Sensorer, Signaler & System Z2
del A SSY041A 130826

1a)

$$x(t) = 2\cos\left(10\pi t + \frac{\pi}{6}\right) + 5\pi\cos\left(17\pi t - \frac{\pi}{4}\right)$$

$\omega_1 = 10\pi$, $T_1 = \frac{2\pi}{\omega_1}$

$\omega_2 = 17\pi$, $T_2 = \frac{2\pi}{\omega_2}$

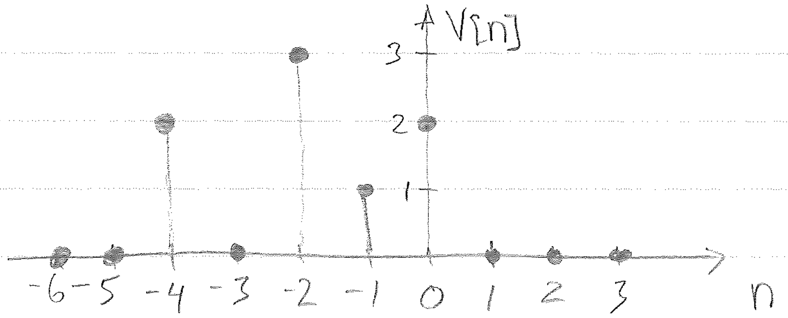
$T = k_1 \cdot T_1 = k_2 T_2$

$$\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{10}{17}$$

Periodisk? Ja! Med $T = k_1 T_1 = k_2 T_2 = 10 \cdot \frac{2\pi}{10\pi} = 17 \cdot \frac{2\pi}{17\pi} = 2\,s$

b)

n	$1-2n$	$v[n] = x[1-2n]$	
-5	11	0	$y[-3]$
-4	9	2	$y[-2]$
-3	7	0	$y[-1]$
-2	5	3	$y[0]$
-1	3	1	$y[1]$
0	1	2	$y[2]$
1	-1	0	$y[3]$



$$v[n] \longrightarrow \underset{H}{\boxed{h[n]}} \longrightarrow y[n] \qquad h[n] = \delta[n-2]$$

Systemet H fördröjer signalen två steg (tidsenheter)

$$y[n] = v[n-2] = \ldots 0, 0, 2, 0, \underline{3}, 1, 2, 0, 0, \ldots$$

$n=0$

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2. $$H(j\omega) = \frac{\omega_c^2}{\omega_c^2 - \omega^2 + j\sqrt{2}\,\omega\omega_c}$$

a/ $$|H(j\omega)| = \frac{\omega_c^2}{\left[(\omega_c^2 - \omega^2)^2 + (\sqrt{2}\,\omega\omega_c)^2\right]^{1/2}} = \frac{\omega_c^2}{\left[\omega_c^4 + \omega^4 - 2\omega^2\omega_c^2 + 2\omega^2\omega_c^2\right]^{1/2}} =$$

$$= \frac{\omega_c^2}{\sqrt{\omega_c^4 + \omega^4}} = \frac{\omega_c^2}{\sqrt{\omega_c^4\left(1 + \left(\frac{\omega}{\omega_c}\right)^4\right)}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^4}}$$

$$\arg\{H(j\omega)\} = -\arctan\left\{\frac{\sqrt{2}\,\omega\omega_c}{\omega_c^2 - \omega^2}\right\} = -\arctan\left\{\frac{\sqrt{2}\,\frac{\omega}{\omega_c}}{1 - \left(\frac{\omega}{\omega_c}\right)^2}\right\} = \phi$$

b/ $$x(t) = \frac{4}{\pi}\left(\sin(\omega_0 t) + \frac{1}{3}\sin(3\omega_0 t)\right)$$

$\omega_c = 600\pi$

$\omega = \omega_0 = 200\pi$ $$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{200\pi}{600\pi}\right)^4}} = 0{,}994$$

$$\phi = -\arctan\left\{\frac{\sqrt{2}\cdot 1/3}{1 - \left(\frac{1}{3}\right)^2}\right\} = -27{,}9^\circ$$

$\omega = 3\omega_0 = 600\pi$ $$|H(j\omega)| = \frac{1}{\sqrt{1 + 1^4}} = \frac{1}{\sqrt{2}} = 0{,}707$$

$$\phi = -\arctan\underbrace{\left\{\frac{\sqrt{2}\cdot 1}{1-1}\right\}}_{\substack{\to\infty \\ \text{då } \omega \to 600\pi}} = -90^\circ$$

$|H(j\omega)|$: Ger amplitudpåverkan
$\arg\{H(j\omega)\} = \phi$: Ger fasförskjutning

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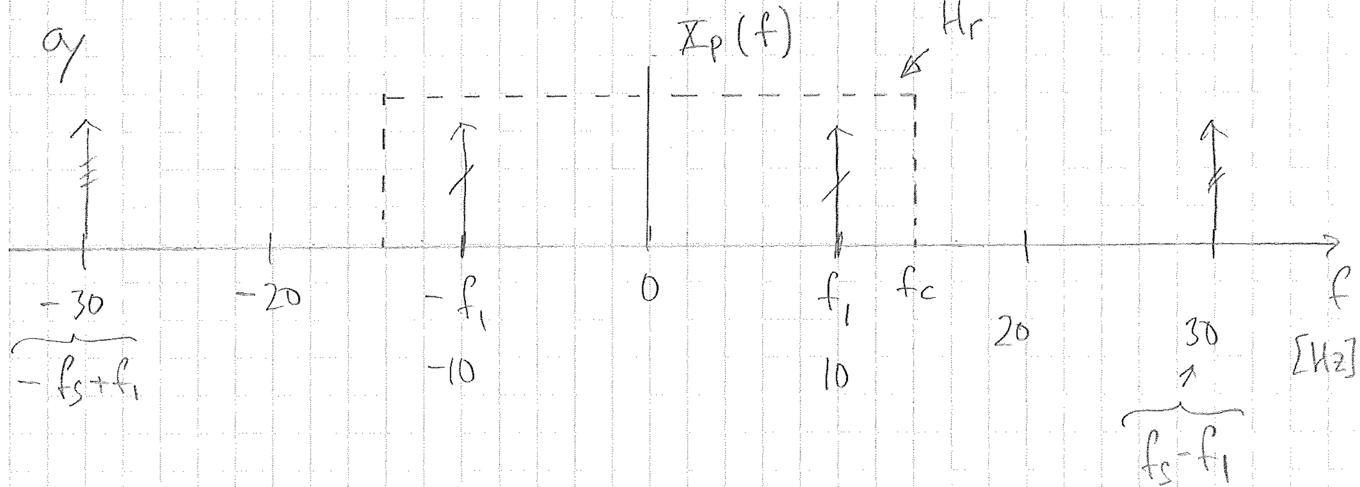
3. $x(t) = 4\cos(\omega_1 t)$ $\quad \omega_1 = 20\pi = 2\pi f_1 \Rightarrow f_1 = 10\text{ Hz}$

Sampelintervall $T = 25\text{ ms} \Rightarrow f_s = \frac{1}{T} = 40\text{ Hz}$ $\quad , \omega_s = 2\pi f_s$

$X(j\omega) = \pi\left[\delta(\omega-\omega_1) + \delta(\omega+\omega_1)\right]$, sampling ger

$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$

a/



Studera figur!

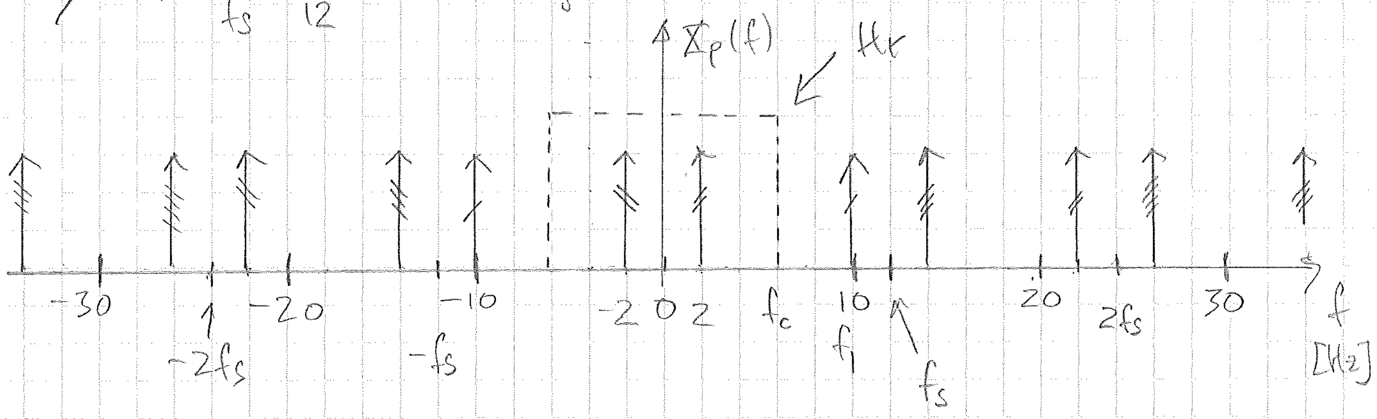
i) $\omega = 2\pi 30 \Rightarrow f = 30\text{ Hz}$

ii) $2\pi\omega_1 < \omega_c < 2\pi\cdot 30$

$10\text{ Hz} < f_c < 30\text{ Hz}$

iii) $f_c < 10\text{ Hz}$

b/ $T = \frac{1}{f_s} = \frac{1}{12}\text{ s} \Rightarrow f_s = 12\text{ Hz}$



Studera figur!

i) $2f_s - f_1 = 24 - 10 = 14\text{ Hz}$

ii) $\omega_s - \omega_1 < \omega_c < \omega_1 \Rightarrow 2\text{ Hz} < f_c < 10\text{ Hz}$

iii) $f_c < 2\text{ Hz}$ $\quad (f_s - f_1)$

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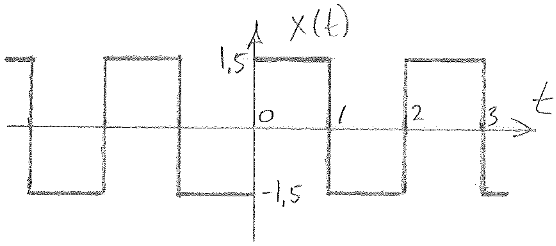
4/

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad ; \quad \omega_0 = \frac{2\pi}{T}$$

$$\omega_0 = \pi \Rightarrow T = \frac{2\pi}{\omega_0} = 2 \text{ s}$$

$$x(t) = \begin{cases} 1{,}5 \ , & 0 \le t < 1 \\ -1{,}5 \ , & 1 \le t < 2 \end{cases}$$



$$c_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{2}\left(-\frac{3}{2}\right)\int_{-1}^{0} e^{-jk\omega_0 t} dt + \frac{1}{2}\frac{3}{2}\int_0^1 e^{-jk\omega_0 t} dt =$$

$$= -\frac{3}{4}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-1}^{0} + \frac{3}{4}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_0^1 = \{\omega_0 = \pi\} =$$

$$= \frac{3}{4\cdot jk\pi}\left\{-\left(1 - e^{-jk\pi}\right) - \left(e^{-jk\pi} - 1\right)\right\} =$$

$$= \frac{3}{4jk\pi}\left(2 - 2e^{-jk\pi}\right) = \frac{3}{2jk\pi}\left(1 - e^{-jk\pi}\right)$$

Kan skrivas om som

$$c_k = \frac{3}{2\pi k}\cdot\frac{1}{j} e^{-jk\frac{\pi}{2}}\left(e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}\right) =$$

$$= \frac{3}{\pi k}\cdot e^{-jk\frac{\pi}{2}}\cdot \sin\left(\frac{k\pi}{2}\right)$$

Studera c_0 separat $\Rightarrow c_0 = 0$ ty medelvärde $= 0$

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5. Total energi $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

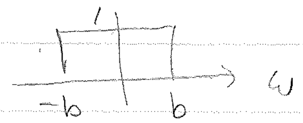
"Parsevals formel" där $x(t) \xleftrightarrow{FT} X(j\omega)$

$$x_1(t) = e^{-at} u(t)$$

$$E_1 = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_0^{\infty} e^{-2at} dt = \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty} =$$

$$= 0 - \left(-\frac{1}{2a}\right) = \frac{1}{2a}$$

$$x_2(t) = \mathrm{sinc}(bt) \xleftrightarrow{FT} \frac{\pi}{b} \underbrace{\left[u(\omega+b) - u(\omega-b) \right]}$$



$$E_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| X_2(j\omega) \right|^2 d\omega =$$

$$= \frac{1}{2\pi} \int_{-b}^{b} \frac{\pi^2}{b^2} d\omega = \frac{1}{2\pi} \cdot \frac{\pi^2}{b^2} \cdot 2b = \frac{\pi}{b}$$

$E_1 = E_2$; $\frac{1}{2a} = \frac{\pi}{b}$

$$\boxed{\frac{b}{a} = 2\pi}$$