Signaler och System, Z2
SSY042 2017-03-16

1a)
$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Insignal	Utsignal
$x_1(t)$	$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$
$x_2(t)$	$y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$
$x(t) = a x_1(t) + b x_2(t)$	$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = \int_{-\infty}^{t} (a x_1(\tau) + b x_2(\tau))\, d\tau =$

$$= a \int_{-\infty}^{t} x_1(\tau)\, d\tau + b \int_{-\infty}^{t} x_2(\tau)\, d\tau =$$

$$= a y_1(t) + b y_2(t)$$

Ja! Systemet är linjärt!

b)

i)
$$x(t) = 4 + 3\sin(20t) + 2\cos\left(30t + \frac{\pi}{4}\right) = \{ \omega_0 = 10 \} =$$

$$= 4 + 3 \cdot \frac{1}{2j}\left(e^{j2\omega_0 t} - e^{-j2\omega_0 t}\right) + 2 \cdot \frac{1}{2}\left(e^{j(3\omega_0 t + \frac{\pi}{4})} + e^{-j(3\omega_0 t + \frac{\pi}{4})}\right) =$$

$$= 4 + \frac{3}{2j}\left(e^{j2\omega_0 t} - e^{-j2\omega_0}\right) + e^{j\frac{\pi}{4}} \cdot e^{j3\omega_0 t} + e^{-j\frac{\pi}{4}} \cdot e^{-j3\omega_0 t} =$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \Rightarrow c_0 = 4,\ c_2 = \frac{3}{2j},\ c_{-2} = -\frac{3}{2j} = c_2^*$$

$$c_3 = e^{j\pi/4} \quad c_{-3} = e^{-j\pi/4} = c_3^*$$

ii)
$$G(s)\Big|_{s=j\omega} = \frac{j\omega}{j\omega + 15} = \frac{1}{1 + \frac{15}{j\omega}} = \{ \omega = 20 \} = \frac{1}{1 - j\frac{15}{20}}$$

$$|G(j \cdot 20)| = \frac{1}{\sqrt{1 + \left(\frac{15}{20}\right)^2}} = 0{,}8 \quad \text{Ny amplitud: } 3 \cdot 0{,}8 = \underline{\underline{2{,}4}}$$

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2 a)

$$T(s) = (s^2+100)(s^2+400)$$

Pöler $s^2+100=0 \Rightarrow s^2=-100 \quad s=\sqrt{-100} = \pm j10$

$s^2+400=0 \Rightarrow s^2=-400 \quad s=\sqrt{-400} = \pm j20$

$$T(s) = (s+j10)(s-j10)(s+j20)(s-j20)$$

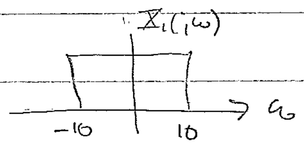
$$T(s)\big|_{s=j\omega} = (j\omega+j10)(j\omega-j10)(j\omega+j20)(j\omega-j20)$$

Svar: Vinkelfrekvenser $\omega=10$ och $\omega=20$ släcks ut

b) Ta fram Fouriertransformen för $x_a(t)$

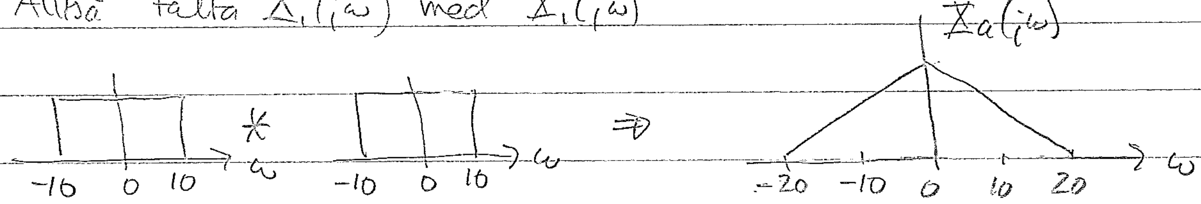
$$x_a(t) = \left[\frac{\sin(10t)}{\pi t}\right]^2 = x_1^2(t)$$

$$x_1(t) = \frac{\sin(10t)}{\pi t} \xleftrightarrow{FT} X_1(j\omega) = u(\omega+10)-u(\omega-10)$$



$$x_1(t)\cdot x_1(t) \xleftrightarrow{FT} \frac{1}{2\pi} X_1(j\omega) * X_1(j\omega)$$

Alltså Falta $X_1(j\omega)$ med $X_1(j\omega)$



$X_a(j\omega) \neq 0$ för $|\omega| < 20$ rad/s

Sampelintervall $T = \frac{2\pi}{30} \Rightarrow \omega_s = \frac{2\pi}{T} = 30$ rad/s

Samplingsvillkoret ej uppfyllt: Vi får vikning!
$(\omega_s < 2\cdot 20)$

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3. $y[n] = x[n-1]$ z-transformera

$$Y(z) = z^{-1} X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = z^{-1}$$

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) =$$

$$= \frac{5}{1-\frac{1}{3}z^{-1}} \cdot \frac{2}{1-\frac{1}{2}z^{-1}} \cdot \left(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}\right) = z^{-1}$$

$$10\left(b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}\right) = z^{-1}\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right) =$$

$$= z^{-1}\left(1 - \left(\frac{1}{2}+\frac{1}{3}\right)z^{-1} + \frac{1}{6}z^{-2}\right) =$$

$$= z^{-1}\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) =$$

$$= z^{-1} - \frac{5}{6}z^{-2} + \frac{1}{6}z^{-3}$$

Jämför VL och HL

$$\left.\begin{aligned} 10\, b_0 &= 0 \\ 10\, b_1 &= 1 \\ 10\, b_2 &= -\frac{5}{6} \\ 10\, b_3 &= \frac{1}{6} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} b_0 &= 0 \\ b_1 &= \frac{1}{10} \\ b_2 &= -\frac{1}{12} \\ b_3 &= \frac{1}{60} \end{aligned}$$

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4. $$m\frac{d^2x(t)}{dt^2}+d\frac{dx(t)}{dt}+kx(t)=F(t)$$

Laplacetransformera

$x(t)$: utsignal
$F(t)$: insignal

$$ms^2X(s)+dsX(s)+kX(s)=F(s)$$

$$H(s)=\frac{X(s)}{F(s)}=\frac{1}{ms^2+ds+k}=\left\{\begin{matrix} m=1.0\text{ kg} \\ d=2.0\text{ Ns/m} \\ k=10\text{ N/m}\end{matrix}\right\}=$$

$$=\frac{1}{s^2+2s+10}=\{\text{komplexa rötter, kvadratkompl.}\}=$$

$$=\frac{1}{(s+1)^2-1+10}=\frac{1}{(s+1)^2+3^2}$$

a) $F(t)=10\delta(t)\Rightarrow F(s)=10$

$$X(s)=H(s)\cdot F(s)=\frac{10}{(s+1)^2+3^2}=\frac{10}{3}\cdot\frac{3}{(s+1)^2+3^2}$$

$$\Rightarrow x(t)=\mathcal{L}^{-1}\{X(s)\}=\frac{10}{3}e^{-t}\cdot\sin(3t)\,u(t)\quad\text{m}$$

b) $F(t)=10\,u(t)\Rightarrow F(s)=\frac{10}{s}$

$$X(s)=\frac{10}{s}\cdot\frac{1}{s^2+2s+10}=\frac{A}{s}+\frac{Bs+C}{s^2+2s+10}$$

$$\Rightarrow\quad 10=A(s^2+2s+10)+(Bs+C)\cdot s$$

s^0: $10=10A\Rightarrow A=1$
s^1: $0=2A+C\Rightarrow C=-2$
s^2: $0=A+B\Rightarrow B=-1$

/forts

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I forts 4

$$X(s) = \frac{1}{s} + \frac{-s-2}{s^2+2s+10} = \frac{1}{S} - \frac{(s+2)}{s^2+2s+10} =$$

$$= \frac{1}{S} - \frac{s+1 \quad +1}{(s+1)^2+3^2} = \frac{1}{S} - \frac{s+1}{(s+1)^2+3^2} - \frac{1}{3} \cdot \frac{3}{(s+1)^2+3^2}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \left[1 - e^{-t}\left(\cos 3t + \frac{1}{3}\sin 3t\right)\right] u(t) \ \text{m}$$

c) I uppg a) $x(t) \to 0$ då $t \to \infty$

Rimligt ty ingen kraft påverkar vagnen då $t > 0$. Vagnen återgår till "viloläge"

I uppg b) $x(t) \to 1 \text{ m}$ då $t \to \infty$

Rimligt ty en konstant kraft på 10 N ligger kvar och motverkas av fjäderkonstanten på 10 N/m. Slutposition blir då $x = 1$ m.

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5. DFT$\{x[n]\} = X[k]$

$x[n]$, $n = 0, 1, 2, \ldots, N-1$

$X[k]$, $k = 0, 1, 2, \ldots, N-1$

$T = \frac{1}{f_s}$; Sampel-intervall

Frekvensupplösning hos DFT: $\Delta f = \frac{f_s}{N} = \frac{1}{NT}$ Hz

$f = k \cdot \Delta f \Rightarrow \frac{f}{f_s} = \frac{k}{N}$

$f = 50\ \text{Hz} \Rightarrow k = \frac{f}{f_s} \cdot N = f \cdot T \cdot N$

k värde som svarar mot f = 50 Hz

(1) $T = 1{,}5\ \text{ms}$, $N = 96 \Rightarrow k = 50 \cdot 1{,}5 \cdot 10^{-3} \cdot 96 \approx 7.2$

(2) $T = 0{,}40\ \text{ms}$, $N = 256 \Rightarrow k = 50 \cdot 0{,}40 \cdot 10^{-3} \cdot 256 \approx 5{,}12$

(3) $T = 1{,}6\ \text{ms}$, $N = 128 \Rightarrow k = 50 \cdot 1{,}6 \cdot 10^{-3} \cdot 128 \approx 10{,}24$

(4) $T = 4{,}0\ \text{ms}$, $N = 64 \Rightarrow k = 50 \cdot 4 \cdot 10^{-3} \cdot 64 \approx 12{,}8$

(5) $T = 3{,}6\ \text{ms}$, $N = 84 \Rightarrow k = 50 \cdot 3{,}6 \cdot 10^{-3} \cdot 84 \approx 15{,}1$

Jämför med max $|X[k]|$ i figur 3

DFT	Undersökning
A	4
B	3
C	1