EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Tuesday 18 October 2011, morning, V.
Jour: Krzysztof Bartoszek, tel. 0700-771 093
Aids: Chalmers approved calculater and at most one (double–sided) A4 page of own notes.
Tables of appropriate statistical distributions are provided.
Grades: Maximal points : 10. You must score at least 3 points on this exam. For the final grade
your score here will be combined with scores from the VLE tests on scale 3: 12 points, 4: 18
points, 5: 24 points.
Motivations: All answers/solutions must be motivated.
Language: There is a Swedish and English version of the questions. You may write your
answers in either of these two languages.

1. (4p)

   a) Provide the definition of an unbiased estimator.

   b) Find an estimator for the mean value of a distribution and show whether it is or is not
   unbiased.

   c) Prove $Cov[X, Y] = E[XY] - E[X]E[Y]$ for random variables X and Y.

   d) Without doing any calculations what will be the covariance between two variables if we
   know that they are independent?

2. (2p)

   a) Provide the definition of the generating function for a sequence $a_0, a_1, a_2, \ldots$.

   b) Define a sequence as $a_n = 5a_{n-1}$, $a_0 = 2$, find its generating function. What do you
   need to assume about its domain and why?

3. (4p) Let $X_1, \ldots, X_n$ be an independent and identically distributed sample from a normal
   distribution with unknown mean μ and known variance σ^2.

   a) Derive a 98% confidence interval for the mean.

   b) Compute the 98% confidence interval for μ, given $n = 3$, $X_1 = 1$, $X_2 = 10$, $X_3 = 7$
   ($\bar{X} = (1 + 10 + 7)/3 = 6$, $1^2 + 10^2 + 7^2 = 150$) and $\sigma^2 = 25$. What is random about this
   interval? What is the exact meaning of it?

Lycka till! Good luck!