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Tentamen

Datum: 200826 kl. 14.30–18.30
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MVE051/MVE055/MSG810 Matematisk Statistik och Diskret Matematik

Hjälpmedel: Alla hjälpmedel är tillåtna men lösningarna ska motiveras på samma sätt som en salstenta.
Betygsgränser: Chalmers: 3:12-17p, 4: 18-23p, 5: 24-30p, GU: G: 12-21, VG: 22-30.
Resultat meddelas via Ladok ca. 15 arbetsdagar efter tentamenstillfället.

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1. (a) A full house in poker is a hand that contains three cards of one rank and two cards of another rank. Knowing that a hand in poker has 5 cards, how many ways are there to get a full-house? What is the probability of getting a full-house? *(A deck of cards consists of 52 cards divided into 4 suits. Each kind consists of 13 ranks).* (1p)

   (b) The average IQ in a population is 100 with standard deviation 15. What is the probability that a randomly selected group of 100 people has an average IQ above 102.7? (2p)

2. A continuous random variable X has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{då } x<0 \\ 3x^2 - 2x^3 & \text{då } 0 \le x \le 1 \\ 1 & \text{då } x>1 \end{cases}$$

   (a) Find $P(X \le \frac{1}{2})$. (0.5p)

   (b) Find the expected value and variance of X. (1.5p)

3. The table below gives measurements of the height and and circumference of 10 eucalyptus.

Height h_i	18.25	19.75	16.5	18.25	19.50	16.25	17.25	19.00	16.25	17.50
Circumference c_i	36	42	33	39	43	34	37	41	27	30

$$\sum_{i=1}^{10} h_i = 178.5 \quad \sum_{i=1}^{10} c_i = 362$$

$$\sum_{i=1}^{10} h_i^2 = 3201.625 \quad \sum_{i=1}^{10} c_i^2 = 13354 \quad \sum_{i=1}^{10} h_i c_i = 6514.75$$

   (a) Find the regression line $\mu_{C|H} = b_0 + b_1 h$ of the circumference C in terms of the height H. (2p)

   (b) Is there any significance linear relation between the height and the circumference? Explain using a hypothesis test with significance level $\alpha = 0.05$. (3p)

   (c) Find a 99% confidence interval on b_0. (1p)

4. Two random variables X and Y have a joint density function

$$f(x,y) = cx^3y(1+y) \qquad \text{for } 0 \le x \le 2 \text{ and } 0 \le y \le 3$$

and $f(x,y) = 0$ otherwise.

(a) Find the value of c. (1p)

(b) Find the probability of $1 \le X \le 3$ knowing that $Y > 2$. (1p)

(c) Find the covariance $Cov(X,Y)$. Can we deduce from the covariance that X and Y are independent? (2p)

5. A random variable X takes the values $0, 1, 2, 3, 4$ and a random variable Y takes the values $0, 1, 2$. X and Y are independent of each other. The following two tables give the density funtions of both variables.

x	0	1	2	3	4
$p_X(x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	c

y	0	1	2
$p_Y(y)$	$\frac{1}{6}$	$\frac{1}{3}$	d

where c and d are unknown. Let Z be the random variable given by $Z = X + 2Y$.

(a) Find the values of c and d. (1p)

(b) Find the probability $P(Z = 4)$. (2p)

(c) Find the expected value Z. (1p)

6. A sample of 12 patients complaining of insomnia was chosen randomly and was given Drug A. An independent sample of 16 patients with same complaint was chosen randomly and was given drug B. The number of hours of sleep experienced during the second night after treatment is given in the following table: (4p)

Drug A:			
3.5	5.7	3.4	6.9
17.8	3.8	3.0	6.4
6.8	3.6	6.9	5.7

Drug B:			
4.5	11.7	10.8	4.5
6.3	3.8	6.2	6.6
7.1	6.4	4.5	5.1
3.2	4.7	4.5	3.0

Assume that the populations are normally distributed with equal variance. Researchers claim that Drug A is better than Drug B. Do you think their claim is valid? Use a hypothesis testing with a level of significance $\alpha = 0.1$.

7. Use generating functions to find an explicit form for a_n where a_n is defined as follows (3p)

$$\begin{cases} a_0 = 1 \\ a_1 = 8 \\ a_n = 6a_{n-1} - 9a_{n-2} + 2^n \quad n > 1 \end{cases}$$

8. (a) Suppose that X follows a binomial distribution $Bin(n, 0.5)$. Find the density function of the random variable Y subject to $2Y = X$. (2p)

   (b) Let B_1 and B_2 two independent random variable following a Bernoulli distribution with $p = 0.5$,i.e. each of B_1 and B_2 takes the values 0 and 1 with $P(1) = 0.5$. Let $X = B_1 + B_2$ and $Y = B_1 - B_2$. Find the joint density function for (X, Y). (2p)