Signaler & System, Z2
SSY042 140313

1 a) $x(t) = 5\cos(\omega t)$

$\omega = 400$ rad/s

$$G(s) = \frac{1}{1+sRC} = \left\{\omega_0 = \frac{1}{RC}\right\} = \frac{1}{1+\frac{s}{\omega_0}}$$

Frekvenssvar: $G(j\omega) = G(s)\Big|_{s=j\omega} = \left(1 + j\frac{\omega}{\omega_0}\right)^{-1}$

$$\omega_0 = \frac{1}{RC} = \frac{1}{10^4 \cdot 0{,}20 \cdot 10^{-6}} = 500 \text{ rad/s}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} = \{\omega = 400\} = \frac{1}{\sqrt{1+\left(\frac{400}{500}\right)^2}} \approx 0{,}78$$

$$\arg\{G(j\omega)\} = -\arctan\left\{\frac{\omega}{\omega_0}\right\} = -\arctan\left\{\frac{400}{500}\right\} \approx -38{,}7^\circ$$

Systemet G påverkar amplitud och fas

$$y(t) = 0{,}78 \cdot 5\cos(400t - 38{,}7^\circ) = 3{,}9\cos(400t - 38{,}7^\circ) \text{ V}$$

b) $\omega = 2\pi f = 2\pi \cdot 500$ rad/s, $T = 0{,}25 \cdot 10^{-3}$ s

$x(t) = A\sin(\omega t)$ samplas $t = nT$

$x[n] = A\sin(\omega nT) = A\sin(\omega T n) = A\sin(\Omega n)$

i)

$$\Omega = \omega T = 2\pi \cdot 500 \cdot 0{,}25 \cdot 10^{-3} = 0{,}25\pi = \frac{\pi}{4}$$

ii) En period motsvarar $2\pi \Rightarrow n = \frac{2\pi}{\Omega} = \frac{2\pi}{\pi/4} = 8$

8 sampel per period

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2.

$$y[n] - 0{,}4y[n-1] = 2x[n]$$

z-transformera

$$Y(z) - 0{,}4z^{-1}Y(z) = 2X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1-0{,}4z^{-1}}$$

$$x[n] = u[n-1] \overset{\mathcal{Z}}{\longleftrightarrow} X(z) = \frac{z}{z-1} \cdot z^{-1} = \frac{1}{z-1}$$

$$Y(z) = X(z) \cdot H(z) = \frac{1}{z-1} \cdot \frac{2z}{z-0{,}4}$$

$$Y(z) = z\underbrace{\left[\frac{2}{(z-1)(z-0{,}4)}\right]}_{\text{PBU}} = z\left[\frac{A}{z-1} + \frac{B}{z-0{,}4}\right]$$

$$2 = A(z-0{,}4) + B(z-1) \quad \Bigg| \quad \begin{array}{l} z=1 \Rightarrow 2 = A \cdot 0{,}6 \\ z=0{,}4 \Rightarrow 2 = B \cdot (-0{,}6) \end{array}$$

$$A = -B = \frac{2}{0{,}6} = \frac{10}{3}$$

$$Y(z) = \frac{10}{3}\left(\frac{z}{z-1} - \frac{z}{z-0{,}4}\right) \overset{\mathcal{Z}^{-1}}{\longrightarrow} y[n] = \frac{10}{3}\left(1-0{,}4^n\right) \cdot u[n]$$

n	$x[n] = u[n-1]$	$y[n] = 0{,}4y[n-1] + 2x[n]$	n	$y[n] = \frac{10}{3}(1-0{,}4^n)$
0	0	0	0	$\frac{10}{3}(1-1) = 0$
1	1	$0+2 = 2$	1	$\frac{10}{3}(1-0{,}4) = 2$
2	1	$0{,}4 \cdot 2 + 2 = 2{,}8$	2	$\frac{10}{3}(1-0{,}4^2) = 2{,}8$
3	1	$0{,}4 \cdot 2{,}8 + 2 = 3{,}12$	3	$\frac{10}{3}(1-0{,}4^3) = 3{,}12$
4	1	$0{,}4 \cdot 3{,}12 + 2 = 3{,}248$	4	$\frac{10}{3}(1-0{,}4^4) = 3{,}248$

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3. $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 13y(t) = -\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 13x(t)$

Laplacetransf.

$s^2Y(s) + 4sY(s) + 13Y(s) = -s^2X(s) + 5sX(s) + 13X(s)$

$Y(s) = \underbrace{\frac{-s^2+5s+13}{s^2+4s+13}}_{H(s)} \cdot X(s)$

Poler: $s_{1,2} = \underbrace{-2 \pm \sqrt{4-13}}_{\text{Komplexa!}}$

Behåll 2:a grads polyn.

$x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$

$Y(s) = \frac{-s^2+5s+13}{s(s^2+4s+13)} = \{\text{P.B.U.}\} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+13}$

$-s^2+5s+13 = A(s^2+4s+13) + Bs^2 + sC$

$s^0 \Rightarrow 13 = A \cdot 13 \Rightarrow A = 1$

$s^1 \Rightarrow 5 = 4A + C \Rightarrow C = 1$

$s^2 \Rightarrow -1 = A + B \Rightarrow B = -2$

$Y(s) = \frac{1}{s} + \frac{-2s+1}{s^2+4s+13} = \frac{1}{s} - 2\frac{s-\frac{1}{2}}{(s+2)^2+13-4} =$

$= \frac{1}{s} - 2\frac{s-\frac{1}{2}}{(s+2)^2+3^2} = \frac{1}{s} - 2\frac{s+2-\frac{1}{2}-2}{(s+2)^2+3^2} =$

$= \frac{1}{s} - 2\frac{s+2}{(s+2)^2+3^2} + \frac{5}{3} \cdot \frac{3}{[(s+2)^2+3^2]}$

Inv. Laplace ger

$y(t) = \left[1 - 2e^{-2t}\cos 3t + \frac{5}{3}\sin 3t\right]u(t)$

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4. Aktuella k index

$$\frac{k}{N} = \frac{\omega}{\omega_s} = \frac{\omega \cdot T}{2\pi} \qquad \text{ty} \quad \omega_s = \frac{2\pi}{T}$$

$$k = \frac{\omega \cdot T \cdot N}{2\pi}$$

reell signal ger "dubbla" bidrag

	k		$N-k$
A:	$\frac{200\pi \cdot 1{,}25 \cdot 10^{-3} \cdot 16}{2\pi}$	$= 2$	14
B:	$\frac{400\pi \cdot 5 \cdot 10^{-3} \cdot 16}{2\pi}$	$= 16$	0
C:	$\frac{50\pi \cdot 30 \cdot 10^{-3} \cdot 16}{2\pi}$ (Aliasing)	$= 12$	4
D:	$\frac{28\pi \cdot 31{,}25 \cdot 10^{-3} \cdot 16}{2\pi}$	$= 7$	9
E:	$\frac{572\pi \cdot 1{,}2 \cdot 10^{-3} \cdot 16}{2\pi}$	$=$ "5,5"	"10,5"

Följande samband gäller

Signal	DFT
A	5
B	2
C	3
D	4
E	1

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5. a) $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ $\qquad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{0.1} = 20\pi$ r/s

$k \neq 0$

$$c_k = \frac{1}{T_0}\int_{-T_1}^{T_1} A e^{-jk\omega_0 t}\,dt = \frac{A}{T_0}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T_1}^{T_1} =$$

$$= \frac{A}{T_0 k\omega_0}\cdot 2\left[\frac{e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}}{-j2}\right] = \frac{A}{\pi k}\sin(k\omega_0 T_1)$$

$$\omega_0 T_1 = \frac{2\pi}{T_0}\cdot T_1 = \pi\frac{2T_1}{T_0} = \pi\frac{2\cdot 12.5}{100} = \frac{\pi}{4}$$

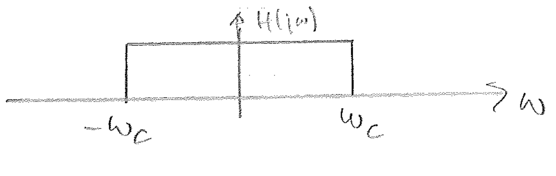
$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \text{med } c_k = \frac{A}{\pi k}\sin\left(\frac{k\pi}{4}\right), \quad k \neq 0$$

$$k = 0 \Rightarrow c_0 = \frac{1}{T_0}\int_{-T_1}^{T_1} A e^{0}\,dt = \frac{A 2T_1}{T_0} = \frac{A}{4}$$

b) Total medeleffekt (Parseval) $\quad \overline{P} = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |c_k|^2$

$$\overline{P} = \frac{1}{T_0}\int_{-T_1}^{T_1} A^2\,dt = \frac{2T_1}{T_0}\cdot A^2 = \frac{A^2}{4} = \{A = 20\} = 100$$

	summera ihop
$k=0, \quad \lvert c_0\rvert^2 = \frac{A^2}{16} = 25$	25
$k=\pm 1, \quad 2\lvert c_1\rvert^2 = 2\left(\frac{A}{1\cdot\pi}\sin\frac{\pi}{4}\right)^2 = 40{,}53$	65.5
$k=\pm 2, \quad 2\lvert c_2\rvert^2 = 2\left(\frac{A}{2\pi}\sin\frac{2\pi}{4}\right)^2 = 20{,}26$	85.8
$k=\pm 3, \quad 2\lvert c_3\rvert^2 = 2\left(\frac{A}{3\pi}\sin\frac{3\pi}{4}\right)^2 = 4{,}50$	90.3



Svar: $\omega_c < 3\omega_0 = 60\pi$ rad/s