Sensorer, Signaler & System, Z2
del B SSY041B 130828

1. $$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 5y(t) = x(t)$$

Laplacetransformera!

$$s^2 Y(s) + 6sY(s) + 5Y(s) = X(s)$$

$$H(s) = \frac{1}{s^2+6s+5} = \ldots = \frac{1}{(s+1)(s+5)}$$

$$x(t) = e^{-7t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+7}$$

$$Y(s) = H(s)\cdot X(s) = \frac{1}{(s+1)(s+5)(s+7)} = \{\text{P.B.U.}\} =$$

$$= \frac{A}{s+1} + \frac{B}{s+5} + \frac{C}{s+7}$$

$$1 = A(s+5)(s+7) + B(s+1)(s+7) + C(s+1)(s+5)$$

$$\begin{cases} s^2: & 0 = A + B + C \quad (1) \\ s^1: & 0 = 12A + 8B + 6C \quad (2) \\ s^0: & 1 = 35A + 7B + 5C \quad (3) \end{cases}$$

"$12\cdot(1) - (2)$": $12B - 8B + 12C - 6C = 0$ ; $4B = -6C$ ; $B = -\frac{3}{2}C$

(2) $12A - \frac{8\cdot3}{2}C + 6C = 0$ ; $12A = 6C$ ; $A = \frac{1}{2}C$

(3) $1 = \frac{35}{2}C - \frac{21}{2}C + 5C$ $\Rightarrow C = \frac{1}{12}$

$A = \frac{1}{24}$ $B = -\frac{1}{8}$

$$Y(s) = \frac{1}{24}\cdot\frac{1}{s+1} - \frac{3}{24}\cdot\frac{1}{s+5} + \frac{2}{24}\cdot\frac{1}{s+7} = \frac{1}{24}\left(\frac{1}{s+1} - \frac{3}{s+5} + \frac{2}{s+7}\right)$$

Inv. Laplacetransf.

$$y(t) = \frac{1}{24}\left(e^{-t} - 3e^{-5t} + 2e^{-7t}\right)\cdot u(t)$$

SSY041B

130828

2. $h[n] = 0{,}2^{n-1} u[n-1] \xleftrightarrow{Z} H(z) = \frac{z}{z-0{,}2} \cdot z^{-1}$

$x[n] = u[n-1] \xleftrightarrow{Z} X(z) = \frac{z}{z-1} \cdot z^{-1}$

$Y(z) = H(z) \cdot X(z) = \frac{1}{(z-1)(z-0{,}2)} = z \cdot \frac{1}{z(z-1)(z-0{,}2)}$

Partialbråksuppdela

$\frac{1}{z(z-1)(z-0{,}2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-0.2}$

$1 = A(z-1)(z-0{,}2) + B\,z(z-0{,}2) + C\,z(z-1)$

$z^2: \quad 0 = A + B + C$ | $A = 5$ | $A = 5$

$z^1: \quad 0 = -1.2A - 0{,}2B - C$ | $B = -5 - C$ | $B = \frac{5}{4}$

$z^0: \quad 1 = 0.2A$ | $0 = -1.2 \cdot 5 - 0{,}2(-5-C) - C$ | $C = -\frac{25}{4}$

$Y(z) = 5 + \frac{5}{4}\,\frac{z}{z-1} - \frac{25}{4} \cdot \frac{z}{z-0{,}2}$ ; Inv. z-transf. ger

$y[n] = 5\delta[n] + \left(\frac{5}{4} - \frac{25}{4} \cdot 0{,}2^n\right) u[n]$

Notera
$y[0] = y[1] = 0$

$\boxed{\text{Alt.}}$ Utan fördröjningar $h'[n] = 0{,}2^n u[n] \xleftrightarrow{Z} \frac{z}{z-0{,}2}$

$x'[n] = u[n] = \frac{z}{z-1}$

$Y'(z) = z\left(\frac{z}{(z-1)(z-0{,}2)}\right) = \ldots = \frac{5}{4} \cdot \frac{z}{z-1} - \frac{1}{4} \cdot \frac{z}{z-0{,}2}$

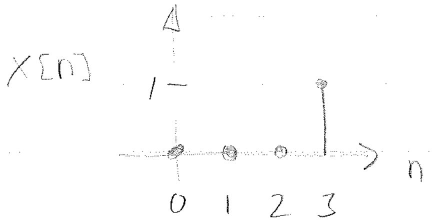
$y'[n] = \left[\frac{5}{4} - \frac{1}{4} \cdot 0{,}2^n\right) u[n]$ Fördröj 2 steg

$y_d = y'[n-2] = \left(\frac{5}{4} - \frac{1}{4} \cdot 0{,}2^{n-2}\right) u[n-2] = \left(\frac{5}{4} - \frac{25}{4} \cdot 0{,}2^n\right) u[n-2]$

$y_d[n] = y[n]$ enligt ovan

SSY041 B
130828

3.




$N=4$

$x[n] = \delta[n-3]$

$$X[k] = \sum_{n=0}^{N} x[n] e^{-j\frac{2\pi}{N}kn} \quad , \quad k = 0, 1, .., N-1$$

$k=0 \qquad X[0] = \sum_{n=0}^{3} \delta[n-3] e^{-j\frac{2\pi}{4}\cdot 0 \cdot n} = 1$

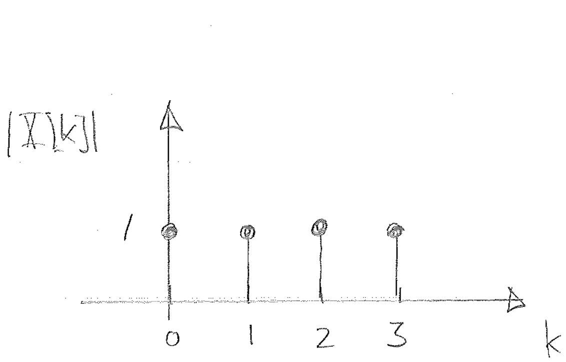
$k=1 \qquad X[1] = \sum_{n=0}^{3} \delta[n-3] e^{-j\frac{2\pi}{4}\cdot 1 \cdot n} = e^{-j\frac{2\pi}{4}\cdot 1\cdot 3} = e^{-j\frac{3\pi}{2}} = j$

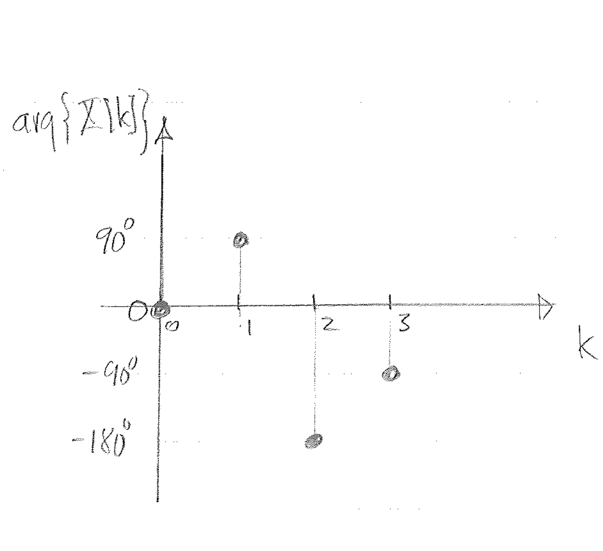
$k=2 \qquad X[2] = \sum_{n=0}^{3} \delta[n-3] e^{-j\frac{2\pi}{4}\cdot 2 \cdot n} = e^{-j\frac{2\pi}{4}\cdot 2\cdot 3} = e^{-j\frac{6\pi}{2}} = -1$

$k=3 \qquad X[3] = \sum_{n=0}^{3} \delta[n-3] e^{-j\frac{2\pi}{4}\cdot 3 \cdot n} = e^{-j\frac{2\pi}{4}\cdot 3\cdot 3} = e^{-j\frac{9\pi}{2}} = -j$

$X[0] = 1 = \underline{1\angle 0^\circ}$ | $X[2] = -1 = \underline{1\angle -180^\circ}$

$X[1] = j = \underline{1\angle 90^\circ}$ | $X[3] = -j = \underline{1\angle -90^\circ}$



SSY041B
130828

4.

$y[n] - 0{,}5\,y[n-1] = x[n]$

z-transformera

$Y(z) - 0{,}5\,z^{-1}Y(z) = X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0{,}5\cdot z^{-1}}$$

För frekvenssvar sätt $z = e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{1}{1-0{,}5\,e^{-j\Omega}} = \frac{1}{1-0{,}5(\cos\Omega - j\sin\Omega)} =$$

$$= \frac{1}{1-0{,}5\cos\Omega + j0{,}5\sin\Omega}$$

$\omega = 200\pi$, $T = 2{,}5$ ms $\qquad \Omega = \omega \cdot T = \frac{\pi}{2}$

Amplitudpåverkan

$$|H(e^{j\Omega})|_{\Omega=\frac{\pi}{2}} = \frac{1}{\left[\left(1-0{,}5\cos\left(\frac{\pi}{2}\right)\right)^2 + \left(0{,}5\sin\left(\frac{\pi}{2}\right)\right)^2\right]^{1/2}}$$

$$= \frac{1}{\sqrt{1^2+0{,}5^2}} = \frac{1}{\sqrt{1+\left(\frac{1}{2}\right)^2}} =$$

$$= \frac{1}{\sqrt{\frac{4+1}{4}}} = \frac{2}{\sqrt{5}} \approx 0{,}89$$

Amplitud multipliceras med $\frac{2}{\sqrt{5}}$.