Signaler & System, Z2.
SSY042 2018-08-29

1a)
A: en period (upprepning) i intervallet
$|X[k]|$ för $k=1$ högt representerat
Sinusformad signal, övriga $X[k]$ små

B: en period (upprepning) i intervallet
$|X[k]|$ för $k=1$ högt representerat
Fyrkantssignal ger övertoner, fler betydande bidrag för $k>1$.

C: Fyra upprepningar i intervallet
$|X[4]|$ dominerar

D: Två perioder i intervallet
$|X[2]|$ dominerar

Svar: A-4, B-2, C-1, D-3

1c)
$$X(z) = \frac{z^3 + 2z^2 - 4z + 8}{z^3} =$$

$$= 1 + 2z^{-1} - 4z^{-2} + 8z^{-3} =$$

$$= \{\text{jämför}\} = \sum_{n=0}^{\infty} x[n] z^{-n}$$

$\Rightarrow$ $x[0]=1$, $x[1]=2$, $x[2]-4$, $x[3]=8$
övriga $x[n]=0$

$$x[n] = \delta[n] + 2\delta[n-1] - 4\delta[n-2] + 8\delta[n-3]$$

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1b) Använd $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$x(t) = 5\sin\left(\omega_0 t + \frac{\pi}{6}\right) + \pi\sin(2\omega_0 t) + 2\cos\left(3\omega_0 t - \frac{\pi}{3}\right) =$$

$$= 5\left[\sin(\omega_0 t)\cos\left(\frac{\pi}{6}\right) + \cos(\omega_0 t)\sin\left(\frac{\pi}{6}\right)\right] + \pi\sin(2\omega_0 t) +$$

$$+ 2\left[\cos(3\omega_0 t)\cos\left(\frac{\pi}{3}\right) + \sin(3\omega_0 t)\cdot\sin\left(\frac{\pi}{3}\right)\right] =$$

$$= \sum_{n=0}^{\infty} a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)$$

Identifiering ger

$$a_1 = 5\sin\left(\frac{\pi}{6}\right) = 5\cdot 0{,}5 = 2{,}5$$

$$b_1 = 5\cos\left(\frac{\pi}{6}\right) = 5\frac{\sqrt{3}}{2} \approx 4{,}33$$

$$b_2 = \pi$$

$$a_3 = 2\cos\left(\frac{\pi}{3}\right) = 2\cdot 0{,}5 = 1$$

$$b_3 = 2\sin\left(\frac{\pi}{3}\right) = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3} \approx 1{,}73$$

övriga a_n och $b_n = 0$

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2.

Stegsvar $y_s(t) = (1-e^{-2t})u(t)$

$$Y_s(s) = \mathcal{L}\{y_s(t)\} = \frac{1}{s} - \frac{1}{s+2} = \frac{s+2-s}{s(s+2)} = \frac{1}{s} \cdot \underbrace{\frac{2}{s+2}}_{H(s)}$$

$$Y_s(s) = \frac{1}{s} \cdot H(s) \Rightarrow H(s) = \frac{2}{s+2}$$

$x(t) = e^{-t}\sin(3t)u(t)$ Laplacetransf.

$$X(s) = \frac{3}{(s+1)^2+3^2} = \frac{3}{s^2+2s+10}$$

$$Y(s) = H(s) \cdot X(s) = \frac{2}{s+2} \cdot \frac{3}{s^2+2s+10} = \{P.B.U\} =$$

$$= \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+10}$$

$$6 = A(s^2+2s+10) + (Bs+C)(s+2)$$

s^2: $0 = A + B \Rightarrow A = -B$

s^1: $0 = 2A + 2B + C \Rightarrow C = 0$

s^0: $6 = 10A + 2C \Rightarrow A = \frac{6}{10} = 0{,}6 = -B$

$$Y(s) = \frac{0{,}6}{s+2} - 0{,}6\frac{s}{s^2+2s+10} = \frac{0{,}6}{s+2} - 0{,}6 \cdot \frac{s+1-1}{(s+1)^2+3^2} =$$

$$= \frac{0{,}6}{s+2} - 0{,}6\frac{s+1}{(s+1)^2+3^2} + \frac{0{,}6}{3} \cdot \frac{3}{(s+1)^2+3^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[0{,}6e^{-2t} - e^{-t}\left(0{,}6\cos(3t) - 0{,}2\sin(3t)\right)\right]u(t)$$

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3.

$$H_1(z) = \frac{6z}{z^2 - 0{,}4z - 0{,}05}$$

$$h_2[n] = \left[5(0{,}5)^{n-1} + (-0{,}1)^{n-1}\right] u[n-1]$$

$$H_2(z) = \mathcal{Z}\{h_2[n]\} = 5 \cdot \frac{z}{z-0{,}5} \cdot z^{-1} + \frac{z}{z+0{,}1} \cdot z^{-1} =$$

$$= \frac{5}{z-0{,}5} + \frac{1}{z+0{,}1} = \frac{5(z+0{,}1) + (z-0{,}5)}{(z-0{,}5)(z+0{,}1)} =$$

$$= \frac{6z}{z^2 - 0{,}4z - 0{,}05} \quad \text{Notera} = H_1(z)$$

Alternativt:

$$H_1(z) = \frac{6z}{z^2 - 0{,}4z - 0{,}05} = \frac{6z}{(z-0{,}5)(z+0{,}1)} =$$

$$= \frac{A}{z-0{,}5} + \frac{B}{z+0{,}1} \quad ; \quad 6z = A(z+0{,}1) + B(z-0{,}5)$$

z^1; $6 = A + B$

z^0: $0 = 0{,}1A - 0{,}5B \Rightarrow A = 5B \Rightarrow B = 1$ och $A = 5$

$$H_1(z) = \frac{5}{z-0{,}5} + \frac{1}{z+0{,}1} = 5\frac{z}{z-0{,}5} \cdot z^{-1} + \frac{z}{z+0{,}1} \cdot z^{-1}$$

$$h_1[n] = \mathcal{Z}^{-1}\{H_1(z)\} = \left[5(0{,}5)^{n-1} + (-0{,}1)^{n-1}\right] u[n-1]$$

$$\left.\begin{array}{l} H_1(z) = H_2(z) \\ h_1[n] = h_2[n] \end{array}\right\} \quad h[n] = h_1[n] - h_2[n] = 0 \ , \ \forall n$$

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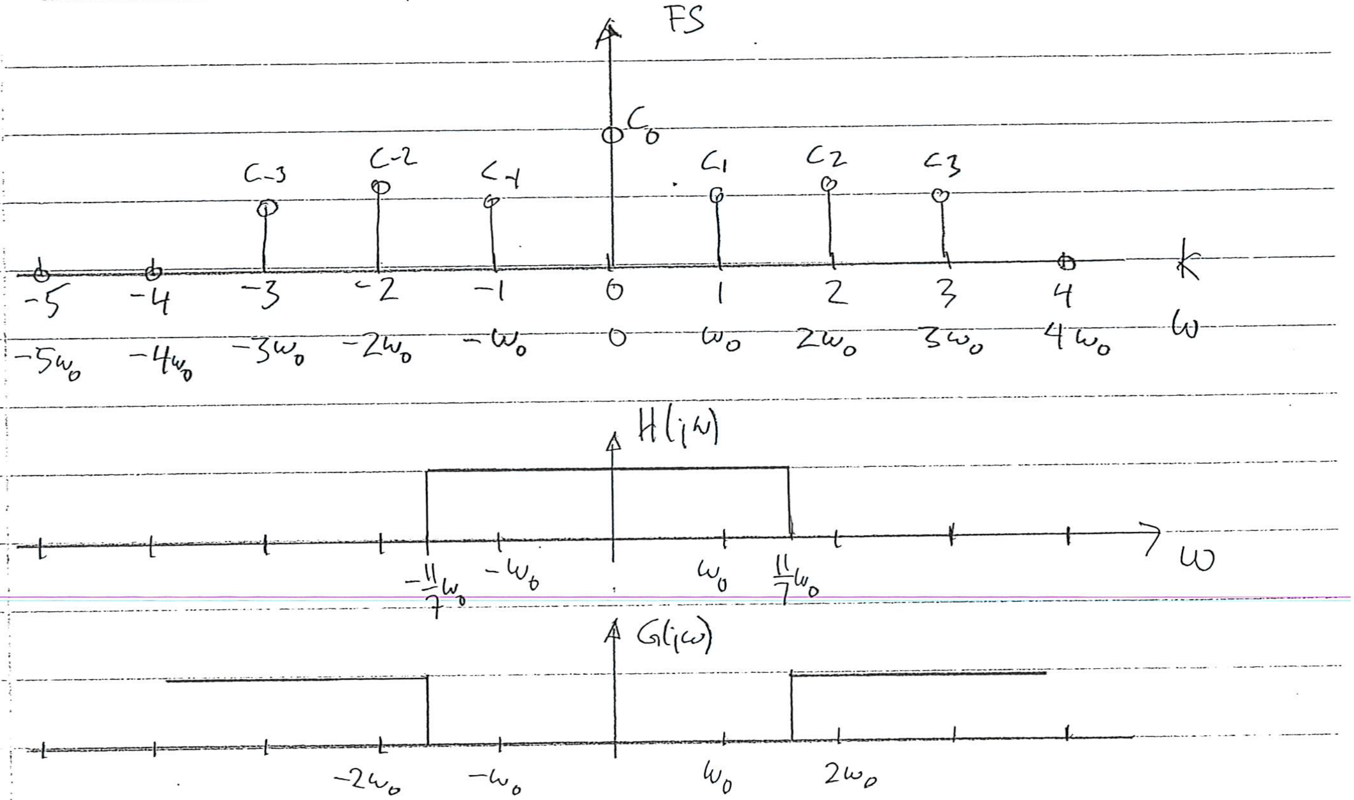
4.



$$x(t) \longrightarrow \boxed{G(j\omega)} \longrightarrow y(t)$$

$G(j\omega)$ släpper endast igenom vinkelfrekv. $|\omega| > \frac{11}{7}\omega_0$

FS-koeff till $y(t)$ blir då $\begin{cases} c_2, c_{-2}, c_3 \text{ och } c_{-3} \\ \text{övriga } c_k = 0 \end{cases}$

Medeleffekt $\quad P = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$P_y = 2|c_2|^2 + 2|c_3|^2 = 2 \cdot 0{,}5^2 + 2 \cdot 0{,}2^2 = 0{,}58$$

$$P_x = P_y + 2|c_1|^2 + |c_0|^2 = P_y + 2 \cdot 1^2 + 2^2 =$$
$$= P_y + 6 = 6{,}58$$

$$\frac{P_y}{P_x} = \frac{P_y}{P_y + 6} = \frac{1}{1 + \frac{6}{0{,}58}} = 0{,}088$$

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5. $x(t) = \sin(10\pi t) + \sin(11\pi t)$

$\omega_1 = 10\pi$ rad/s ,

$\omega_2 = 11\pi$ rad/s

$f_s = 100$ Hz $\quad T = \frac{1}{f_s} \qquad \omega_s = 2\pi f_s$

Upplösning i DFT är $\Delta\omega = \frac{\omega_s}{N}$

$\omega_1 \xleftrightarrow{DFT} |X[k_1]|$ ger "topp"

$\omega_2 \xleftrightarrow{DFT} |X[k_2]|$ ger "topp"

$|k_1 - k_2| \geq 10$

$$\frac{\omega_2 - \omega_1}{\Delta\omega} = \frac{(11-10)\pi}{\omega_s/N} = \frac{\pi}{\omega_s} \cdot N =$$

$$= \frac{\pi}{2\pi f_s} \cdot N = \frac{N}{200} \geq 10$$

$N \geq 2000 \Rightarrow n = 11$ ty $2^{11} = 2048$

Svar: $n = 11$