EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Wednesday 11 January 2012, kl. 14.00–16.00, V.
Jour: Fredrik Boulund, tel. 0737-706 629.
Aids: Chalmers approved calculater and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: Maximal points : 10. You must score at least 3 points on this exam. For the final grade your score here will be combined with score from the VLE re–exam on scale 3: 12 points, 4: 18 points, 5: 24 points.
Motivations: All answers/solutions must be motivated.
Language: There is a Swedish and English version of the questions. You may write your answers in either of these two languages.
IMPORTANT: There are two versions of the exam. One for current students and the other for students from previous years who choose to take the "old style" exam. This is the "new style" exam.

1. (1p) If $0 \leq \alpha \leq 1$ and $g(x)$, $h(x)$ are continuous probability density functions show that, $d(x) = \alpha g(x) + (1-\alpha)h(x)$ is also a continuous probability density function.

2. (3p) Let $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ be a vector of fixed numbers (weights). For a random sample $(X_1, X_2, \ldots, X_n)$ define the **weighted sum** by,

$$\bar{X}_w \quad = \quad \textstyle\sum_{i=1}^{n} w_i X_i.$$

   Answer the following by recalling that a random sample means that the points are independent and identically distributed.

   a) Find conditions on $\mathbf{w}$ that make $\bar{X}_w$ an unbiased estimator of the mean of the distribution from which the X_is are sampled. If these conditions hold then $\bar{X}_w$ is called a **weighted mean** or **weighted average**.

   b) (2p) Determine the variance of a weighted average (in terms of the weights $\mathbf{w}$) if each X_i has variance σ^2.

3. (3p) Consider a simple gambling game where you place a bet of $n$kr, flip a coin (with probability of heads p) and win $n$kr if it comes up heads and lose the $n$kr if it comes up tails. Consider now the following strategy for playing this game.

   - Start by betting 1kr.
   - Each time you lose, bet twice as much in the next coin flip.
   - As soon as you win, quit.

   a) Let K be the number of times you bet in a game. What is the distribution of K and derive the density function of K.

   b) Let X be the amount of money bet in the last round (when you win). What possible values can X take and why? Derive a formula for X's density function, $P(X = x)$.

   c) Calculate, using the definition of the expected value, the expected values of K and X depending on the value of p. What is strange about them? What problem do they reveal about this betting scheme?

4. (3p)

   a) Provide the definition of the Moment Generating Function of a random variable X and discuss how this is related to a generating function.

   b) Calculate the mgf of a geometric random variable with parameter p.

   c) Using the mgf calculate the mean and variance of a geometric random variable.

Lycka till! Good luck!