

Lösningar till tentamen i Matematisk statistik och diskret matematik D2 (MVE055/MSG810).

These are sketches of the solutions.

1. Lösning:

- a) $P(A \cap B) = P(A)P(B)$
- b) $P(A \cap B) = 0.12 = 0.3 \cdot 0.4 = P(A)P(B)$ they are independent
- c) $P(A \cap B^C) = 0.3 - 0.12 = 0.18$ and $P(B \setminus A) = 0.4 - 0.12 = 0.28$.
- d) $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- e) $P(A|B) = P(A) = 0.3$, $P(B|A) = P(B) = 0.4$ by independence

2. Lösning:

- a) $P(1) = P(3) = P(5) = \frac{1}{9}$, $P(2) = P(4) = P(6) = \frac{2}{9}$
- b) $\frac{1}{3}$
- c) $P(X_1 + X_2 \in \{3, 4, 5, \dots\}) = 1 - P(X_1 + X_2 \in \{2\}) = 1 - \frac{1}{81} = \frac{80}{81}$

3. Lösning:

- a) $33\mu_X + 12\mu_Y - 11$
- b) $25\sigma_X^2 + 4\sigma_Y^2 + 20\sigma_{XY}$
- c) $\sigma_{XY} = 0$

4. Lösning:

- a) $\mathbf{E}\hat{\theta} = \theta$, $\mathbf{E}X = \lambda$, $\hat{\lambda} = \bar{X} = 19$
- b) same as a), 19
- c) $19/4 = 4.75$

5. Lösning:

- a) $M_X(t) = \mathcal{E}[e^{tX}]$
- b) $e^{\lambda(e^t - 1)}$
- c) $\frac{\mu}{\mu - t}$
- d) $e^{\lambda(e^t - 1)} \cdot \frac{\mu}{\mu - t}$

6. Lösning:

- a) $\bar{X} = 3.698$
- b) $\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \mathbf{E}X)^2 = \frac{n \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2}{n(n-1)} = \frac{1}{n-1} (\sum_{i=1}^n X_i^2 - \frac{1}{n} (\sum_{i=1}^n X_i)^2) \approx 0.0487$
- c) $\hat{\sigma} = \sqrt{\hat{\sigma}^2} \approx 0.2206$, no it is biased

7. Lösning:

- a) 0.356, 0.489, -0.133
- b) -0.133 ± 0.062
- c) yes as 0 is not contained in the confidence interval

8. Lösung:

a) Draw graph

b)

$$P = \begin{bmatrix} q & p \\ p & q \end{bmatrix}$$

c) $P^n[01]^T$

d) $p \cdot p + q \cdot q$