EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Tuesday, January 15, 2013, morning, V.
Telefonvakt: Alexey Lindo, tel. 772 82 94
Aids: Chalmers approved calculator and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: Maximal points: 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18 points, 5: 24 points.
Motivations: All answers/solutions must be motivated.
Language: You may write your answers in either English or Swedish.

1. (2p)

   a) Provide the definition of the moment-generating function (mgf). Calculate the mgf of a normal random variable having mean μ and variance σ^2.

   b) Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ be two independent normal random variables. Find the mgf of the random variable $S = X + Y$.

2. (2p) The random variable X takes values in the non-negative integers ($\mathbb{Z}_0^+ = \{0, 1, 2, \dots\}$). Show that X has mean satisfying

$$\mathbb{E}(X) = \sum_{k=0}^{\infty} \mathbb{P}(X > k),$$

   whenever this series converges.

3. (3p) Let X_n, $n = 1, 2, \dots$ be a sequence of independent identically distributed random variables, $\mathbb{P}(X_n = 1) = 1 - \mathbb{P}(X_n = -1) = p$. Do the following sequences of random variables form a Markov chain:

   a) $Y_n = X_n \cdot X_{n+1}$;

   b) $Y_n = X_1 \cdot X_2 \cdot \ldots \cdot X_n$;

   c) $Y_n = f(X_n, X_{n+1})$, where $f(-1,-1) = 1$, $f(-1,1) = 2$, $f(1,-1) = 3$, $f(1,1) = 4$?

   If a sequence forms a Markov chain provide its transition matrix.

4. (3p) Let $X_1, \dots, X_n$ be a sample from the distribution $N(\theta, \theta^2)$, $\theta > 0$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Prove that $(1-\alpha)100\%$ confidence interval for the parameter θ has the form

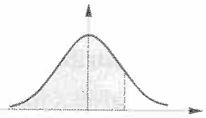
$$\left(\frac{\bar{X}}{1 + z_{\alpha/2}/\sqrt{n}}, \frac{\bar{X}}{1 - z_{\alpha/2}/\sqrt{n}}\right)$$

   where $\mathbb{P}(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1 - \alpha$ and Z is normal random variable with mean 0 and variance 1. Find a similar solution for the case $\theta < 0$.

Lycka till! Good luck!

Normal distribution

Distribution function



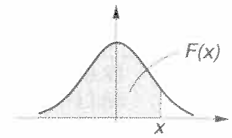
The table gives $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$. For $x < 0$ values of $\Phi(x)$ can be obtained from $\Phi(-x) = 1 - \Phi(x)$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9^303	0.9^306	0.9^310	0.9^313	0.9^316	0.9^318	0.9^321	0.9^324	0.9^326	0.9^329
3.2	0.9^331	0.9^334	0.9^336	0.9^338	0.9^340	0.9^342	0.9^344	0.9^346	0.9^348	0.9^350
3.3	0.9^352	0.9^353	0.9^355	0.9^357	0.9^358	0.9^360	0.9^361	0.9^362	0.9^364	0.9^365
3.4	0.9^366	0.9^368	0.9^369	0.9^370	0.9^371	0.9^372	0.9^373	0.9^374	0.9^375	0.9^376

For large values of x the following approximation can be used.

$$\frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \cdot \left(\frac{1}{x} - \frac{1}{x^3}\right) < 1 - \Phi(x) < \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \cdot \frac{1}{x}$$

$\Phi(4) = 0.9^468329$ $\quad$ $\Phi(5) = 0.9^67133$



The t-distribution

The table gives x for given values of the distribution function $F(x)$ for a t-distribution with r degrees of freedom. For $x < 0$ values of $F(x)$ can be obtained from $F(-x) = 1 - F(x)$.

$F(x)$	**0.75**	**0.90**	**0.95**	**0.975**	**0.990**	**0.995**	**0.9975**	**0.9995**
$r=$ 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	636.6
2	0.8165	1.886	2.920	4.303	6.965	9.925	14.09	31.60
3	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	12.92
4	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	8.610
5	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	6.869
6	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.959
7	0.7111	1.415	1.895	2.365	2.998	3.499	4.029	5.408
8	0.7064	1.397	1.860	2.306	2.896	3.355	3.832	5.041
9	0.7027	1.383	1.833	2.262	2.821	3.250	3.690	4.781
10	0.6998	1.372	1.812	2.228	2.764	3.169	3.581	4.587
11	0.6974	1.363	1.796	2.201	2.718	3.106	3.497	4.437
12	0.6955	1.356	1.782	2.179	2.681	3.055	3.428	4.318
13	0.6938	1.350	1.771	2.160	2.650	3.012	3.372	4.221
14	0.6924	1.345	1.761	2.145	2.624	2.977	3.326	4.140
15	0.6912	1.341	1.753	2.131	2.602	2.947	3.286	4.073
16	0.6901	1.337	1.746	2.120	2.583	2.921	3.252	4.015
17	0.6892	1.333	1.740	2.110	2.567	2.898	3.222	3.965
18	0.6884	1.330	1.734	2.101	2.552	2.878	3.197	3.922
19	0.6876	1.328	1.729	2.093	2.539	2.861	3.174	3.883
20	0.6870	1.325	1.725	2.086	2.528	2.845	3.153	3.850
21	0.6864	1.323	1.721	2.080	2.518	2.831	3.135	3.819
22	0.6858	1.321	1.717	2.074	2.508	2.819	3.119	3.792
23	0.6853	1.319	1.714	2.069	2.500	2.807	3.104	3.767
24	0.6848	1.318	1.711	2.064	2.492	2.797	3.090	3.745
25	0.6844	1.316	1.708	2.060	2.485	2.787	3.078	3.725
26	0.6840	1.315	1.706	2.056	2.479	2.779	3.069	3.707
27	0.6837	1.314	1.703	2.052	2.473	2.771	3.056	3.690
28	0.6834	1.313	1.701	2.048	2.467	2.763	3.047	3.674
29	0.6830	1.311	1.699	2.045	2.462	2.756	3.038	3.659
30	0.6828	1.310	1.697	2.042	2.457	2.750	3.030	3.646
34	0.6818	1.307	1.691	2.032	2.441	2.728	3.002	3.601
39	0.6808	1.304	1.685	2.023	2.426	2.708	2.976	3.559
44	0.6801	1.301	1.680	2.015	2.414	2.692	2.956	3.526
49	0.6795	1.299	1.677	2.010	2.405	2.680	2.940	3.501
59	0.6787	1.296	1.671	2.001	2.391	2.662	2.916	3.464
69	0.6781	1.294	1.667	1.995	2.382	2.649	2.900	3.438
79	0.6776	1.292	1.664	1.990	2.374	2.640	2.888	3.418
89	0.6773	1.291	1.662	1.987	2.369	2.632	2.879	3.404
99	0.6770	1.290	1.660	1.984	2.365	2.626	2.871	3.392
∞	0.6745	1.282	1.645	1.960	2.326	2.576	2.807	3.291