Nancy Abdallah
Chalmers tekniska högskola - Göteborgs universitet
Tentamen

Datum: 200107 kl. 14.00–18.00
Telefonvakt: Tobias Magnusson
5325

MVE051/MVE055/MSG810 Matematisk statistik och diskret matematik

Hjälpmedel: Chalmers godkända miniräknare samt Betaboken **eller** ett datorskrivet eller handskrivet A4-papper med egna anteckningar (båda sidor får användas).
Tentan rättas och bedöms anonymt. **Skriv tentamenskoden tydligt på placeringlista**. Fyll i omslaget ordentligt.
För godkänt på tentan krävs 12 poäng på tentamen.
För betyg VG för GU studenter krävs 22 poäng.
För betyg 4 resp. 5 för Chalmers studenter krävs dessutom 18 resp. 24 poäng.
Alla svar ska vara motiverade.

---

1. A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a (false) positive when applied to a non-sufferer. It is estimated that 0.5 % of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). Calculate the following probabilities: (5p)

(a) that the test result will be positive;

(b) that, given a positive result, the person is a sufferer;

(c) that, given a negative result, the person is a non-sufferer.

2. In a packing plant, a machine packs cartons with jars. It is supposed that a new machine will pack faster on the average than the machine currently used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results, in seconds, are shown in the tables. (5p)

New Machine	42.1	41.3	42.4	43.2	41.8	41.0	41.8	42.8	42.3	42.7
Old Machine	42.7	43.8	42.5	43.1	44.0	43.6	43.3	43.5	41.7	44.1

Denote by X and Y the variables that give the time for the New Machine and Old Machine respectively. Suppose that the two population variances are equal.
Do the data provide sufficient evidence to conclude that, on the average, the new machine packs faster? Use a hypothesis test with a level of significance $\alpha = 5\%$ and the data below to answer the question.

New machine, X**:** $\sum_{i=1}^{10} x_i = 421.4, \ \sum_{i=1}^{10}(x_i - \overline{x})^2 = 4.204$

Old machine Y**:** $\sum_{i=1}^{10} y_i = 432.3, \ \sum_{i=1}^{10}(y_i - \overline{y})^2 = 5.061,$

where $\overline{x}$ and $\overline{y}$ are the sample means for the new and the old machine respectively.

3. Let A, B and C be three events such that A and B are independent and A and C are disjoint. Suppose that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B \cup C) = 0.9$. Compute $P(B^c \cap C)$ where B^c is the complement of B. (2.5p)

VÄND!

4. Let X be a random variable with moment generating function

$$m_X(t) = (1-t)^{-2}$$

Find the expected value $E[X]$ and the variance $V[X]$ of X. (2p)

5. The density function of a continuous random variable X is given by

$$f(x) = \frac{1}{39}x^2$$

for $2 \leqslant x \leqslant 5$ and 0 otherwise.

(a) Find the cumulative distribution function for X. (2p)

(b) Use the cumulative distribution function to compute the following probabilities: (1.5p)

i. the probability that $X \leqslant 1$;

ii. the probability that $X \geqslant 4$.

6. Consider a company that markets and repairs small computers. The following table illustrates the relationship between the length of a service call, in minutes, and the number of electronic components in the computer that must be repaired or replaced.

Number of Components	1	2	3	4	4	5	6	6	7	8	9	9
Length of Service Call	23	29	49	64	74	87	96	97	109	119	149	145

Denote by X the variable that gives the number of components and Y be the variable that gives the length of service call. The following information can be calculated using the above table:

$$\sum_{i=1}^{12} x_i = 64 \qquad \sum_{i=1}^{12} y_i = 1041$$

$$\sum_{i=1}^{10} x_i^2 = 418 \qquad \sum_{i=1}^{10} y_i^2 = 108805 \qquad \sum_{i=1}^{10} x_i y_i = 6734$$

(a) Estimate the linear regression curve $\mu_{Y|X} = \beta_0 + \beta_1 x$ of the number of components in terms of the length of service calls. (2p)

(b) Find a confidence interval with a level of significance 5% on β_0. (3p)

7. Let X be a random variable that takes on the four values 0, 1, 3, and 4 with probabilities 0.2, 0.4, 0.1, and c, respectively. Let Y be an independent random variable that takes on the three values 0, 1, and -1 with probabilities 0.4, 0.3, and 0.3, respectively. Calculate: (2p)

(a) The value of c.

(b) $P(XY > 1)$.

8. Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$. Find the distribution of $2X - 5$ and compute $P(2X - 5 > 0)$. (2p)

VÄND!

9. Multiple choice questions. Choose the correct answer for each of the following questions.

(3p)

(a) The moment generating function of a random variable X is given by $m_X(t) = e^{2(e^t-1)}$ and that of Y is $m_Y(t) = e^{3(e^t-1)}$. If X and Y are independent then the moment generating function of $X+Y$ is given by

- ☐ $m_{X+Y}(t) = e^{2(e^t-1)} + e^{3(e^t-1)}$
- ☐ $m_{X+Y}(t) = e^{6(e^t-1)}$
- ☐ $m_{X+Y}(t) = e^{5(e^t-1)}$
- ☐ None of the above

(b) A bag contains letter tiles. Forty-four of the tiles are vowels, and 56 are consonants. Seven tiles are picked at random. What is the probability that four of the seven tiles are vowels.

☐ 0.235 ☐ $0.85 \cdot 10^{-5}$ ☐ 0.00658 ☐ None of the above

(c) According to Chebyshev's inequality, if X is a random variable of mean 0 and variance 4, then $P(|X| \geq 3)$ is less than or equal than

☐ 0.5 ☐ 0.2 ☐ 0.1 ☐ 0.43

(d) On a certain Friday evening 1% of the car drivers are intoxicated. At a road check 50 drivers are tested. The probability that at least one intoxicated driver is caught is equal to

☐ 0.605 ☐ 0.395 ☐ 0.694 ☐ 0.306

(e) Let X be the continuous random variable that gives the time of the occurence of the first event in a Poisson process. Then X has a ... distribution.

☐ Poisson ☐ Uniform ☐ Normal ☐ Exponential

(f) Roll a fair die and denote by X the random variable that gives the number of rolls needed until we get 5 for the first time. Then X has a ... distribution.

☐ Hypergeometric ☐ Binomial ☐ Geometric ☐ Bernoulli

VÄND!

T-distribution table

(The values on the first row represent the area to the left of the T values.)

df	0.75	0.90	0.95	0.975	0.99	0.995	0.999
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891