1a/

$$x(t) \longrightarrow \bigotimes \longrightarrow y(t)$$
$$\uparrow \cos(\pi t)$$

(i) Tidsinvariant ?

$$x(t) \to \boxed{\text{System}} \xrightarrow{y(t)=\cos(\pi t)x(t)} \boxed{\text{Delay}} \to y(t-t_0) = \cos(\pi(t-t_0))\,x(t-t_0)$$

$$x(t) \to \boxed{\text{Delay}} \xrightarrow{x(t-t_0)} \boxed{\text{System}} \to y_d = \cos(\pi t)\cdot x(t-t_0)$$

$$y(t-t_0) \neq y_d(t)$$ Ej tidsinvariant

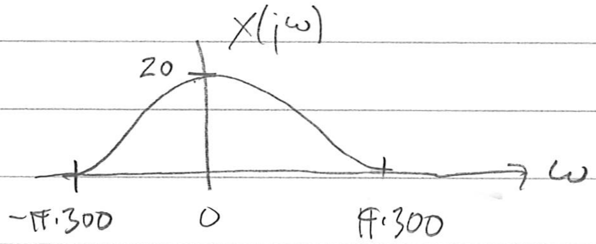
(ii) Linjärt ?

| Insignal | Utsignal |
|---|---|
| $x_1(t)$ | $y_1(t) = \cos(\pi t)\,x_1(t)$ |
| $x_2(t)$ | $y_2(t) = \cos(\pi t)\,x_2(t)$ |
| $x(t) = ax_1(t) + bx_2(t)$ | $y(t) = \cos(\pi t)\cdot x(t) =$ |

$$= \cos(\pi t)(ax_1(t) + bx_2(t)) =$$
$$= a\cos(\pi t)x_1(t) + b\cos(\pi t)x_2(t) =$$
$$= ay_1(t) + by_2(t)$$

Linjärt? Ja !

b/




Högsta signalfrekvens (Nyqvist) = $\omega_M = \pi\cdot 300$

Samplingsfrekvens : $\omega_s > 2\omega_M = 2\pi\cdot 300$ rad/s

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2. $$H(s) = \frac{K}{(s-s_1)(s-s_2)} \qquad s_1 = -500 \quad s_2 = -1000$$

a/

$$H(s)\Big|_{\substack{s=j\omega \\ \omega\to 0}} = \frac{K}{s_1 s_2} = \frac{K}{500\cdot 10^3} = 2 \Rightarrow K = 10^6$$

$$H(s) = \frac{10^6}{(s+500)(s+1000)} = \frac{10^6}{s^2 + s\,1500 + 500\cdot 10^3}$$

b/ Insignal $x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$

$$Y(s) = X(s)\cdot H(s) = \frac{10^6}{s(s+500)(s+1000)} = \{\text{P.B.U.}\} = \frac{A}{s} + \frac{B}{s+500} + \frac{C}{s+1000}$$

$$10^6 = A(s+500)(s+1000) + Bs(s+1000) + Cs(s+500)$$

$s = 0 \Rightarrow 10^6 = A\cdot 500\cdot 1000 \Rightarrow A = 2$

$s = -500 \Rightarrow 10^6 = B(-500)(-500+1000) \Rightarrow B = \frac{10^6}{-500\cdot 500} = -4$

$s = -1000 \Rightarrow 10^6 = C(-1000)(-1000+500) \Rightarrow C = \frac{10^6}{1000\cdot 500} = 2$

$$Y(s) = \frac{2}{s} - \frac{4}{s+500} + \frac{2}{s+1000}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left(2 - 4e^{-500t} + 2e^{-1000t}\right)u(t)$$

3.

$$y[n] + a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]$$

z - transformera

$$Y(z) + a_1 Y(z) \cdot z^{-1} = b_0 X(z) + b_1 X(z) z^{-1}$$

$$Y(z)(1 + a_1 z^{-1}) = X(z)(b_0 + b_1 z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} = \frac{b_0 z + b_1}{z + a_1} =$$

$$= b_0 \frac{z}{z + a_1} + b_1 \frac{z}{z + a_1} \cdot z^{-1}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = b_0 (-a_1)^n \cdot u[n] + b_1 (-a_1)^{n-1} u[n-1]$$

$$h[0] = b_0 = 2$$

$$h[1] = b_0(-a_1 + b_1)$$

$$h[2] = b_0 (-a_1)^2 + b_1(-a_1) = (-a_1)[-b_0 a_1 + b_1]$$

$$h[3] = b_0((-a_1)^3 + b_1(-a_1)^2) = (-a_1)^2[-b_0 a_1 + b_1] = -0{,}5$$

$$h[4] = b_0(-a_1)^4 + b_1(-a_1)^3) = (-a_1)^3[-b_0 a_1 + b_1] = 0{,}25$$

$$\frac{h[4]}{h[3]} = -a_1 = \frac{0{,}25}{-0{,}5} = -0{,}5 \Rightarrow \boxed{a_1 = 0{,}5}$$

$$h[3] = (-0{,}5)^2 [-2 \cdot 0{,}5 + b_1] = -0{,}5$$

$$-1 + b_1 = -0{,}5/((-0{,}5)^2 = -2$$

$$\Rightarrow \boxed{b_1 = -1}$$

Svar: $b_0 = 2$, $b_1 = -1$, $a_1 = 0{,}5$

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4.

$$G(s) = \frac{20s}{s^2+2s+101}$$

Studera frekvenssvaret $(s=j\omega)$

$$G(j\omega) = \frac{j20\omega}{101-\omega^2+j2\omega}$$

a) $\omega = 6$

$$|G(j\omega)|_{\omega=6} = \frac{20\cdot 6}{\sqrt{(101-6^2)^2+(2\cdot 6)^2}} =$$

$$= \frac{120}{\sqrt{65^2+12^2}} \approx 1{,}82 \text{ ggr}$$

$$20\cdot \log|G(j\cdot 6)| = 5{,}2 \text{ dB}$$ (Stämmer med figur)

b)

Im del > 0
Re del < 0
Addera 180°

$$\arg\{G(j\omega)\}\Big|_{\omega=16} =$$

$$= \arg\{j20\cdot 16\} - \operatorname{atan}\left\{\frac{2\cdot 16}{101-256}\right\} =$$

$$= 90^\circ - (-11{,}7+180) = -78{,}3^\circ$$

(Stämmer med figur)

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5.

$\omega_s = 2000\pi = 1000 \cdot 2\pi = f_s \cdot 2\pi$ rad/s

$f_s = 1000$ Hz

$N = 2^8 = 256$

$|X[k]|$ har sitt första "toppvärde" vid $k = 11$

Det motsvarar grundfrekvens $f_0$

a/

$\frac{f_0}{f_s} = \frac{k}{N}$ ; $f_0 = \frac{k}{N} f_s = \frac{11}{256} \cdot 1000 \approx 43$ Hz

Fundamental period $T_0 = \frac{1}{f_0} = 23$ ms

b/ $T_s = \frac{1}{f_s}$ ; sampelintervall

Antal perioder $m = \frac{T_s \cdot N}{T_0} = N \frac{T_s}{T_0} = N \frac{f_0}{f_s} = k$

Sampel per period: $\frac{N}{m} = \frac{N}{K} = 23,3$ "i medeltal"

c/ $\Delta f = \frac{f_s}{N} = \frac{1000}{256} = 3,9$ Hz