EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time: Tuesday, October 25, 2016, morning.
Jour: Roza Maghsood, tel. 0737747320
Aids: Chalmers approved calculator and at most one (double-sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: Maximal points: 30. Chalmers: 12-17.5: 3; 18-23.5: 4; 24-30: 5, GU: 12-21.5: G; 22-30: VG.
Motivations: All answers/solutions must be motivated.
Language: Please write your answers in English.

1. (4p) Let $P(A) = 0.9$, $P(B) = 0.15$ and $P(A|B) = 0.9$. Find $P(A|B^c)$.

2. (6p)

   (a) Provide the definition of the moment-generating function (mgf). Calculate the mgf of a normal random variable having mean μ and variance σ^2.

   (b) Assume that $X_i \sim N(\mu_i, \sigma_i^2)$ for $i = 1, 2, ..., n$ and X_i's are independent random variables. Find the mgf of $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

3.(4p) Let X and Y be iid random variables and have uniform distribution $U[0, 1]$. Define $U = min(X, Y)$ and $V = max(X, Y)$. Find $cov(U, V)$.

4.(6p) Let X and Y be independent and have Poisson distributions with parameters λ_1 and λ_2, respectively.

   (a) Show that $X + Y$ has Poisson distribution with parameter $\lambda_1 + \lambda_2$. (**Hint: use mgf function**)

   (b) Show that the conditional distribution of $X = x$ given $X + Y = n$ is binomial and identify the parameters. (**Hint: use part (a) to find** $P(X + Y = n)$)

5.(3p) Let X and Y be independent and have U[0, 1]. Find (a) $E[XY]$, (b) $E[X/Y]$ and (c) $E[log(XY)]$.

6.(3p) Two different types of plants were evaluated in an experiment and the vitamin E content of the ripe berries was measured.

   Type A: 416, 492, 444, 404, 325, 286, 403
   Type B: 279, 352, 320, 385, 315

Assume that the samples are normally distributed with equal variances. Find 90% confidence intervals for the difference in mean vitamin E content

between the two types.

7.(4p) Assume that we have 5 different levels of depth and we measured the water temperature:

Depth (x): 1, 2.5, 4, 6, 8
Temperature (y): 14.6, 13.5, 13.2, 12.0, 11.2

Is there any significant linear relation between the level of depth and temperature? Find the estimated regression line. Do the test whether β_1 is significantly non-zero. Consider $\alpha = 0.05$ and find the P-value.