}5} + \dfrac{1}{6}\,\dfrac{z}{z+0{,}5}$

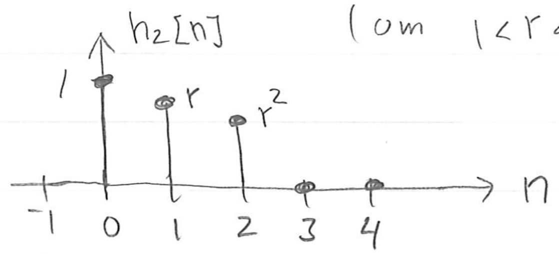
Stegsvar $y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \left[\dfrac{4}{3} - \dfrac{1}{2}(0{,}5)^n + \dfrac{1}{6}(-0{,}5)^n\right]u[n]$

b) Stabilt? Ja! Ty bägge polerna $z_1 = 0{,}5$ och $z_2 = -0{,}5$ ligger innanför enhetscirkeln

SSY042
2019-06-11

5. $x[n] \longrightarrow \boxed{h_1[n]} \xrightarrow{x_1[n]} \boxed{h_2[n]} \longrightarrow y[n]$

$h_1[n] = \delta[n-2]$ (Fördröjning med två "steg")
$h_2[n] = r^n(u[n]-u[n-3])$



(om $1<r<1$)

Insignal (enligt figur) $x[n] = \delta[n] + \delta[n-1] - \delta[n-2]$
Det ger $x_1[n] = \delta[n-2] + \delta[n-3] - \delta[n-4]$
och $y[n] = h_2[n-2] + h_2[n-3] - h_2[n-4]$

n	0	1	2	3	4	5	6	7
$h_2[n-2]$			1	r	r^2			
$h_2[n-3]$				1	r	r^2		
$-h_2[n-4]$					-1	$-r$	$-r^2$	
$y[n]$			1	$1+r$	r^2+r-1	r^2-r	$-r^2$	

Svar:

$y[2] = 1$
$y[3] = 1+r$
$y[4] = r^2+r-1$
$y[5] = r^2-r$
$y[6] = -r^2$

övriga $y[n] = 0$