Nancy Abdallah
Chalmers tekniska högskola - Göteborgs universitet
Datum: 191029 kl. 8.30–12.30
Tentamen
Telefonvakt: Felix Held
5325

# MVE055/MSG810 Matematisk statistik och diskret matematik

**Hjälpmedel**: Chalmers godkända miniräknare och Betaboken.
Tentan rättas och bedöms anonymt. **Skriv tentamenskoden tydligt på placeringlista och samtliga inlämnade papper.** Fyll i omslaget ordentligt.
För godkänt på tentan krävs 12 poäng på tentamen.
För betyg VG för GU studenter krävs 22 poäng.
För betyg 4 resp. 5 för Chalmers studenter krävs dessutom 18 resp. 24 poäng.
Alla svar ska vara motiverade.
Uppgift 9 finns på separat blad på vilket lösning och svar kan skrivas. Detta blad inlämnas tillsammans med övriga lösningar.

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**1**. The experiment consists of rolling a fair die several times. Answer the following questions. (3p)

(a) Find the probability of getting number 5 for the first time at the sixth roll.
(b) Find the probability of getting number 5 for the third time at the sixth roll.
(c) Find the probability of getting number 5 at least once if we roll the die exactly 10 times.

**2**. Let $A$ and $B$ be two events such that $P(A) = 0.2$, $P(B) = 0.3$ and $P(A|B) = 0.6$. (3p)

(a) Are $A$ and $B$ independent? Motivate your answer.
(b) Are $A$ and $B$ disjoint? Motivate your answer.
(c) Compute $P(A \cup B)$.

**3**. In 1980, out of 700 men aged between 20 and 34, 130 were found to be overweight. Whereas, in 1990, out of 750 men aged between 20 and 34, 160 were found to be overweight.

(a) Find an unbiased point estimate to $p_1$, the proportion of overweight men in 1980, and an unbiased point estimate to $p_2$, the proportion of overweight men in 1990, based on the samples given above. (1p)
(b) Find an unbiased point estimate for $p_1 - p_2$. (0.5p)
(c) Find a 95% confidence interval on the difference of proportions.
*(Hint: The estimator for $p_1 - p_2$ is $\mathcal{N}(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2})$ where $n_1$ and $n_2$ are the corresponding sample sizes.* (1.5p)
(d) Could you conclude based on the confidence interval computed in (c) that, for men 20-34 years old, a higher percentage were overweight in 1990 than 10 years earlier? Motivate your answer. (1p)

**4**. The amount of fuel that a vessel has in its tank at the beginning of a voyage has a normal distribution with mean 1200 litres and standard deviation 80 litres. The amount which it will use on the voyage has a normal distribution with mean 800 litres and standard deviation 50 litres. Find the probability that it has less than 500 litres in the tank at the end of the voyage. (3p)

VÄND!

**5**. Let $X$ be a random variable with moment generating function

$$m_X(t) = (pe^t + q)^{10}$$

where $p$ is constant between 0 and 1 and $q = 1 - p$.
Use $m_X(t)$ to find the expected value $E[X]$ and the variance $V[X]$ in terms of $p$. (3p)

**6**. The following table consists of one student athlete's time (in minutes) to swim 2000 yards and the student's heart rate (beats per minute) after swimming on a random sample of 10 days:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Swim time | 34.12 | 35.72 | 34.72 | 34.05 | 34.13 | 35.73 | 36.17 | 35.57 | 35.37 | 35.57 |
| Heart rate | 144 | 152 | 124 | 140 | 152 | 146 | 128 | 136 | 144 | 148 |

Denote by $X$ the variable that gives the Swim time and $Y$ the variable that gives the heart rate. The following information can be calculated using the above table:

$$\sum_{i=1}^{10} x_i = 351.15 \qquad \sum_{i=1}^{10} x_i^2 = 12\ 336.22 \qquad \sum_{i=1}^{10}(x_i - \overline{x})^2 = 5.5869$$

$$\sum_{i=1}^{10} y_i = 1\ 414 \qquad \sum_{i=1}^{10} y_i^2 = 200\ 756 \qquad \sum_{i=1}^{10}(y_i - \overline{y})^2 = 816.4$$

$$\sum_{i=1}^{10} x_i y_i = 49644.26 \qquad \sum_{i=1}^{10}(x_i - \overline{x})(y_i - \overline{y}) = -8.35$$

(a) Find the regression line $\hat{\mu}_{Y|X} = b_0 + b_1 x$ of the heart rate in terms of swim time. (2p)

(b) Is there a significant linear relation between the heart rate in terms of swim time? Test $H_1 : \beta_1 \neq 0$ at the 0.1 level of significance. (3p)

**7**. The purpose of this exercise is to find how many integer solutions the equation

$$y_1 + y_2 + y_3 = n \qquad (1)$$

has, where $y_1 \geq 2$ and $0 \leq y_2 \leq 5$, using generating functions.

(a) Prove that the generating function corresponding to the problem can be written as $f(x) = \frac{x^2(1-x^6)}{(1-x)^3}$. (2p)

(b) How many integer solutions does equation (1) have for (2p)
*(Hint: Use the n generating functions given on the next page).*

i. n=3?

ii. n=15?

**8**. Let

$$f(x) = cx^2$$

for $0 \leq x \leq 6$.

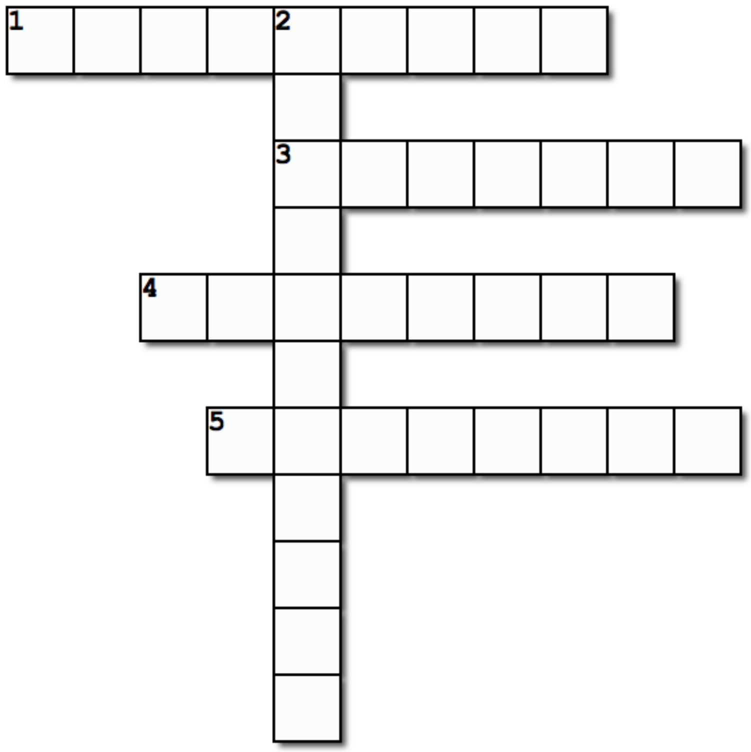
(a) Find the value of $c$ that makes $f(x)$ a density function for a continuous random variable $X$. (2p)

(b) Find the probability that $X \leq 5$ given that $4 \leq X \leq 7$. (2p)

VÄND!

| Anonym kod | MVE055/MSG810 191029 | sid.nummer | Poäng |
|---|---|---|---|
| | | | |

**9**. The following crossword consists of five words enumerated from 1 to 5 and whose clues are given below. Solve the crosswords or write your answers on the answer sheet. (1p)




1. Roll a fair die and denote by $X$ the random variable that gives the number of rolls needed until we get 5 for the first time. Then $X$ has a ... distribution.
2. Let $X$ be the random variable that gives the time of the occurence of the first event in a Poisson process. Then $X$ has a ... distribution.
3. If $X_1$ and $X_2$ are random variables having Poisson distributions, then $X_1 + X_2$ has a ... distribution.
4. Roll a fair die 5 times and denote by $X$ the number of times number 6 appears, then $X$ has a ... distribution.
5. Let $\hat{\theta}$ be a point estimator for a parameter $\theta$. If $E[\hat{\theta}] = \theta$, then $\hat{\theta}$ is said to be ....

Lycka till!

**Common generating functions**

(a) $\sum\limits_{n=0}^{\infty} (cx)^n = \frac{1}{1-cx}$, for $|x| < 1$.

(b) $\sum\limits_{k=0}^{\infty} \binom{n}{k} x^k = (1+x)^n$.

(c) $\sum\limits_{n=0}^{\infty} \binom{k+n}{k} x^n = \frac{1}{(1-x)^{k+1}}$.

$T$-distribution table

| df | 0.60 | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
|---|---|---|---|---|---|---|---|---|
| **6** | 0.265 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| **7** | 0.263 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 |
| **8** | 0.262 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 |
| **9** | 0.261 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 |
| **10** | 0.260 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 |

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| **0.1** | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| **0.2** | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| **0.3** | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| **0.4** | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| **0.5** | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| **0.6** | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| **0.7** | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| **0.8** | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| **0.9** | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| **1.0** | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| **1.1** | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| **1.2** | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| **1.3** | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| **1.4** | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| **1.5** | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| **1.6** | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| **1.7** | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| **1.8** | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| **1.9** | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| **2.0** | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| **2.1** | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| **2.2** | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| **2.3** | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| **2.4** | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| **2.5** | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| **2.6** | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| **2.7** | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| **2.8** | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| **2.9** | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| **3.0** | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| **3.1** | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| **3.2** | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| **3.3** | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| **3.4** | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| **3.5** | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| **3.6** | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| **3.7** | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| **3.8** | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| **3.9** | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |