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Applied Mathematics and Statistics
Chalmers and GU

MVE055 / MSG810 / MVE051 Matematisk statistik och diskret matematik

Exam 7 January 2019, 14:00 - 18:00

Allowed aids: Chalmers-approved calculator
and one (two-sided) A4 sheet of paper with your own notes.
Total number of points: 30. To pass, at least 12 points are needed.
Note: All answers should be motivated.

1. **(5 points)** A company would like to investigate the length of the aluminium bars they produce. The average length of 16 samples resulted in $\bar{X} = 94.32$ cm, and the sample variance resulted in $s^2 = 1.5$ cm. Assume that the length is normally distributed with mean μ and variance σ^2.

   (a) State and perform a hypothesis test to check if the mean μ is equal to 95 cm at level $\alpha = 0.01$.

   (b) Assume that the variance is known to be $\sigma^2 = 1.2$ cm. State and perform a hypothesis test to check if the mean μ is equal to 95 cm at level $\alpha = 0.01$.

2. **(5 points)** Let A and B be independent events such that $P(A \cap B) = 0.144$ and $P(A \cup B) = 0.626$. Assuming that $P(A) > P(B)$, determine the probabilities $P(A)$ and $P(B)$.

3. **(5 points)** A scientist has observed the following values of x = force applied to an elastic spring (N) and y = extension of the elastic spring (cm). The data collected have been summarized below and visualized in a scatterplot in Figure 1.

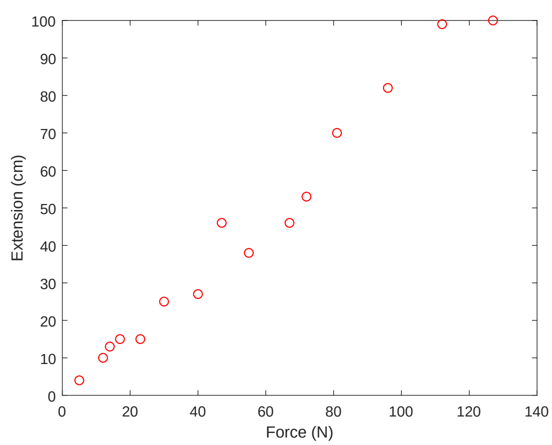

Figure 1: Scatterplot of the data observed by the scientist.

x: 5,12,14,17,23,30,40,47,55,67,72,81,96,112,127.
y: 4,10,13,15,15,25,27,46,38,46,53,70,82,99,100.

$$\sum_{i=1}^{15} x_i = 798$$

$$\sum_{i=1}^{15} x_i^2 = 63040$$

$$\sum_{i=1}^{15} y_i = 643$$

$$\sum_{i=1}^{15} y_i^2 = 41999$$

$$\sum_{i=1}^{15} x_i y_i = 51232$$

(a) Does the data support the use of the linear regression model?

(b) Calculate point estimates of the slope and intercept of the regression line.

(c) Compute the coefficient of determination and comment on its value.

4. **(5 points)** Consider a Markov chain with three possible states $s_1 = 1, s_2 = 2, s_3 = 3$ and the following transtion matrix (columns and rows corresponds in the order to the states s_1, s_2, and s_3.)

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Find the probabilty that the Markov chain will be in state s_1 in two steps if the chain is currently in the state s_1.

5. **(5 points)**

(a) Give the definition of moment generating function of a random variable.

(b) Find the moment generating function for a Poisson random variable X with parameter λ.

(c) Using the moment generating function, compute $\mathbb{E}[X]$, where X is the random variable of the previous point of this exercise.

6. **(5 points)** Students in Mathematics, Physics and Mechanical engineering take a calculus course given at Chalmers University of Technology. 60% of the students taking the course are mechanical engineers, 25% physicists and the remaining 15% are mathematicians. The probability to pass the exam for a mechanical engineer is 60%, for a physicist is 75%, and for a mathematician is 99%. What is the probability that a randomly chosen student among those that passed the exam is a mathematician?