EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Tuesday 23 August 2011, kl. 08.30–12.30, V.
Jour: José Sánchez, tel. 0767-683 090
Aids: Chalmers approved calculater and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: 3: 12 points, 4: 18 points, 5: 24 points. Maximal points : 30.
Motivations: All answers/solutions must be motivated.
Language: There is a Swedish and English version of the questions. You may write your answers in either of these two languages.

1. (5p) Let A and B be two events, such that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cup B) = 0.58$.
   a) What is the definition of independence of two events?
   b) Are A and B independent and why if they are or are not?
   c) Calculate $P(A \cap B^C)$ and $P(B \setminus A)$.
   d) Give the defintion of conditional probability.
   e) Calculate $P(A|B)$ and $P(B|A)$.

2. (3p) Let X be the outcome of a throw of a six sided dice, *i.e.* X is a random variable taking values in the set $\{1, 2, 3, 4, 5, 6\}$. However the dice is not fair, we have the probability of observing an even number is twice as large as the probability of observing an odd number, the probability of observing any of the odd numbers is the same and also the probability of observing any even number is the same.
   a) Calculate the probabilities of observing the individual numbers.
   b) What is $P(X \in \{3, 6\})$?
   c) Let us make two independent throws of this dice and then sum the result. What is the chance of observing an outcome of at least 3?

3. (3p) Let X and Y be two random variables with respective means : μ_X, μ_Y, variances σ_X^2, σ_Y^2 and covariance σ_{XY}.
   a) Find the expected value of $33X + 12Y - 11$.
   b) Calculate the variance of $5X + 2Y + 4$.
   c) If in addition we know that X and Y are independent what can we say about σ_{XY}?

4. (4p) An interactive computer system is available at a large installation. Let X denote the number of requests for this system received per hour. Assume that X has a Poisson distribution with parameter λ. These data are obtained :

25	20	20
30	24	15
10	23	4,

   the moments of this data are $\overline{X} = 19$, $\overline{X^2} = 419$.
   a) (2p) Write the definition of an unbiased estimator and find an unbiased estimate for λ.
   b) Find an unbiased estimator for the average number of requests received per hour and calculate its value.
   c) Find an unbiased estimator for the average number of requests received per quarter hour and calculate its value.

5. (4p)

   a) Provide the definition of the Moment Generating Function of a random variable X.

   b) Calculate the mgf of a Poisson random variable with mean value λ.

   c) Calculate the mgf of an exponential random variable with mean $\frac{1}{\mu}$

   d) Let X be Poisson distributed with mean value λ, Y be an exponential random variable with mean $\frac{1}{\mu}$ and assume that X and Y are independent. Calculate the mgf of $X + Y$.

6. (3p) Let X denote the unit price of a 3.5–inch floppy diskette. These observations are obtained from a random sample of 10 suppliers:

\$3.83	3.54	3.44	3.89	3.65
3.70	3.59	3.37	4.04	3.93

   the moments of this data are $\overline{X} = 3.698$, $\overline{X^2} = 13.71902$.

   a) Find an unbiased estimator for the mean price of these diskettes and calculate its value.

   b) Find an unbiased estimator for the variance in the price of these diskettes and calculate its value.

   c) Find the sample standard deviation. Is this an unbiased estimate for the true standard deviation σ?

7. (4p) A random sample of 500 workers engaged in research and development (R&D) last year is selected. Of these, 178 earn over \$72 000 per year. Of the 450 workers in R&D studied during the current year, 220 earn in excess of \$72 000 per year.

   a) (2p) Let p_1 and p_2 denote the proportion of workers engaged in research and development who earned over \$72 000 per year last year and this year respectively. Find points estimates for p_1, p_2 and $p_1 - p_2$.

   b) Find a 95% confidence interval for $p_1 - p_2$ (we assume a person's salary this year is independent of the one from last year).

   c) Would you be surprised to hear someone claim that the proportion of R&D workers earning over \$72 000 was the same this year as it was last year. Explain using the confidence interval of part b).

8. (4p) A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage there is a probability p that the digit that enters this stage will be changed when it leaves and a probability $q = 1 - p$ that it won't.

   a) Form a Markov chain to represent the process of transmission by taking as states the digits 0 and 1.

   b) Write down the matrix of transition probabilities.

   c) If the digit 1 was put into the machine, write down the formula for the probability distribution after n stages.

   d) What is the probability that a 0 remains a 0 after two stages?

Lycka till! Good luck!