EXAM: Matematisk statistik och diskret matematik D (MVE055/MSG810)
Time and place: Wednesday 12 January 2011, kl. 14.00–18.00, V.
Jour: Krzysztof Bartoszek, tel. 0700-771 093.
Aids: Chalmers approved calculater and at most one (double–sided) A4 page of own notes. Tables of appropriate statistical distributions are provided.
Grades: 3: 12 points, 4: 18 points, 5: 24 points. Maximal points : 30.
Motivations: All answers/solutions must be motivated.
Language: There is a Swedish and English version of the questions. You may write your answers in either of these two languages.

1. (5p) Let A and B be two disjoint events, with $P(A) = 0.3$ and $P(B) = 0.4$.
   a) Are A and B independent and why if they are or are not?
   b) Calculate $P(A \cup B)$ and $P(A \cap B^C)$.
   c) (2p) State and prove Bayes' theorem.
   d) Let $C \subset A$ such that $P(C) = 0.1$. What is $P(C|A)$ and $P(C|B)$?

2. (5p)
   a) State the Central Limit Theorem
   b) Discuss what the CLT can be useful for.
   c) (3p) You are a system administrator at a university and sampled the amount of disk spaces from 1000 employees. The summary information is : $\sum_i x_i = 10221.31$, $\sum_i x_i^2 = 125820.8$. One of your responsibilities is of course to make sure that employees have quotas large enough to be able to do their work. Estimate the average disk usage and calculate the confidence interval at a level you think is sensible. Why are you allowed to use the formula and why do you have to ask yourself this question? When a new employee arrives how much quota would you give him at the start and why?

3. (3p) Assume that $Y = \beta_0 + \beta_1 X$, where X is some random variable with mean μ_X and variance σ_X^2.
   a) Find the expected value of Y
   b) Calculate the variance of Y
   c) Calculate the covariance between X and Y

4. (4p) In order to be effective, reflective highway signs must be picked up by the automobile's headlights. To do so at long distances requires that the beams be on "high". A study conducted by highway engineers reveals that 45 of 50 randomly selected cars in a high–traffic–volume area have the headlights on low beam.
   a) Find a point estimate for p, the proportion of cars in this type area that use low beams.
   b) Find a 90% confidence interval for p
   c) (2p) How large a sample would have been required to estimate p to within 0.02 with 90% confidence if the engineers had not performed their study?

   The formula for a $1 - \alpha$ level confidence interval on p is $\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$ and the formula for sample size IF we have some prior knowledge of p is $n = \frac{z_{\alpha/2}^2 \hat{p}(1-\hat{p})}{d^2}$.

5. (4p) The joint density for the pair of discrete random variables (X, Y) is given by $f_{XY}(x, y) = 2/(n(n+1))$ where $1 \leq y \leq x \leq n$ and $n > 0$ is a positive integer
   a) Verify that $f_{XY}(x, y)$ satisfies the conditions necessary to be a density.

b) Find the marginal densities for X and Y.

c) Are X and Y independent?

d) What does the knowledge of the independence between X and Y tell us about the value of covariance between them?

6. (2p)

a) Give the definition of the generating function for a sequence $\{a_n\}_{n=0}^{\infty}$. Calculate the generating function of the sequence $a_n = 2^n + 3^{n-1}$

b) Express $\frac{x^2}{(x-1)(x+3)(x-5)}$ as a sum of partial fractions.

7. (4p)

a) State the Law of Large Numbers and discuss why it is an important result.

b) We have two coins : one is a fair coin and the other is a coin that produces heads with probability 0.75. One of the two coins is picked at random and this coin is tossed n times. Let S_n be the number of heads that turns up in these n tosses. Does the Law of Large Numbers allow us to predict the proportion of heads that will turn up in the long run? After we have observed a large number of tosses, can we tell which coin was chosen?

8. (3p) State and prove Markov's inequality.

Lycka till! Good luck!