Signaler och System, Z2
SSY042 2018-03-15

1. a) Allmänt

$$H(s) = b_0 \frac{(s-c_1)(s-c_2)\dots(s-c_M)}{(s-d_1)(s-d_2)\dots(s-d_N)}$$

$$|H(j\omega)| = \left|H(s)|_{s=j\omega}\right| = |b_0| \frac{|j\omega-c_1||j\omega-c_2|\cdots|j\omega-c_M|}{|j\omega-d_1||j\omega-d_2|\cdots|j\omega-d_N|}$$

c_i : nollställe "o" , d_i : pol "x"

Utvärdera $H(s)$ på Im-axeln ($s=j\omega$)

$\boxed{1}$	Nollställe i $\omega=0$ och $\omega=8$	D
$\boxed{2}$	Nollställe i $\omega=4$	C
$\boxed{3}$	Inga nollställen	A
$\boxed{4}$	Nollställe i $\omega=0$	B

Matchning mot grafer ger $\longrightarrow$

b) $$x(t) = \sum_{k=0}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

Ifrån vår lab vet vi att om max/min värde på fyrkantsvågen är +1/-1 blir
$a_k = 0$, $\forall k$, $b_k = \frac{4}{\pi k}$, $k=1,3,5,7,\dots$

Total medeleffekt , Parsevals formel (Beta)

$$\bar{P} = \frac{1}{T}\int_T x^2(t)\,dt = \frac{1}{T}\int_T 1\cdot dt = \frac{1}{T}\left[t\right]_0^T = 1$$

men $\bar{P} = \frac{1}{2}\sum_{k=1}^{\infty} b_k^2$; Grundtonen ($k=1$) $P_1 = \frac{1}{2}\left(\frac{4}{\pi}\right)^2 = 0{,}81$

$$\frac{P_1}{\bar{P}} = 0{,}81 \;\hat{=}\; 81\%$$

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2. $\frac{dy(t)}{dt} + 9y(t) = x(t)$

Laplace transf. $sY(s) + 9Y(s) = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+9}$$

$$x(t) = 41 \sin(t) u(t) \xrightarrow{\mathcal{L}} X(s) = \frac{41}{s^2+1}$$ (komplexa poler)

$$Y(s) = H(s) \cdot X(s) = \frac{41}{(s+9)(s^2+1)} = \frac{A}{s+9} + \frac{Bs+C}{s^2+1}$$

$$41 = A(s^2+1) + (s+9)(Bs+C)$$

s^2:	$0 = A+B$	$A = -B$
s^1:	$0 = 9B+C$	$C = -9B = 9A$
s^0:	$41 = A+9C$	$41 = A + 9(9A) = 82A$

$$\Rightarrow A = \frac{1}{2} \; ; \; B = -\frac{1}{2} \; ; \; C = \frac{9}{2}$$

$$Y(s) = \frac{1}{2} \cdot \frac{1}{s+9} - \frac{1}{2} \frac{s}{s^2+1} + \frac{9}{2} \frac{1}{s^2+1}$$

Inv. Laplacetransf. ger

$$y(t) = \left(\frac{1}{2} e^{-9t} - \frac{1}{2} \cos(t) + \frac{9}{2} \sin(t) \right) u(t)$$

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3. $x(t) = 6{,}0 \sin(3t) + 2{,}0 \cos(6t - \frac{\pi}{3}) = \{\text{Euler}\} =$

$$= \frac{6}{2j}\left(e^{j3t} - e^{-j3t}\right) + \frac{2}{2}\left(e^{j(6t-\frac{\pi}{3})} + e^{-j(6t-\frac{\pi}{3})}\right) =$$

$$= \frac{3}{j} e^{j3t} - \frac{3}{j} e^{-j3t} + e^{-j\frac{\pi}{3}} \cdot e^{j6t} + e^{j\frac{\pi}{3}} e^{-j6t} =$$

$$= \sum_{k=-\infty}^{\infty} c_{x,k} e^{jk\omega_0 t} \qquad \omega_0 = 3$$

Fouriertransformera

$$X(j\omega) = \frac{2\pi \cdot 3}{j} \delta(\omega - 3) - \frac{2\pi \cdot 3}{j} \delta(\omega + 3) + e^{-j\frac{\pi}{3}} \cdot 2\pi \delta(\omega - 6) + e^{j\frac{\pi}{3}} \cdot 2\pi \delta(\omega + 6)$$

$$h(t) = e^{-9t} u(t) \xleftrightarrow{FT} H(j\omega) = \frac{1}{9 + j\omega}$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) =$$

$$= 2\pi \boxed{\frac{3}{j} \cdot \frac{1}{9+j3}} \delta(\omega - 3) - 2\pi \cdot \boxed{\frac{3}{j} \frac{1}{9-j3}} \cdot \delta(\omega + 3) +$$

$$+ 2\pi \boxed{e^{-j\frac{\pi}{3}} \cdot \frac{1}{9+j6}} \cdot \delta(\omega - 6) + 2\pi \boxed{e^{j\frac{\pi}{3}} \cdot \frac{1}{9-j6}} \delta(\omega + 6)$$

Identifiera

k	$c_{y,k}$
1	$\frac{3}{j} \cdot \frac{1}{9+j3} = -j/(3+j) = \frac{1}{-1+3j}$
-1	$-\frac{3}{j} \cdot \frac{1}{9-j3} = j/(3-j) = \frac{-1}{1+3j}$
2	$\frac{e^{-j\pi/3}}{9+j6}$
-2	$\frac{e^{j\pi/3}}{9-j6}$

övriga $c_{y,k} = 0$

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$$x[n] \xrightarrow{} \boxed{\begin{matrix} h[n] \\ H(z) \end{matrix}} \xrightarrow{} y[n]$$

$X(z)$ $\qquad$ $Y(z)$

$$Y(z) = H(z) \cdot X(z)$$

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] \longleftrightarrow X(z) = \frac{z}{z - \frac{1}{2}}$$

$$y[n] = \delta[n] + 0{,}3\left(\tfrac{1}{2}\right)^{n-1} u[n-1] - \frac{2}{15}\left(-\tfrac{1}{3}\right)^{n-1} u[n-1]$$

z-transformera

$$Y(z) = 1 + \frac{0{,}3}{\left(z - \frac{1}{2}\right)} - \frac{2}{15} \cdot \frac{1}{\left(z + \frac{1}{3}\right)} =$$

$$= \frac{15\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right) + 0{,}3 \cdot 15\left(z + \frac{1}{3}\right) - 2\left(z - \frac{1}{2}\right)}{15\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)} =$$

$$= \frac{15\left(z^2 + z\left(\frac{1}{3} - \frac{1}{2}\right) - \frac{1}{6}\right) + 4{,}5z + 1{,}5 - 2z + 1}{15\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)} =$$

$$= \frac{15\left(z^2 + z\left[\left(-\frac{1}{6}\right) \cdot 15 + 4{,}5 - 2\right] - \frac{15}{6} + 1{,}5 + 1\right.}{15\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)} =$$

$$= \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 \quad \left(z - \frac{1}{2}\right)}{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right) \cdot z} = \frac{z}{z + \frac{1}{3}}$$

Invers z-transf.

$$h[n] = \left(-\tfrac{1}{3}\right)^n \cdot u[n]$$

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5. Sampelintervall $T = 6.0$ ms

Samplingsvinkelfrekvens $\omega_s = \frac{2\pi}{T}$ rad/s

Samplingsfrekvens $f_s = \frac{1}{T}$ Hz [s^{-1}]

$N = 2^{10} = 1024$

$X[k]$ har värden för $k = 0, 1, 2, \ldots, N-1$

där $X[N]$ motsvarar värdet vid samplingsfrekvensen

Fig 5 Grundtonen svarar mot grundvinkelfrekvensen som ges av pulsen

$k = 10$ ger max $|X[k]|$

Frekvens i Hz vid $k = 10$

$$\frac{k}{N} = \frac{f}{f_s} \Rightarrow f = \frac{k}{N} \cdot f_s = \frac{k}{T \cdot N} =$$

$$= \frac{10}{6 \cdot 10^{-3} \cdot 1024} = 1{,}63 \text{ Hz } [s^{-1}]$$

Puls: $f \cdot 60 \approx 98$ slag/min