Signaler och System, Z2

SSY042 160317

1a/

$$x(t) = 5\cos(16t) + 4\left|\sin\left(\frac{96}{10}t\right)\right|$$

$$\omega_1 = 16 \Rightarrow T_1 = \frac{2\pi}{16}$$

$$\omega_2 = \frac{96}{10} \Rightarrow T_2' = \frac{2\pi}{\omega_2} = \frac{2\pi \cdot 10}{96}$$

Belopp ger $T_2 = \frac{1}{2}T_2' = \frac{\pi 10}{96}$

Gemensam periodtid

$$T = k_1 T_1 = k_2 T_2 \qquad \frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{\pi \cdot 10}{96} \cdot \frac{16}{2\pi} = \frac{5 \cdot 2 \cdot 16}{6 \cdot 16 \cdot 2} = \frac{5}{6}$$

$$T = k_1 T_1 = 5 \cdot \frac{2\pi}{16} = \frac{5\pi}{8}$$

$$T = k_2 T_2 = 6 \cdot \frac{10\pi}{96} = \frac{6 \cdot 10\pi}{6 \cdot 16} = \frac{5\pi}{8} \text{ s}$$

Svar: Periodisk? Ja! $T = \frac{5\pi}{8}$ s

b/ Periodtid $T \Rightarrow$ fundamental frekvens

$f_1 = \frac{1}{T}$ (ger max i $|X[k]|$)

Samplingsfrekvens: $f_s = 200$ Hz

$N = 2^{10} = 1024$

$$\frac{k}{N} = \frac{f_1}{f_s} \; ; \; k = \frac{f_1}{f_s} \cdot N = \frac{N}{T f_s} = \frac{2^{10}}{640 \cdot 10^{-3} \cdot 200} = 8$$

Reell signal ger samma värde vid $N-k = 1016$

Svar: $k = 8$ och 1016

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2.

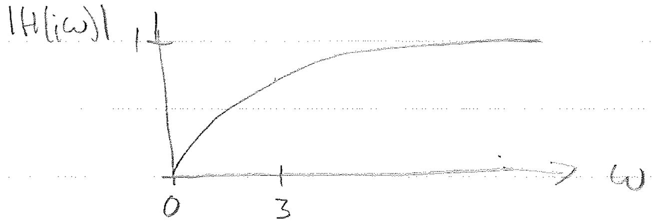
$$h(t) = \delta(t) - 3e^{-3t}u(t)$$

Laplacetransf.

$$H(s) = 1 - \frac{3}{s+3} = \frac{s+3-3}{s+3} = \frac{s}{s+3}$$

Frekv. svar $s = j\omega$ (Fouriertransf.)

a) $H(j\omega) = \dfrac{j\omega}{3+j\omega}$; $|H(j\omega)| = \dfrac{\omega}{\sqrt{3^2+\omega^2}}$



b) Högpass

c) I stationärtillstånd $\omega = \frac{3}{\sqrt{8}}$ rad/s

Amplitudförändring $|H(j\omega)| = \dfrac{3/\sqrt{8}}{\sqrt{9+\frac{9}{8}}} =$

$$= \frac{3}{\sqrt{8}} \cdot \frac{1}{\sqrt{\frac{81}{8}}} = \frac{1}{3}$$

Fasbidrag: $\arg\{H(j\omega)\} = 90 - \arctan\frac{\omega}{3} = \left\{\omega = \frac{3}{\sqrt{8}}\right\} = 70{,}5^\circ \hat{=}$

$= \frac{\pi}{2} - 0{,}34 = 0{,}39\pi$ rad

$$y(t) = 6\,|H(j\omega)| \cos\left(\omega t - \frac{\pi}{2} + \arg\{H(j\omega)\}\right) = \left\{\omega = \frac{3}{\sqrt{8}}\right\} =$$

$$= 2\cos\left(\frac{3}{\sqrt{8}}t - 0{,}11\pi\right)$$

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3. $y[n] = \left(2\left(\frac{1}{2}\right)^n + 3\left(-\frac{3}{4}\right)^n\right) u[n]$

z-transformera

$$Y(z) = \frac{2z}{z-\frac{1}{2}} + \frac{3z}{z+\frac{3}{4}} \qquad \Big| \quad x[n] = 5u[n], \quad X(z) = \frac{5z}{z-1}$$

$$Y(z) = z \cdot \frac{2\left(z+\frac{3}{4}\right) + 3\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)} = z \cdot \frac{2z + \frac{3}{2} + 3z - \frac{3}{2}}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)} =$$

$$= z \cdot \frac{5z}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)}$$

$$H(z) = \frac{Y(z)}{X(z)} = z \cdot \frac{5z}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)} \cdot \frac{z-1}{5z} = z \cdot \underbrace{\frac{z-1}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)}}_{\text{P.B.U}}$$

$$\frac{z-1}{\left(z-\frac{1}{2}\right)\left(z+\frac{3}{4}\right)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z+\frac{3}{4}}$$

$$z-1 = A\left(z+\frac{3}{4}\right) + B\left(z-\frac{1}{2}\right)$$

$$z = +\frac{1}{2}: \quad -\frac{1}{2} = A\left(\frac{2}{4}+\frac{3}{4}\right) \Rightarrow -\frac{2}{4} = A\left(\frac{5}{4}\right) \Rightarrow A = -\frac{2}{5} = -0{,}4$$

$$z = -\frac{3}{4}: \quad -\frac{3}{4} - 1 = B\left(-\frac{3}{4}-\frac{1}{2}\right) \Rightarrow -\frac{7}{4} = B\left(-\frac{5}{4}\right) \Rightarrow B = \frac{7}{5} = 1{,}4$$

$$H(z) = -\frac{2}{5} \cdot \frac{z}{z-\frac{1}{2}} + \frac{7}{5} \cdot \frac{z}{z+\frac{3}{4}}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \frac{1}{5}\left(-2\left(\frac{1}{2}\right)^n + 7\left(-\frac{3}{4}\right)^n\right) u[n] \quad \text{Impulssvar!}$$

$$H(z) = \frac{z^2 - z}{z^2 + z\left(\frac{3}{4}-\frac{1}{2}\right) - \frac{3}{8}} = \frac{z^2 - z}{z^2 + z \cdot \frac{1}{4} - \frac{3}{8}} = \frac{1 - z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z)\left(1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}\right) = X(z)\left(1 - z^{-1}\right) \Rightarrow y[n] + \frac{1}{4}y[n-1] - \frac{3}{8}y[n-2] =$$

$$= x[n] - x[n-1]$$

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4. Poler: -8 och -6

Nollst: -5 och -12

$$H(s) = H_0 \cdot \frac{(s-s_1)(s-s_2)}{(s-s_3)(s-s_4)} = H_0 \frac{(s+5)(s+12)}{(s+6)(s+8)}$$

Frekv. svar $(s = j\omega)$

$$H(j\omega) = H_0 \frac{(j\omega+5)(j\omega+12)}{(j\omega+6)(j\omega+8)} = \{\omega \to 0\} = H_0 \frac{5 \cdot 12}{6 \cdot 8} = 10$$

$$\Rightarrow H_0 = 8$$

Stegsvar: Insignal $X(s) = \frac{1}{s} = \mathcal{L}\{u(t)\}$

$$Y(s) = X(s)\,H(s) = 8 \frac{(s+5)(s+12)}{s(s+6)(s+8)}$$ Partialbråksuppdela

$$\frac{8(s+5)(s+12)}{s(s+6)(s+8)} = \frac{A}{s} + \frac{B}{s+6} + \frac{C}{s+8} = Y(s)$$

$$8(s+5)(s+12) = A(s+6)(s+8) + Bs(s+8) + Cs(s+6)$$

$s=0$: $8 \cdot 5 \cdot 12 = A \cdot 6 \cdot 8 \Rightarrow A = 10$

$s=-6$: $8(-1) \cdot 6 = B(-6) \cdot 2 \Rightarrow B = 4$

$s=-8$: $8(-3)4 = C(-8)(-2) \Rightarrow C = -6$

$$Y(s) = \frac{10}{s} + \frac{4}{s+6} - \frac{6}{s+8}$$

Stegsvar $y(t) = \mathcal{L}^{-1}\{Y(s)\} =$

$$= \left(10 + 4e^{-6t} - 6e^{-8t}\right) u(t)$$

5. $T = \frac{1}{12}$ ms , sampelintervall

Frekvens $f = 2{,}0$ kHz motsvarar efter sampling

diskret frekvens $\Omega = \omega T = 2\pi \cdot f \cdot T =$

$$= \frac{2\pi \cdot 2000 \cdot}{12 \cdot 1000} = \frac{\pi}{3} \text{ rad}$$

System: $y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2]$

z-transformera

$$Y(z) = X(z)\left(1 + b_1 z^{-1} + b_2 z^{-2}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + b_1 z^{-1} + b_2 z^{-2} = \frac{z^2 + b_1 z + b_2}{z^2}$$

Faktorisera!

$$H(z) = \frac{(z - z_1)(z - z_2)}{z^2}$$ med frekvenssvar

$$H(z)\Big|_{z = e^{j\Omega}} = H(e^{j\Omega}) = \frac{(e^{j\Omega} - z_1)(e^{j\Omega} - z_2)}{(e^{j\Omega})^2}$$

$H(e^{j\Omega}) = 0$ för $\Omega = \frac{\pi}{3} \Rightarrow z_1 = e^{j\frac{\pi}{3}}$

b_1 och b_2 reella koeff $\Rightarrow z_2 = z_1^*$

Täljarpolynom: $(z - z_1)(z - z_2) = z^2 - z(z_1 + z_2) + z_1 z_2$

Alltså $b_1 = -(z_1 + z_2) = -\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right) = -1$

$b_2 = e^{j\pi/3} \cdot e^{-j\pi/3} = 1$

Svar: $b_1 = -1$, $b_2 = 1$

1a/ $x(t) = 5\cos(16t) + 4\left|\sin\left(\frac{96\pi}{10}t\right)\right|$

$\omega_1 = 16 \Rightarrow T_1 = \frac{2\pi}{16}$

$\omega_2 = \frac{96\pi}{10} \Rightarrow T_2' = \frac{2\pi}{\omega_2} = \frac{2\pi \cdot 10}{96\pi}$

Belopp ger $T_2 = \frac{1}{2}T_2' = \frac{10}{96}$

Gemensam periodtid
$T = k_1 T_1 = k_2 T_2$; $\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{10}{96} \cdot \frac{16}{2\pi} = \frac{5}{6\pi}$

$\frac{k_1}{k_2}$ kan ej bildas som kvot mellan heltal

Signalen ej periodisk

1b/ $x(t) = 8\cos(90t)\cos(50t)$

(i) Använd: $\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

$\Rightarrow x(t) = 4\left[\cos(40t) + \cos(140t)\right]$

Högsta signalfrekvens $\omega_M = 140$ r/s

För att undvika aliasing krävs $\omega_s > 2\omega_M = 280$ r/s

(ii) Frekvensupplösning $\Delta\omega = \frac{\omega_s}{N}$

$\Rightarrow N = \frac{\omega_s}{\Delta\omega} = \frac{300}{0,5} = 600$

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2)

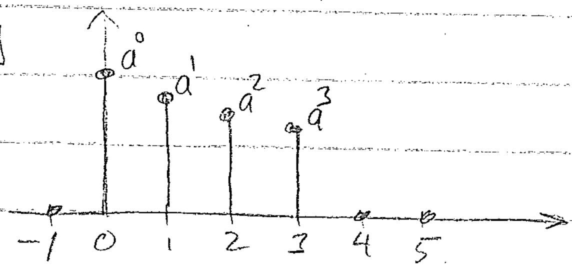
h_1: Fördröjer insignalen: Utsignal: $x[n-2]$

Insignal till h_2 blir $x[n-2] = \delta[n-2] - \delta[n-3] + \delta[n-4]$

Superposition ger

$$y[n] = h_2[n-2] - h_2[n-3] + h_2[n-4]$$

$h_2[n]$



n	-1	0	1	2	3	4	5	6	7
$h_2[n-2]$	0	0	0	a^0	a^1	a^2	a^3	0	0
$-h_2[n-3]$	0	0	0	0	$-a^0$	$-a^1$	$-a^2$	$-a^3$	0
$h_2[n-4]$	0	0	0	0	0	a^0	a^1	a^2	a^3

$\sum \Rightarrow y[n]$

$$y[2] = a^0 = 1$$
$$y[3] = a^1 - a^0 = a - 1$$
$$y[4] = a^2 - a^1 + a^0 = 1 + a^2 - a^1$$
$$y[5] = a^3 - a^2 + a^1 = a + a^3 - a^2$$
$$y[6] = -a^3 + a^2 = a^2 - a^3$$
$$y[7] = a^3$$
$y[n] = 0$ för övriga n

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3./ $y[n] - 0{,}9y[n-1] + 0{,}2y[n-2] = x[n]$

z-transf. $Y(z) - 0{,}9z^{-1}Y(z) + 0{,}2z^{-2}Y(z) = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0{,}9z^{-1}+0{,}2z^{-2}} = \frac{z^2}{z^2-0{,}9z+0{,}2} =$$

$$= \left\{\text{Faktorisera nämnaren}\right\} = \ldots = \frac{z^2}{(z-0{,}4)(z-0{,}5)}$$

Insignal: $X(z) = \frac{z}{z-0{,}6} \cdot z^{-1}$ fy $x[n] = 0{,}6^{n-1}u[n-1]$

$$Y(z) = X(z)\cdot H(z) = z \cdot \underbrace{\frac{z}{(z-0{,}4)(z-0{,}5)(z-0{,}6)}}_{\text{P.B.U}}$$

$$\frac{z}{(z-0{,}4)(z-0{,}5)(z-0{,}6)} = \frac{A}{z-0{,}4} + \frac{B}{z-0{,}5} + \frac{C}{z-0{,}6}$$

$$z = A(z-0{,}5)(z-0{,}6) + B(z-0{,}4)(z-0{,}6) + C(z-0{,}4)(z-0{,}5)$$

$z=0{,}4$: $0{,}4 = A(0{,}4-0{,}5)(0{,}4-0{,}6) \Rightarrow A = 20$

$z=0{,}5$: $0{,}5 = B(0{,}5-0{,}4)(0{,}5-0{,}6) \Rightarrow B = -50$

$z=0{,}6$: $0{,}6 = C(0{,}6-0{,}4)(0{,}6-0{,}5) \Rightarrow C = 30$

$$Y(z) = 20\,\frac{z}{z-0{,}4} - 50\,\frac{z}{z-0{,}5} + 30\,\frac{z}{z-0{,}6}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \left(20\cdot 0{,}4^n - 50\cdot 0{,}5^n + 30\cdot 0{,}6^n\right)u[n]$$

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Forts 3/ Alternativ lösning

Gör P.B.U. av

$$\frac{z^2}{(z-0{,}4)(z-0{,}5)(z-0{,}6)} = \frac{A}{z-0{,}4} + \frac{B}{z-0{,}5} + \frac{C}{z-0{,}6}$$

$$z^2 = A(z-0{,}5)(z-0{,}6) + B(z-0{,}4)(z-0{,}6) + C(z-0{,}4)(z-0{,}5)$$

$z=0{,}4$: $\quad 0{,}4^2 = A(0{,}4-0{,}5)(0{,}4-0{,}6) \Rightarrow A = 8$

$z=0{,}5$: $\quad 0{,}5^2 = B(0{,}5-0{,}4)(0{,}5-0{,}6) \Rightarrow B = -25$

$z=0{,}6$: $\quad 0{,}6^2 = C(0{,}6-0{,}4)(0{,}6-0{,}5) \Rightarrow C = 18$

$$Y(z) = 8\frac{1}{z-0{,}4} - 25\frac{1}{z-0{,}5} + 18\frac{1}{z-0{,}6} =$$

$$= 8\frac{z}{z-0{,}4}\cdot z^{-1} - 25\frac{z}{z-0{,}5}\cdot z^{-1} + 18\frac{z}{z-0{,}6}\cdot z^{-1}$$

$$y[n] = \mathcal{Z}^{-1}\{Y(z)\} = \left[8\cdot 0{,}4^{n-1} - 25\cdot 0{,}5^{n-1} + 18\cdot 0{,}6^{n-1}\right) u[n-1]$$

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4)

$$H(s) = \frac{H_0}{(s-s_1)(s-s_2)} = \left\{ \begin{array}{l} s_1 = -6+j8 \\ s_2 = -6-j8 \end{array} \right\} = \frac{H_0}{(s+6-j8)(s+6+j8)} =$$

$$= \frac{H_0}{(s+6)^2+64} = \frac{H_0}{s^2+s12+100}$$

Frekvsvar $H(s)\Big|_{\substack{s=j\omega \\ \omega\to 0}} = \frac{H_0}{100} = 8 \Rightarrow H_0 = 800$

Insignal $x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$

Utsignal (stegsvar): $Y(s) = X(s) \cdot H(s)$

$$Y(s) = \frac{800}{s(s^2+s12+100)} = \frac{A}{s} + \frac{Bs+C}{s^2+s12+100}$$

$$800 = A(s^2+s12+100) + Bs^2 + Cs$$

s^2: $0 = A+B \Rightarrow A = -B$

s^1: $0 = 12A + C$

s^0: $800 = A \cdot 100$

$A = 8$

$B = -8$

$C = -12A = -96$

$$Y(s) = \frac{8}{s} - \frac{8s+96}{s^2+s12+100} = 8\left(\frac{1}{s} - \frac{s+12}{(s+6)^2+64}\right) =$$

$$= 8\left(\frac{1}{s} - \frac{s+6+6}{(s+6)^2+8^2}\right) = 8\left(\frac{1}{s} - \frac{s+6}{(s+6)^2+8^2} - \frac{6}{8}\cdot\frac{8}{(s+6)^2+8^2}\right)$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = 8\left(1 - e^{-6t}\left(\cos 8t + \frac{3}{4}\sin 8t\right)\right) \cdot u(t)$$

(5)

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Denna lösning använder Beta för
att bestämma Fourierkoeff. till x(t).

$x(t)$: Medelvärde = 0, topp-topp värde $|h| = 2$ $(2L = T)$

$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = \frac{2\pi}{T}$ Från Beta fås

$$x(t) = -\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left\{\frac{(2n-1)2\pi}{\underbrace{2L}_{T}}\cdot t\right\} =$$

$$= -\frac{8}{\pi^2}\sum_{k=1,3,5,..}^{\infty}\frac{1}{k^2}\cos(k\omega_0 t) \quad ; \quad A'_k = \frac{1}{k^2}, \ k = 1,3,5,\ldots$$

$$H(j\omega) = \frac{j\frac{\omega}{\omega_0}}{\left(j\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0} + 1}$$

Studera frekvenserna
$\omega = \omega_0, 3\omega_0$ och $5\omega_0$

$$H(j\omega)\Big|_{\omega=\omega_0} = \frac{j}{-1+j+1} = 1 \qquad |H(j\omega_0)| = 1, \quad \arg\{H(j\omega_0)\} = 0$$

$$H(j\omega)\Big|_{\omega=3\omega_0} = \frac{3j}{-9+3j+1} = \frac{j3}{-8+j3} \qquad |H(j3\omega_0)| = \frac{3}{\sqrt{64+9}} = \frac{3}{\sqrt{73}}$$

$$\arg\{H(j3\omega_0)\} = 90^\circ - 159.4^\circ = -69.4^\circ$$

$$H(j\omega)\Big|_{\omega=5\omega_0} = \frac{5j}{-25+j5+1} = \frac{j5}{-24+j5} \qquad |H(j5\omega_0)| = \frac{5}{\sqrt{601}}$$

$$\arg\{H(j5\omega_0)\} = 90^\circ - 168.2^\circ = -78.2^\circ$$

$$A_k = -\frac{8}{\pi^2}\cdot A'_k \cdot |H(jk\omega_0)| \qquad \varphi_k = \arg\{H(jk\omega_0)\}$$

$$A_1 = -\frac{8}{\pi^2}\cdot 1\cdot 1 = -\frac{8}{\pi^2} \qquad \varphi_1 = 0^\circ$$

$$A_3 = -\frac{8}{\pi^2}\cdot\frac{1}{9}\cdot\frac{3}{\sqrt{73}} \qquad \varphi_3 = -69.4^\circ$$

$$A_5 = -\frac{8}{\pi^2}\cdot\frac{1}{25}\cdot\frac{5}{\sqrt{601}} \qquad \varphi_5 = -78.2^\circ$$